Synchronization of spatiotemporal chaos in complex networks via backstepping*

Chai Yuan(柴 元)[a], Lü Ling(吕 翎)[b], and Chen Li-Qun(陈立群)[a,c,†]

[a] *Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai* 200072, *China*
[b] *College of Physics and Electronic Technology, Liaoning Normal University, Dalian* 116029, *China*
[c] *Department of Mechanics, Shanghai University, Shanghai* 200444, *China*

(Received 12 July 2011; revised manuscript received 9 October 2011)

A backstepping approach is proposed for the synchronization of chain networks of multi-spatiotemporal chaotic systems with topologically equivalent structures. The synchronization of multi-spatiotemporal chaotic systems is implemented by adding the control only to a terminal node, and the controller is designed via a corresponding update law. The control law is applied to spatiotemporal Gray–Scott systems. Numerical results demonstrate the effectiveness and the feasibility of the proposed approach.

Keywords: spatiotemporal chaos, backstepping design, Lyapunov function, chain complex networks

PACS: 05.45.Xt, 05.45.Pq **DOI:** 10.1088/1674-1056/21/3/030506

1. Introduction

Since the pioneering work of Ott *et al.*[1] and Pecora and Carroll (PC),[2] who proposed a successful method to synchronize two identical chaotic systems in the early 1990s, chaos synchronization has aroused considerable interest in many areas owing to its great potential for applications. Various types of approaches have been revealed to investigate chaos synchronization, such as adaptive control,[3] impulsive control,[4] sliding mode control,[5] open-plus-closed-loop control,[6,7] and backstepping control. In particular, the backstepping-design technique has become one of the most popular methods in chaos control. In the backstepping control of two systems, Mascolo and Grassi[8] controlled chaos via the backstepping design. Njah *et al.*[9] proposed the generalized control and synchronization of chaos in the RCL-shunted Josephson junction using the backstepping design. Laoye *et al.*[10] realized the chaos control of four-dimensional (4D) chaotic systems using a recursive backstepping nonlinear controller. Sharma and Kar[11] studied the contraction theory based adaptive synchronization of chaotic systems using the backstepping design. Pishkenari *et al.*[12] discussed the robust adaptive backstepping control of the uncertain Lorenz system. Yu and Li[13] introduced the adaptive hybrid projective synchronization of uncertain chaotic systems based on the backstepping design.

The small-world model was proposed by Watts and Strogatz[14] in 1998. Then, Newman and Watts[15] (1999) modified it to generate another variant of the small-world model. The scale-free network was introduced by Barabási and Albert[16] in 1999. Since then, the synchronization of complex networks has attracted much attention among scientists.[17−23] In the backstepping control of complex networks, Chen et al.[24] discussed backstepping control for periodically time-varying systems using the high-order neural network and the Fourier series expansion. Lin *et al.*[25] introduced the dynamic fuzzy neural network model and the adaptive backstepping tracking control of uncertain chaotic systems.

Time-varying chaotic systems are selected as network nodes for demonstration in most papers, though a large number of natural systems are spatiotempo-

*Project supported by the National Outstanding Young Scientists Foundation of China (Grant No. 10725209), the National Natural Science Foundation of China (Grant Nos. 90816001 and 10902064), the Shanghai Subject Chief Scientist Project, China (Grant No. 09XD1401700), the Shanghai Leading Talent Program and the Shanghai Leading Academic Discipline Project, China (Grant No. S30106), the Program for Changjiang Scholars and Innovative Research Team in University, China (Grant No. IRT0844), the Natural Science Foundation of Liaoning Province, China (Grant No. 20082147), and the Innovative Team Program of Liaoning Educational Committee, China (Grant No. 2008T108).

†Corresponding author. E-mail: lqchen@staff.shu.edu.cn

© 2012 Chinese Physical Society and IOP Publishing Ltd

ral chaos systems. Spatiotemporal chaos synchronization has potential applications in many fields, such as secure communication, physics, auto control, fluid, chemistry, and biological systems. Therefore, the synchronization of spatiotemporal chaos has attracted much attention among scientists.[26−28] However, the backstepping synchronization of complex networks with spatiotemporal chaos is still rarely reported. Research into spatiotemporal chaos synchronization via backstepping design has a vast unknown space to explore.

