Analytical solutions and rogue waves in (3+1)-dimensional nonlinear Schrödinger equation*

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Analytical solutions in terms of rational-like functions are presented for a (3+1)-dimensional nonlinear Schrödinger equation with time-varying coefficients and a harmonica potential using the similarity transformation and a direct ansatz. Several free functions of time t are involved to generate abundant wave structures. Three types of elementary functions are chosen to exhibit the corresponding nonlinear rogue wave propagations.

Keywords: nonlinear Schrödinger equation, similarity transformation, rational-like solution, rogue wave

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1. Introduction

Rogue waves, also known as freak waves, monster waves, extreme waves, and hundred year waves, are relatively large and spontaneous ocean surface waves, which are a threat even to large ships and ocean liners.[1−4] The theoretical study of rogue waves, one of the essential topics in nonlinear science, has received more and more attention in recent years. The Darboux transformation,[5−6] the similarity transformation, and the numerical simulation[7−13] are used. For describing the natural nonlinear phenomenon, the nonlinear Schrödinger (NLS) equation is a fundamental model, which is widely used in optics of nonlinear media,[14−16] photonics,[17] plasmas,[18] mean-field theory of Bose–Einstein condensates,[19] condensed matter physics,[20] and many other fields. However, describing rogue waves with a higher-dimensional NLS equation has seldom been studied so far. In this paper, the rational-like solutions of a (3+1)-dimensional NLS equation with time-varying coefficients and a harmonica potential are derived through the similarity transformation,[6,8,9] and several rogue structures are exhibited to describe these wave propagations through the obtained solutions by choosing three types of elementary functions.

2. Explicit solutions through similarity transformation

To illustrate the above idea, we focus on the (3+1)-dimensional nonautonomous NLS equation with varying coefficients and a harmonica potential

$$\mathrm{i}\partial_t\Psi + \frac{\beta_1(t)}{2}\boldsymbol{\nabla}_\perp\Psi + \frac{\beta_2(t)}{2}\partial_z^2\Psi + \gamma(t)|\Psi|^2\Psi - \mathrm{i}g(t)\Psi + \frac{\omega(t)}{2}r^2\Psi = 0, \tag{1}$$

where $\Psi \equiv \Psi(x,y,z,t)$ is the amplitude of a beam propagating in an inhomogenous nonlinear medium, $\boldsymbol{\nabla}_\perp = \partial_x^2 + \partial_y^2$ is the transverse Laplacian operator, and $r^2 = x^2 + y^2 + z^2$. The dimensionless coefficients $\beta_{1,2}$, γ, g, and ω are all functions of time t, $\beta_1(t)$ and $\beta_2(t)$ are the dispersion coefficients, $\gamma(t)$ is the measure of the Kerr nonlinearity, $g(t)$ is the distributed gain (loss) coefficient, and $\omega(t)r^2/2$ is a harmonica potential. The notation ∂ means the partial derivative with respect to the given variable.[21−24]

We first construct the following transformation for the envelope field Ψ:

$$\Psi = (\Psi_\mathrm{R} + \mathrm{i}\Psi_\mathrm{I})\,\mathrm{e}^{\mathrm{i}\varphi}, \tag{2}$$

where $\Psi_\mathrm{R} \equiv \Psi_\mathrm{R}(x,y,z,t), \Psi_\mathrm{I} \equiv \Psi_\mathrm{I}(x,y,z,t)$, and $\varphi \equiv \varphi(x,y,z,t)$ are real functions.[9,25−27]

Substituting Eq. (2) into Eq. (1) yields the following coupled partial differential equations with variable

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coefficients:

$$
\begin{aligned}
&-2\Psi_{\mathrm{I},t}-2\varphi_t\Psi_{\mathrm{R}}+\beta_1(t)[\Psi_{\mathrm{R},xx}+\Psi_{\mathrm{R},yy}\\
&\quad-(\varphi_{xx}+\varphi_{yy})\Psi_{\mathrm{I}}-(\varphi_x^2+\varphi_y^2)\Psi_{\mathrm{R}}\\
&\quad-2(\varphi_x\Psi_{\mathrm{I},x}+\varphi_y\Psi_{\mathrm{I},y})]\\
&\quad+\beta_2(t)(\Psi_{\mathrm{R},zz}-\Psi_{\mathrm{R}}\varphi_z^2-2\Psi_{\mathrm{I},z}\varphi_z-\Psi_{\mathrm{I}}\varphi_{zz})\\
&\quad+2\gamma(t)(\Psi_{\mathrm{R}}^2+\Psi_{\mathrm{I}}^2)\Psi_{\mathrm{R}}+2g(t)\Psi_{\mathrm{I}}\\
&\quad+(x^2+y^2+z^2)\omega(t)\Psi_{\mathrm{R}}=0, \qquad (3)\\
&2\Psi_{\mathrm{R},t}-2\varphi_t\Psi_{\mathrm{I}}+\beta_1(t)[\Psi_{\mathrm{I},xx}+\Psi_{\mathrm{I},yy}\\
&\quad+(\varphi_{xx}+\varphi_{yy})\Psi_{\mathrm{R}}-(\varphi_x^2+\varphi_y^2)\Psi_{\mathrm{I}}\\
&\quad+2(\varphi_x\Psi_{\mathrm{R},x}+\varphi_y\Psi_{\mathrm{R},y})]\\
&\quad+\beta_2(t)(\Psi_{\mathrm{I},zz}-\Psi_{\mathrm{I}}\varphi_z^2+2\Psi_{\mathrm{R},z}\varphi_z+\Psi_{\mathrm{R}}\varphi_{zz})\\
&\quad+2\gamma(t)(\Psi_{\mathrm{R}}^2+\Psi_{\mathrm{I}}^2)\Psi_{\mathrm{I}}-2g(t)\Psi_{\mathrm{R}}\\
&\quad+(x^2+y^2+z^2)\omega(t)\Psi_{\mathrm{I}}=0. \qquad (4)
\end{aligned}
$$

For real functions $\Psi_{\mathrm{R}},\Psi_{\mathrm{I}}$ and phase φ, applying the following similarity transformation:

$$
\begin{aligned}
&\Psi_{\mathrm{R}}=A+BP(\eta,\tau),\\
&\Psi_{\mathrm{I}}=C+DQ(\eta,\tau),\\
&\varphi=\chi+\mu\tau, \qquad (5)
\end{aligned}
$$

to systems (3) and (4), where the new variables are $A\equiv A(t)$, $B\equiv B(t)$, $C\equiv C(t)$, $D\equiv D(t)$, $\tau\equiv\tau(t)$, $\eta\equiv\eta(x,y,z,t)$, $\chi\equiv\chi(x,y,z,t)$, and μ is a constant, we obtain the following equations:

$$
\begin{aligned}
&\Delta_1BP_\eta-2D\Delta_2Q_\eta+(A+BP)\Delta_3-\{2D_t\\
&\quad+[\beta_1(t)(\chi_{xx}+\chi_{yy})+\beta_2(t)\chi_{zz}-2g(t)\\
&\quad-4\gamma(t)(A+BP)C]D\}Q\\
&\quad-\{2C_t+[\beta_1(t)(\chi_{xx}+\chi_{yy})+\beta_2(t)\chi_{zz}-2g(t)\\
&\quad-2\gamma(t)AC]C\}-2[\mu(A+BP)+DQ_\tau]\tau_t\\
&\quad+[\beta_1(t)(\eta_x^2+\eta_y^2)+\beta_2(t)\eta_z^2]BP_{\eta\eta}\\
&\quad+2\gamma(t)[(A+BP)^3+(A+BP)D^2Q^2\\
&\quad+C^2BP]=0, \qquad (6)\\
&\Delta_1DQ_\eta+2B\Delta_2P_\eta+(C+DQ)\Delta_3+\{2B_t\\
&\quad+[\beta_1(t)(\chi_{xx}+\chi_{yy})+\beta_2(t)\chi_{zz}-2g(t)\\
&\quad+4\gamma(t)(C+DQ)A]B\}P\\
&\quad+\{2A_t+[\beta_1(t)(\chi_{xx}+\chi_{yy})+\beta_2(t)\chi_{zz}\\
&\quad-2g(t)]A\}+2[BP_\tau-\mu(C+DQ)]\tau_t\\
&\quad+[\beta_1(t)(\eta_x^2+\eta_y^2)+\beta_2(t)\eta_z^2]DQ_{\eta\eta}+2\gamma(t)\\
&\quad\times(C+DQ)^2[(C+DQ)^2+A^2+B^2P^2]=0, \qquad (7)
\end{aligned}
$$

where Δ_i $(i=1,2,3)$ are given by

$$
\begin{aligned}
&\Delta_1=\beta_1(t)(\eta_{xx}+\eta_{yy})+\beta_2(t)\eta_{zz}, \qquad (8)\\
&\Delta_2=\eta_t+\beta_1(t)(\chi_x\eta_x+\chi_y\eta_y)+\beta_2(t)\chi_z\eta_z, \qquad (9)\\
&\Delta_3=\omega(t)(x^2+y^2+z^2)-2\chi_t\\
&\quad-\beta_1(t)(\chi_x^2+\chi_y^2)-\beta_2(t)\chi_z^2. \qquad (10)
\end{aligned}
$$