Many colossal complex networks, such as railway networks, the world wide web (WWW), the food web, communication networks, and biological neural networks, each are composed of a chain structure. Therefore, the research of the synchronization of a chain network is valuable. However, it is impossible to add controllers to a lot of nodes in the network to achieve synchronization. To reduce the number of controllers becomes a key issue, very few results exist in this crucial aspect of network synchronization.[29−31] In this paper, a backstepping approach is proposed for the synchronization of chain networks between multi-spatiotemporal chaotic systems with topologically equivalent structures. The synchronization of the multi-spatiotemporal chaotic systems is implemented by adding the control only to a terminal node, and the controller is designed via a corresponding update law. The control law is applied to spatiotemporal Gray–Scott systems. Numerical results demonstrate the effectiveness and the feasibility of the proposed approach.

2. Design of the synchronization controller

The backstepping approach of complex networks is a multi-systematic control technique, which is characterized by a step-by-step interlacing procedure. At each step, a coordinate transformation and a design of the virtual control based on the Lyapunov technique are required. Finally, the true terminal controller is obtained under the global stability. Here, our attention is focused on the spatiotemporal chaos systems. In this section, we give a terminal synchronization controller via the backstepping design.

Suppose that a complex chain network consists of N nodes, and the state equations of the nodes are spatiotemporal chaos systems. When the coupling action is not considered, each node can be described by

$$\dot{\boldsymbol{X}}_i(r,t) = \boldsymbol{F}(\boldsymbol{X}_i(r,t)), \quad i = 1,2,\ldots,N, \tag{1}$$

where $\boldsymbol{X}_i(r,t) = [X_{i1}(r,t),\, X_{i2}(r,t),\ldots,X_{in}(r,t)]^{\mathrm{T}} \in \mathrm{R}^n$ is the n-dimensional state variable of node i at time t, r is the spatial variable, and $\boldsymbol{F}: \mathrm{R}^n \to \mathrm{R}^n$ is continuously differentiable. Coupling to other nodes is added by using single state variables, and the state equation (1) of the i-th node is rewritten as

$$\begin{aligned}\dot{\boldsymbol{X}}_i(r,t) \;=\; & \boldsymbol{F}(\boldsymbol{X}_i(r,t)) + \alpha\sum_{j=1}^{N} P_{ij}\boldsymbol{X}_j(r,t) \\ & + \beta_i \boldsymbol{U}(r,t),\end{aligned} \tag{2}$$

where α is the coupling strength between nodes of the network, and $\alpha = 1$ in this work; P_{ij} is the matrix element of the inner coupling matrix, whose specific form depends on the connection type of the network; β_i is the terminal adding design, if there is a terminal adding, then $\beta_i = 1$, otherwise $\beta_i = 0$; and $\boldsymbol{U}(r,t)$ is the terminal synchronization controller. The unidirectional chain connection is discussed in the following parts.

The first error state variable of the chain network between the first and the second nodes is defined as

$$\boldsymbol{e}_1(r,t) = \boldsymbol{X}_2(r,t) - \boldsymbol{X}_1(r,t). \tag{3}$$

The first partial Lyapunov function is constructed as

$$V_1 = \frac{1}{2}\boldsymbol{e}_1(r,t)^2. \tag{4}$$

The derivative of V_1 is

$$\begin{aligned}\dot{V}_1 \;=\; & \boldsymbol{e}_1(r,t)\dot{\boldsymbol{e}}_1(r,t) \\ =\; & -\boldsymbol{e}_1(r,t)^2 + \boldsymbol{e}_1(r,t)(\boldsymbol{e}_1(r,t) + \dot{\boldsymbol{e}}_1(r,t)).\end{aligned} \tag{5}$$

Letting

$$\begin{aligned}\boldsymbol{e}_2(r,t) \;=\; & \boldsymbol{e}_1(r,t) + \dot{\boldsymbol{e}}_1(r,t) \\ =\; & \boldsymbol{F}(\boldsymbol{X}_2(r,t)) - \boldsymbol{F}(\boldsymbol{X}_1(r,t)) \\ & + (\boldsymbol{X}_3(r,t) - \boldsymbol{X}_2(r,t)),\end{aligned} \tag{6}$$

and introducing function $\boldsymbol{W}_2$ as

$$\boldsymbol{W}_2(r,t) = \boldsymbol{F}(\boldsymbol{X}_2(r,t)) - \boldsymbol{F}(\boldsymbol{X}_1(r,t)) + \boldsymbol{X}_3(r,t), \tag{7}$$

we have

$$\boldsymbol{e}_2(r,t) = \boldsymbol{W}_2(r,t) - \boldsymbol{X}_2(r,t). \tag{8}$$