From the reduction equations $\Delta_i=0$ and $2\sigma_t+[\beta_1(t)(\chi_{xx}+\chi_{yy})+\beta_2(t)\chi_{zz}-2g(t)]\sigma=0$ $(\sigma=A,B,C,D)$, we obtain

$$
\begin{aligned}
\eta&=\delta_1(t)x+\delta_2(t)y+\delta_3(t)z+\delta_0(t),\\
\omega(t)&=\frac{2\chi_t+\beta_1(t)(\chi_x^2+\chi_y^2)+\beta_2(t)\chi_z^2}{x^2+y^2+z^2}, \qquad (11)\\
\chi&=-\frac{1}{\beta_1(t)}\Bigg(\frac{\delta_1(t)\delta_1(t)_t-\delta_2(t)\delta_2(t)_t}{2\delta_1(t)^2}x^2\\
&\quad+\frac{\delta_2(t)_ty+\delta_0(t)}{\delta_1(t)}x\Bigg)\\
&\quad-\frac{\delta_3(t)_t}{2\beta_2(t)\delta_3(t)}z^2+d_0(t), \qquad (12)\\
A&=a_0\exp\Bigg[\frac{1}{2}\int_0^t\Bigg[\frac{\delta_1(s)_s}{\delta_1(s)}-\frac{\delta_2(s)\delta_2(s)_s}{\delta_1(s)^2}\\
&\quad+\frac{\delta_3(s)_s}{\delta_3(s)}+g(s)\Bigg]\mathrm{d}s\Bigg],\\
B&=bA,\quad D=dA, \qquad (13)
\end{aligned}
$$

where a_0, b, and d are arbitrary constants, $\delta_0(t)$, $\delta_1(t)$, $\delta_2(t)$, $\delta_3(t)$, and $d_0(t)$ are functions of time t, and $C=0$. When variable τ and nonlinearity $g(t)$ are given by

$$
\begin{aligned}
\tau&=\frac{1}{2}\int_0^t[\beta_1(s)(\delta_1^2(s)+\delta_2^2(s))+\beta_2(s)\delta_3^2(s)]\mathrm{d}s,\\
\gamma(t)&=\frac{\beta_1(t)(\delta_1^2(t)+\delta_2^2(t))+\beta_2(t)\delta_3^2(t)}{2A^2}, \qquad (14)
\end{aligned}
$$

the coupled system of the constant coefficients from Eqs. (6) and (7) is reduced to

$$
\begin{aligned}
&(d^2Q^2+b^2P^2+2bP+1-\mu)dQ\\
&\quad+bP_\tau+dQ_{\eta\eta}=0, \qquad (15)\\
&(1+bP)[d^2Q^2+(1+bP)^2-\mu]\\
&\quad-dQ_\tau+bP_{\eta\eta}=0. \qquad (16)
\end{aligned}
$$

Therefore, according to the direct method developed in Refs. [9] and [25]–[27] $(\mu=1)$, the first-order rational solution of Eqs. (15) and (16) is

$$
P=-\frac{4}{R_1(\eta,\tau)b},\quad Q=-\frac{8\tau}{R_1(\eta,\tau)d}, \qquad (17)
$$

where $R_1(\eta,\tau)=1+2\eta^2+4\tau^2$, and the second-order rational solution is

$$
P=\frac{P_1(\eta,\tau)}{R_2(\eta,\tau)b},\quad Q=-\frac{Q_1(\eta,\tau)\tau}{R_2(\eta,\tau)d}, \qquad (18)
$$

where $P_1(\eta,\tau)=\frac{3}{8}-9\tau^2-\frac{3}{2}\eta^2-6\eta^2\tau^2-10\tau^4-\frac{1}{2}\eta^4$, $Q_1(\eta,\tau)=-\frac{15}{4}+2\tau^2-3\eta^2+4\eta^2\tau^2+4\tau^4+\eta^4$, and

$R_2(\eta,\tau) = \frac{3}{32} + \frac{33}{8}\tau^2 + \frac{9}{16}\eta^2 - \frac{3}{2}\eta^2\tau^2 + \frac{9}{2}\tau^4 + \frac{1}{8}\eta^4 + \frac{2}{3}\tau^6 + \eta^2\tau^4 + \frac{1}{2}\eta^4\tau^2 + \frac{1}{12}\eta^6$.

Finally, a direct reduction solution of Eq. (1) can be derived as

$$\Psi = A(1 + bP + \mathrm{i}dQ)\,\mathrm{e}^{\mathrm{i}(\chi+\tau)}, \tag{19}$$

the known functions $A \equiv A(t)$, $P \equiv P(\eta,\tau)$, $Q \equiv Q(\eta,\tau)$, $\chi \equiv \chi(x,y,z,t)$, and $\tau \equiv \tau(t)$ are expressed by Eqs. (13), (17), (18), (12), and (14), respectively.