The second partial Lyapunov function is constructed as

$$V_2 = V_1 + \frac{1}{2}\boldsymbol{e}_2(r,t)^2. \tag{9}$$

The derivative of V_2 is

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \boldsymbol{e}_2(r,t)\dot{\boldsymbol{e}}_2(r,t)\\ &= -\boldsymbol{e}_1(r,t)^2 - \boldsymbol{e}_2(r,t)^2 + \boldsymbol{e}_2(r,t)(\boldsymbol{e}_1(r,t)\\ &\quad + \boldsymbol{e}_2(r,t) + \dot{\boldsymbol{e}}_2(r,t)).\end{aligned} \tag{10}$$

Letting

$$\boldsymbol{e}_3(r,t) = \boldsymbol{e}_1(r,t) + \boldsymbol{e}_2(r,t) + \dot{\boldsymbol{e}}_2(r,t), \tag{11}$$

and introducing function $\boldsymbol{W}_3$ as

$$\begin{aligned}\boldsymbol{W}_3(r,t) &= \boldsymbol{e}_1(r,t) + \boldsymbol{e}_2(r,t) + \dot{\boldsymbol{F}}(\boldsymbol{x}_2(r,t))\\ &\quad - \dot{\boldsymbol{F}}(\boldsymbol{x}_1(r,t)) + \boldsymbol{F}(\boldsymbol{x}_3(r,t))\\ &\quad + \boldsymbol{X}_4(r,t) - \dot{\boldsymbol{X}}_2(r,t),\end{aligned} \tag{12}$$

we have

$$\boldsymbol{e}_3(r,t) = \boldsymbol{W}_3(r,t) - \boldsymbol{X}_3(r,t). \tag{13}$$

.....

The $i-1$ partial Lyapunov function is constructed as

$$V_{i-1} = V_{i-2} + \frac{1}{2}\boldsymbol{e}_{i-1}(r,t)^2. \tag{14}$$

The derivative of V_{i-1} is

$$\begin{aligned}\dot{V}_{i-1} &= -\sum_{q=1}^{i-1}\boldsymbol{e}_q(r,t)^2 + \boldsymbol{e}_{i-1}(r,t)\\ &\quad \times (\boldsymbol{e}_{i-2}(r,t) + \boldsymbol{e}_{i-1}(r,t) + \dot{\boldsymbol{e}}_{i-1}(r,t)).\end{aligned} \tag{15}$$

Letting

$$\boldsymbol{e}_i(r,t) = \boldsymbol{e}_{i-2}(r,t) + \boldsymbol{e}_{i-1}(r,t) + \dot{\boldsymbol{e}}_{i-1}(r,t), \tag{16}$$

and introducing function $\boldsymbol{W}_i$ as

$$\begin{aligned}&\boldsymbol{W}_i(r,t)\\ &= \boldsymbol{e}_{i-2}(r,t) + \boldsymbol{e}_{i-1}(r,t) + \sum_{l_1=1}^{i-3}\frac{\partial^{i-2-l_1}}{\partial t^{i-2-l_1}}\boldsymbol{e}_{l_1}(r,t)\\ &\quad + \sum_{l_2=2}^{i-2}\frac{\partial^{i-1-l_2}}{\partial t^{i-1-l_2}}\boldsymbol{e}_{l_2}(r,t) - \frac{\partial^{i-2}}{\partial t^{i-2}}\boldsymbol{F}(\boldsymbol{X}_1(r,t))\\ &\quad + \sum_{j_1=2}^{i}\frac{\partial^{i-j_1}}{\partial t^{i-j_1}}\boldsymbol{F}(\boldsymbol{X}_{j_1}(r,t)) + \boldsymbol{X}_{i+1}(r,t)\\ &\quad - \sum_{j_2=3}^{i-1}\frac{\partial^{i-j_2}}{\partial t^{i-j_2}}\boldsymbol{X}_{j_2}(r,t) - \boldsymbol{X}_2^{i-2}(r,t),\end{aligned} \tag{17}$$

we have

$$\boldsymbol{e}_i(r,t) = \boldsymbol{W}_i(r,t) - \boldsymbol{X}_i(r,t), \tag{18}$$

where $\frac{\partial^{i-2-l_1}}{\partial t^{i-2-l_1}}\boldsymbol{e}_{l_1}(r,t)$ is the $i-2-l_1$ order derivative of $\boldsymbol{e}_{l_1}(r,t)$ with respect to t, and $\boldsymbol{X}_2^{i-2}(r,t)$ is the $i-2$ order derivative of $\boldsymbol{X}_2(r,t)$ with respect to t.

.....

The $N-2$ partial Lyapunov function is constructed as

$$V_{N-2} = V_{N-3} + \frac{1}{2}\boldsymbol{e}_{N-2}(r,t)^2. \tag{19}$$

The derivative of V_{N-2} is

$$\begin{aligned}\dot{V}_{N-2} &= -\sum_{i=1}^{N-2}\boldsymbol{e}_i(r,t)^2 + \boldsymbol{e}_{N-2}(r,t)\\ &\quad \times (\boldsymbol{e}_{N-3}(r,t) + \boldsymbol{e}_{N-2}(r,t)\\ &\quad + \dot{\boldsymbol{e}}_{N-2}(r,t)).\end{aligned} \tag{20}$$

Letting

$$\begin{aligned}\boldsymbol{e}_{N-1}(r,t) &= \boldsymbol{e}_{N-3}(r,t) + \boldsymbol{e}_{N-2}(r,t)\\ &\quad + \dot{\boldsymbol{e}}_{N-2}(r,t),\end{aligned} \tag{21}$$

and introducing function $\boldsymbol{W}_{N-1}$ as

$$\begin{aligned}&\boldsymbol{W}_{N-1}(r,t)\\ &= \boldsymbol{e}_{N-3}(r,t) + \boldsymbol{e}_{N-2}(r,t) + \sum_{l_1=1}^{N-4}\frac{\partial^{N-3-l_1}}{\partial t^{N-3-l_1}}\boldsymbol{e}_{l_1}(r,t)\\ &\quad + \sum_{l_2=2}^{N-3}\frac{\partial^{N-2-l_2}}{\partial t^{N-2-l_2}}\boldsymbol{e}_{l_2}(r,t) - \frac{\partial^{N-3}}{\partial t^{N-3}}\boldsymbol{F}(\boldsymbol{X}_1(r,t))\\ &\quad + \sum_{j_1=2}^{N-1}\frac{\partial^{N-1-j_1}}{\partial t^{N-1-j_1}}\boldsymbol{F}(\boldsymbol{X}_{j_1}(r,t)) + \boldsymbol{X}_N(r,t)\\ &\quad - \sum_{j_2=3}^{N-2}\frac{\partial^{N-1-j_2}}{\partial t^{N-1-j_2}}\boldsymbol{X}_{j_2}(r,t) - \boldsymbol{X}_2^{N-3}(r,t),\end{aligned} \tag{22}$$

we have

$$\boldsymbol{e}_{N-1}(r,t) = \boldsymbol{W}_{N-1}(r,t) - \boldsymbol{X}_{N-1}(r,t). \tag{23}$$

The $N-1$ partial Lyapunov function is constructed as

$$V_{N-1} = V_{N-2} + \frac{1}{2}\boldsymbol{e}_{N-1}(r,t)^2. \tag{24}$$

The derivative of V_{N-1} is

$$\begin{aligned}\dot{V}_{N-1} &= -\sum_{i=1}^{N-1}\boldsymbol{e}_i(r,t)^2 + \boldsymbol{e}_{N-1}(r,t)\\ &\quad \times (\boldsymbol{e}_{N-2}(r,t) + \boldsymbol{e}_{N-1}(r,t)\\ &\quad + \dot{\boldsymbol{e}}_{N-1}(r,t)).\end{aligned} \tag{25}$$

Letting

$$\boldsymbol{e}_N(r,t) = \boldsymbol{e}_{N-2}(r,t) + \boldsymbol{e}_{N-1}(r,t) + \dot{\boldsymbol{e}}_{N-1}(r,t), \tag{26}$$

we have

$$\dot{V}_{N-1} = -\sum_{i=1}^{N-1}\boldsymbol{e}_i(r,t)^2 + \boldsymbol{e}_{N-1}(r,t)\boldsymbol{e}_N(r,t). \tag{27}$$

A terminal node of controller $\boldsymbol{U}(r,t)$ is designed in the chain network as

$$\begin{aligned}
&\boldsymbol{e}_N(r,t)\\
&= \boldsymbol{e}_{N-2}(r,t)+\boldsymbol{e}_{N-1}(r,t)+\dot{\boldsymbol{e}}_{N-1}(r,t)\\
&= \boldsymbol{e}_{N-2}(r,t)+\boldsymbol{e}_{N-1}(r,t)+\sum_{l_1=1}^{N-3}\frac{\partial^{N-2-l_1}}{\partial t^{N-2-l_1}}\boldsymbol{e}_{l_1}(r,t)\\
&\quad+\sum_{l_2=2}^{N-2}\frac{\partial^{N-1-l_2}}{\partial t^{N-1-l_2}}\boldsymbol{e}_{l_2}(r,t)-\frac{\partial^{N-2}}{\partial t^{N-2}}\boldsymbol{F}(\boldsymbol{X}_1(r,t))\\
&\quad+\sum_{j_1=2}^{N}\frac{\partial^{N-j_1}}{\partial t^{N-j_1}}\boldsymbol{F}(\boldsymbol{X}_{j_1}(r,t))+\boldsymbol{U}(r,t)\\
&\quad-\sum_{j_2=3}^{N-1}\frac{\partial^{N-j_2}}{\partial t^{N-j_2}}\boldsymbol{X}_{j_2}(r,t)-\boldsymbol{X}_2^{N-2}(r,t). \qquad (28)
\end{aligned}$$