3. Rogue waves in solutions

Now, we focus on rogue structures in the solutions of the (3+1)-dimensional nonautonomous NLS equation (1).

According to Eq. (19), the first-order rational-like solution of Eq. (1) can be rewritten as

$$\begin{aligned}\Psi = {} & a_0\left(\frac{-3 + 2\eta^2 + 4\tau^2 - 8\mathrm{i}\tau}{1 + 2\eta^2 + 4\tau^2}\right) \\ & \times \exp\left[\frac{1}{2}\int_0^t \left(\frac{\delta_1(s)_s}{\delta_1(s)} - \frac{\delta_2(s)\delta_2(s)_s}{\delta_1(s)^2} \right.\right. \\ & \left.\left. + \frac{\delta_3(s)_s}{\delta_3(s)} + g(s)\right)\mathrm{d}s\right]\mathrm{e}^{\mathrm{i}(\chi+\tau)},\end{aligned} \tag{20}$$

where variables η, τ, and χ are expressed by Eqs. (11), (14), and (12), respectively. As the time functions $\delta_0(t)$, $\delta_1(t)$, $\delta_2(t)$, $\delta_3(t)$, $d_0(t)$, $g(t)$, $\beta_1(t)$, and $\beta_2(t)$ are arbitrary, the time-modulated excitations of the intensity

$$\begin{aligned}U \equiv |\Psi|^2 = {} & \frac{a_0^2\{[-3 + 2(\delta_1(t)x + \delta_2(t)y + \delta_3(t)z + \delta_0(t))^2 + 4\tau^2]^2 + 64\tau^2\}}{[1 + 2(\delta_1(t)x + \delta_2(t)y + \delta_3(t)z + \delta_0(t))^2 + 4\tau^2]^2} \\ & \times \exp\left[\int_0^t \left(\frac{\delta_1(s)_s}{\delta_1(s)} - \frac{\delta_2(s)\delta_2(s)_s}{\delta_1(s)^2} + \frac{\delta_3(s)_s}{\delta_3(s)} + g(s)\right)\mathrm{d}s\right],\end{aligned} \tag{21}$$

can be derived, where $\tau = \int_0^t [\beta_1(s)(\delta_1^2(s) + \delta_2^2(s)) + \beta_2(s)\delta_3^2(s)]\mathrm{d}s/2$.

At the same time, the second-order rational-like solution of Eq. (1) can be rewritten as

$$\begin{aligned}\Psi = {} & a_0\left(1 + \frac{P_1(\eta,\tau)}{R_2(\eta,\tau)} - \frac{Q_1(\eta,\tau)\tau\mathrm{i}}{R_2(\eta,\tau)}\right) \\ & \times \exp\left[\frac{1}{2}\int_0^t \left(\frac{\delta_1(s)_s}{\delta_1(s)} - \frac{\delta_2(s)\delta_2(s)_s}{\delta_1(s)^2} \right.\right. \\ & \left.\left. + \frac{\delta_3(s)_s}{\delta_3(s)} + g(s)\right)\mathrm{d}s\right]\mathrm{e}^{\mathrm{i}(\chi+\tau)},\end{aligned} \tag{22}$$

its intensity is

$$\begin{aligned}U \equiv {} & |\Psi|^2 \\ = {} & a_0^2\frac{(P_1(\eta,\tau) + R_2(\eta,\tau))^2 + (Q_1(\eta,\tau)\tau)^2}{(R_2(\eta,\tau))^2} \\ & \times \exp\left[\int_0^t \left(\frac{\delta_1(s)_s}{\delta_1(s)} - \frac{\delta_2(s)\delta_2(s)_s}{\delta_1(s)^2} \right.\right. \\ & \left.\left. + \frac{\delta_3(s)_s}{\delta_3(s)} + g(s)\right)\mathrm{d}s\right],\end{aligned} \tag{23}$$

where $P_1(\eta,\tau) = \frac{3}{8} - 9\tau^2 - \frac{3}{2}\eta^2 - 6\eta^2\tau^2 - 10\tau^4 - \frac{1}{2}\eta^4$, $Q_1(\eta,\tau) = -\frac{15}{4} + 2\tau^2 - 3\eta^2 + 4\eta^2\tau^2 + 4\tau^4 + \eta^4$, $R_2(\eta,\tau) = \frac{3}{32} + \frac{33}{8}\tau^2 + \frac{9}{16}\eta^2 - \frac{3}{2}\eta^2\tau^2 + \frac{9}{2}\tau^4 + \frac{1}{8}\eta^4 + \frac{2}{3}\tau^6 + \eta^2\tau^4 + \frac{1}{2}\eta^4\tau^2 + \frac{1}{12}\eta^6$, variables η, τ, and χ are expressed by Eqs. (11), (14), and (12), respectively, and the time functions $\delta_0(t)$, $\delta_1(t)$, $\delta_2(t)$, $d_0(t)$, $\gamma(t)$, and $\alpha(t)$ are arbitrary.

It can be seen that the solution structure (20) is different from the solution of the generalized (3+1)D Gross–Pitaevskii (GP) equation (20) or (21),[28] whereby applying a novel similarity transformation, the (3+1)D GP equation is reduced to a (3+1)D standard NLS equation, and the solution of the GP equation is thus constructed via that of the NLS equation. For simplification, we fix the parameters $a_0 = \delta_1(t) = \delta_2(t) = \delta_3(t) = 1$. When $\delta_0(t) = \mathrm{e}^{1+0.1\sin(t)}, \beta_1(t) = \beta_2(t) = \cos^2(0.02t)$, and $g(t) = \sin^3(0.005t)$, a rogue wave with the dromion structure in the symmetric time interval $[-2, 2]$ shows its propagation along the L axis (Figs. 1(a) and 1(b)),[10] where $L \equiv x\delta_1(t) + y\delta_2(t) + z\delta_3(t)$. When $\delta_0(t) = 1 + \mathrm{e}^{1+\sin(t)}, \beta_1(t) = 0.1\tanh(\sin(t)), \beta_2(t) = \tanh(t)$, and $g(t) = 0.1\mathrm{sech}(t)$, a chirping rogue structure with a snaking behavior in the symmetric time interval $[-8, 8]$ shows oscillatory variation during its propagation along the L axis (Figs. 2(a) and 2(b)).[28]

Figures 1(c), 1(d), and 2(c), 2(d) show that (i) although the same functions as those in Figs. 1(a), 1(b), and 2(a), 2(b) are taken, respectively, the excitations have double structures compared with the former single ones; (ii) the second-order rational-like solution has an even higher amplitude, just as the rogue waves in the deep ocean and the high intensity rogue light wave pulses in optical fibers.[6]

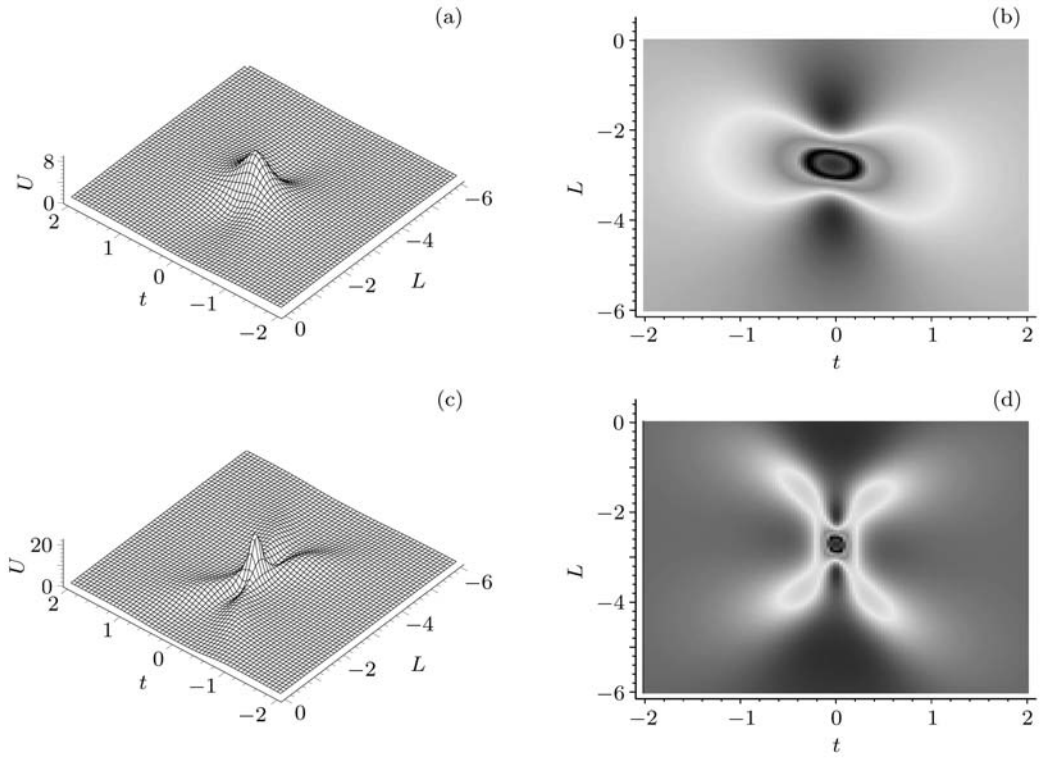


Fig. 1. Rogue wave propagations with dromion structures ((a) and (c)), and contour plots ((b) and (d)) for intensity U given in Eqs. (21) and (23).

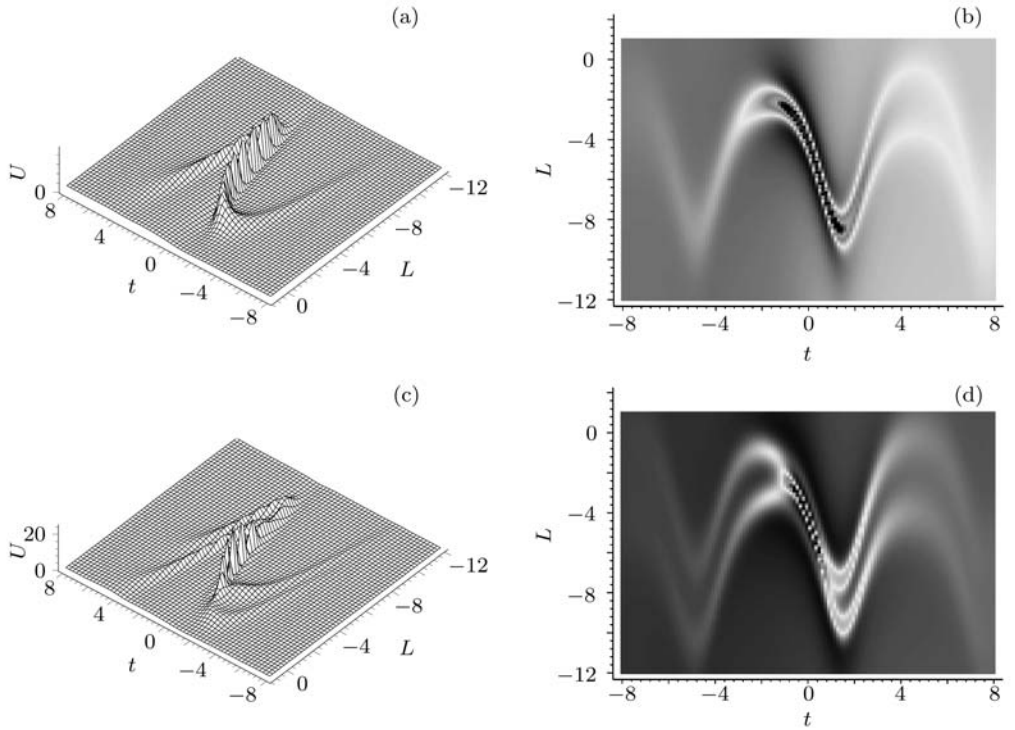


Fig. 2. Rogue wave propagations with chirped structures ((a) and (c)), and contour plots ((b) and (d)) for intensity U given in Eqs. (21) and (23).

Now, we focus on the periodic propagating wave pattern in terms of some elementary functions for Eq. (21). Considering solution (20), for convenience, we restrict our attention to some basic cases, in which the quantities $\delta_0(t)$, $\beta_1(t)$, $\beta_2(t)$, and $g(t)$ can be evaluated in simple closed forms and the complicated integrals are not involved. The triangle, the hyperbolic, and the exponential functions satisfy these re-

quirements, and can lead to new rogue wave patterns for the (3+1)-dimensional nonautonomous NLS equation (1). Figures 3(a) and 3(b) show periodic wave propagation with the chirped structure and the corresponding sectional view for intensity U of the first-order rational-like solution (20), respectively, where $\delta_0(t) = 1 + \mathrm{e}^{1+\sin(t)}, \beta_1(t) = \cos(t), \beta_2(t) = \sin(t)$, and $g(t) = 0.1\sin(t)$. Figures 3(c) and 3(d) show disordered wave propagation with the chirped structure and the corresponding sectional view for intensity U when the dispersion coefficients are $\beta_1(t) = \cos(t)$ and $\beta_2(t) = \sin^2(t)$. Figures 3(e) and 3(f) show single rogue wave propagation with the chirped structure and the corresponding sectional view when the dispersion coefficients are substituted by $\beta_1(t) = \cos^2(t)$ and $\beta_2(t) = \sin(t)$.

Several other circumstances of rogue wave propagation are presented in Fig. 4. When the coefficients are taken as $\delta_0(t) = \sin^2(t)$, $\beta_1(t) = \beta_2(t) = 0.5\tanh(t)$, and $g(t) = 0.1\sin(t)$, the periodic chirping structure for intensity U of the first-order rational-like solution (20) as a function of propagation distance L is constructed (Figs. 4(a) and 4(b)). Figures 4(c) and 4(d) show coherent chirping wave propagation and the corresponding sectional view for intensity U given in Eq. (21) when the dispersion coefficients

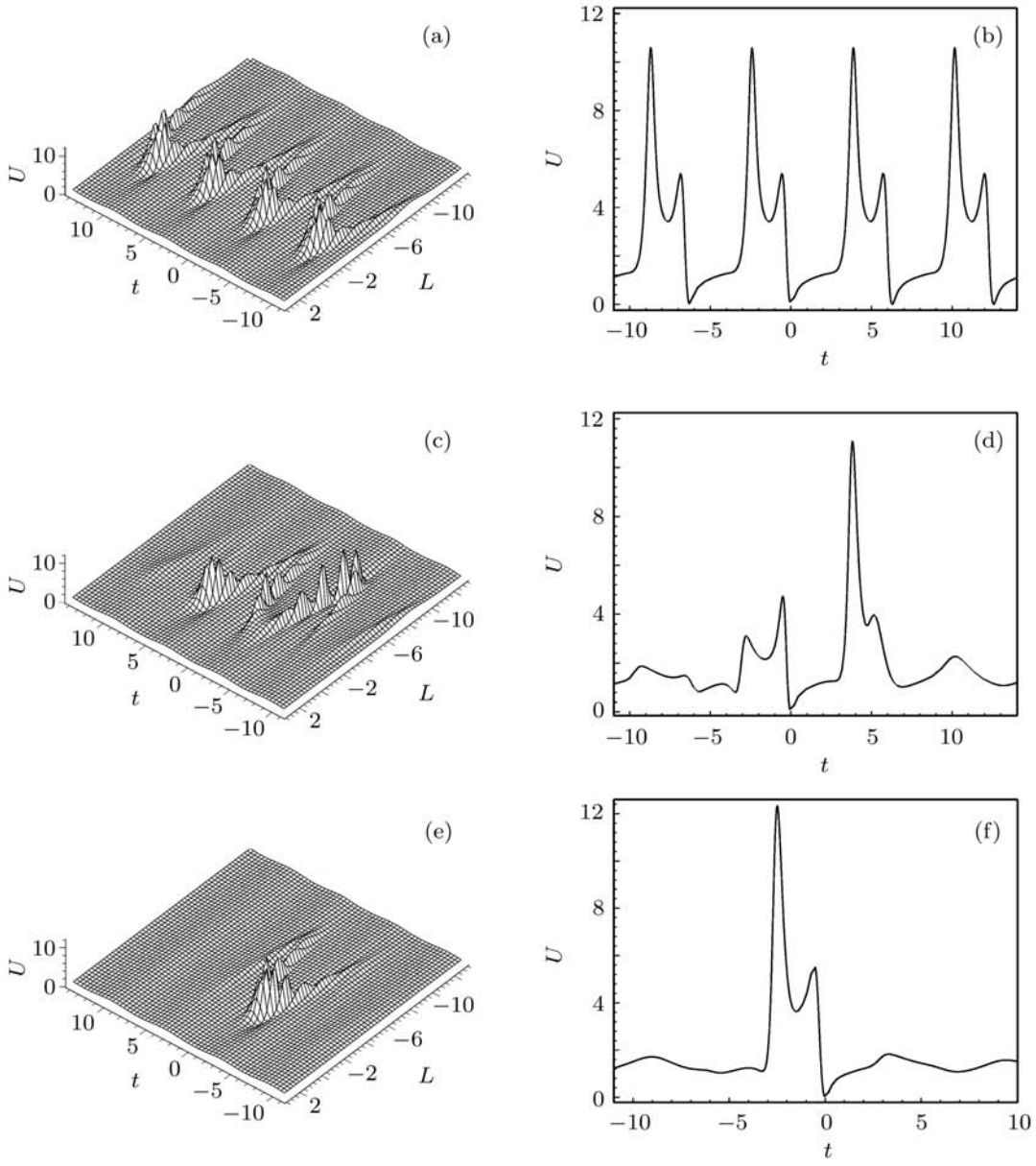


Fig. 3. Chirped structures and corresponding sectional views at $L = -2.5$ for intensity U given in Eq. (21). Panels (a) and (b) show periodic rogue wave propagation, panels (c) and (d) show disordered rogue wave propagation, and panels (e) and (f) show single rogue wave propagation.

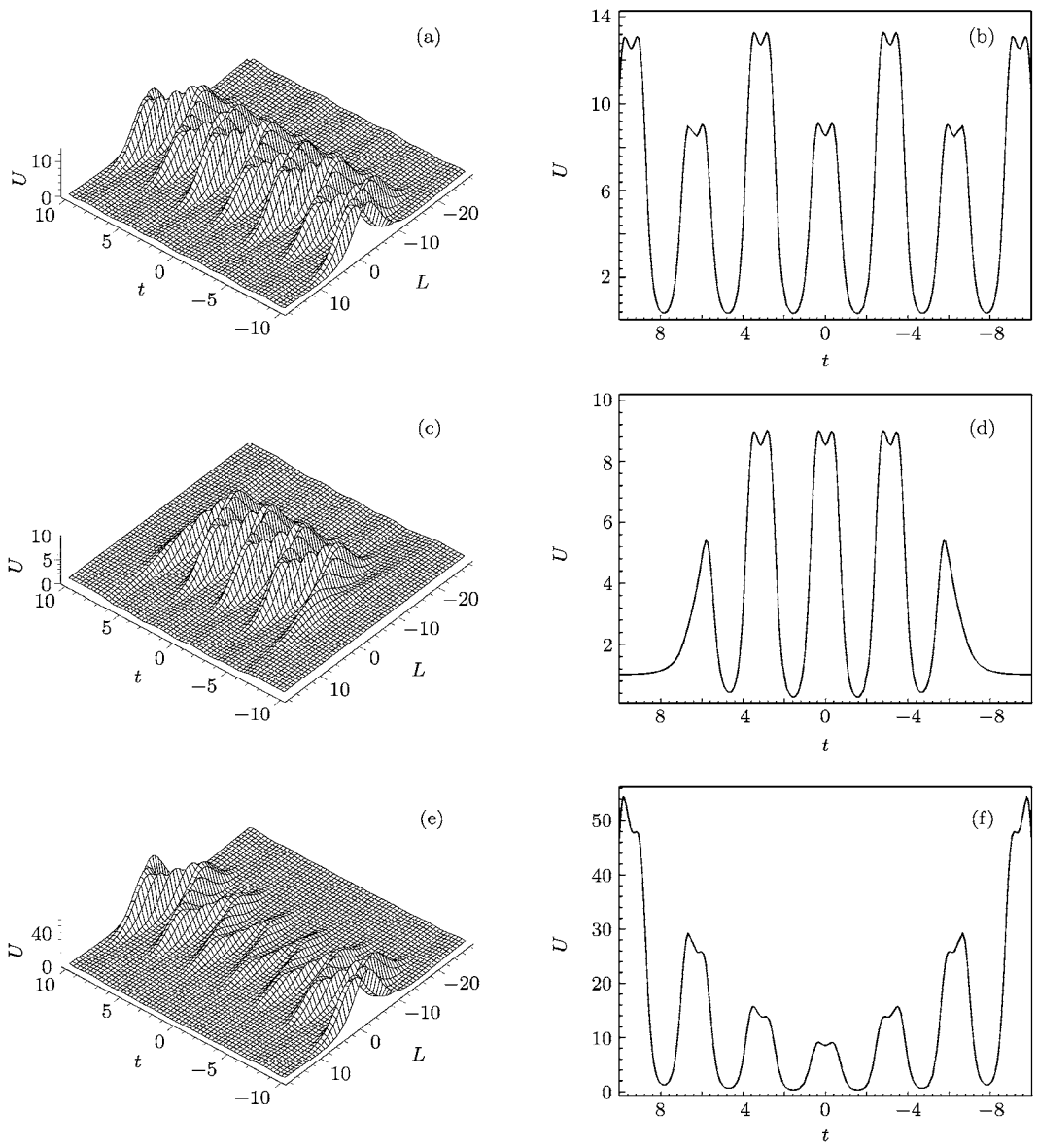


Fig. 4. Chirped structures and corresponding sectional views at $L = -1$ for intensity U given in Eq. (21). Panels (a) and (b) show periodic rogue wave propagation, panels (c) and (d) show coherent rogue wave propagation, and panels (e) and (f) show dispersive rogue wave propagation.

and the distributed gain (loss) coefficient are selected as $\beta_1(t) = 0.5\tanh(t), \beta_2(t) = 0.5\sinh(t)$, and $g(t) = 0.001\tanh(t)$. Figures 4(e) and 4(f) show dispersive rogue wave propagation with a chirped structure and the corresponding sectional view respectively when the dispersion coefficients and the distributed gain (loss) coefficient are substituted by $\beta_1(t) = \beta_2(t) = 0.5\tanh(t)$ and $g(t) = 0.1\tanh(t)$. Figure 4 displays the pattern behavior in the symmetric time interval $[-10, 10]$. The chirping amplitude changes in three different ways as time t increases from -10 to 10. Obviously, all these structures are different from those in our previous work.[29−35]

4. Conclusion

We obtain analytical solutions in terms of rational-like functions for the (3+1)-dimensional nonlinear Schrödinger equation with time-varying coefficients using a similarity transformation and a direct ansatz. The obtained solutions contain several free functions of time t, which provide us with more choices of functions to generate abundant wave structures. Here, we chose three types of elementary functions to exhibit rogue wave propagations in different ways. The solutions may be helpful for the study of nonlinear science.

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