The structure of $\boldsymbol{U}(r,t)$ is

$$\begin{aligned}
&\boldsymbol{U}(r,t)\\
&=-\Bigg(\boldsymbol{e}_{N-2}(r,t)+\boldsymbol{e}_{N-1}(r,t)+\sum_{l_1=1}^{N-3}\frac{\partial^{N-2-l_1}}{\partial t^{N-2-l_1}}\boldsymbol{e}_{l_1}(r,t)\\
&\quad+\sum_{l_2=2}^{N-2}\frac{\partial^{N-1-l_2}}{\partial t^{N-1-l_2}}\boldsymbol{e}_{l_2}(r,t)-\frac{\partial^{N-2}}{\partial t^{N-2}}\boldsymbol{F}(\boldsymbol{X}_1(r,t))\\
&\quad+\sum_{j_1=2}^{N}\frac{\partial^{N-j_1}}{\partial t^{N-j_1}}\boldsymbol{F}(\boldsymbol{X}_{j_1}(r,t))\\
&\quad-\sum_{j_2=3}^{N-1}\frac{\partial^{N-j_2}}{\partial t^{N-j_2}}\boldsymbol{X}_{j_2}(r,t)-\boldsymbol{X}_2^{N-2}(r,t)\Bigg). \qquad (29)
\end{aligned}$$

When the structure of synchronization controller $\boldsymbol{U}(r,t)$ is given in Eq. (29), we obtain

$$\dot{V}_{N-1}=-\sum_{i=1}^{N-1}\boldsymbol{e}_i(r,t)^2. \qquad (30)$$

According to the Lyapunov stability theory, the synchronization of the chain network is achieved when equation (30) is fulfilled.

3. Numerical results

In this section, we present numerical results to verify the analytical predictions obtained in the previous section. We use a terminal node controller to control the spatiotemporal chaos of a complex chain network. The numerical simulation results provide excellent validations of our theoretical analysis.

The Gray–Scott system as a producer of the spatiotemporal chaos is described as[32,33]

$$\begin{cases}
\dot{X}_1(r,t)=-X_1(r,t)X_2(r,t)^2+a(1-X_1(r,t))\\
\qquad\qquad+d_1\boldsymbol{\nabla}^2X_1(r,t),\\
\dot{X}_2(r,t)=X_1(r,t)X_2(r,t)^2-(a+b)X_2(r,t)\\
\qquad\qquad+d_2\boldsymbol{\nabla}^2X_2(r,t),
\end{cases} \qquad (31)$$

where a and b are the system parameters; $X_1(r,t)$ and $X_2(r,t)$ are the state variables of the system, $X_1(r,t)\in[0,1]$, and $X_2(r,t)\in[0,1]$; d_1 and d_2 are the diffusion coefficients. System size L is 2.5×2.5, and $r\in[0,L]$. The initial conditions are given as $X_1(r,0)=1$, $X_2(r,0)=1$. The time step is $\Delta t=1$, the space step is $\Delta l=0.01$, and the diffusion coefficients are $d_1=2\times10^{-5}$ and $d_2=2\times10^{-5}$. The periodic boundary conditions are $X_1(0,t)=X_1(L,t)=1$ and $X_2(0,t)=X_2(L,t)=0$. The spatiotemporal evolutions of state variables $X_1(r,t)$ and $X_2(r,t)$ are shown in Figs. 1 and 2, respectively.

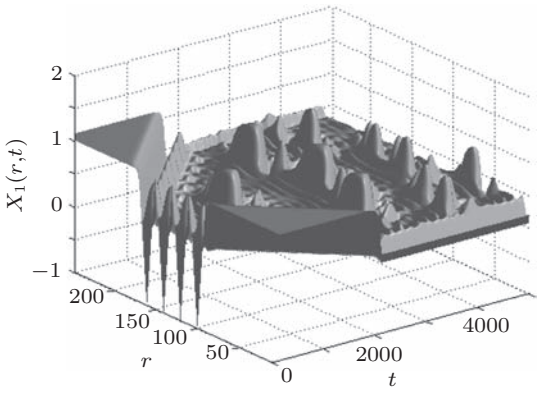


Fig. 1. Spatiotemporal evolution of $X_1(r,t)$.

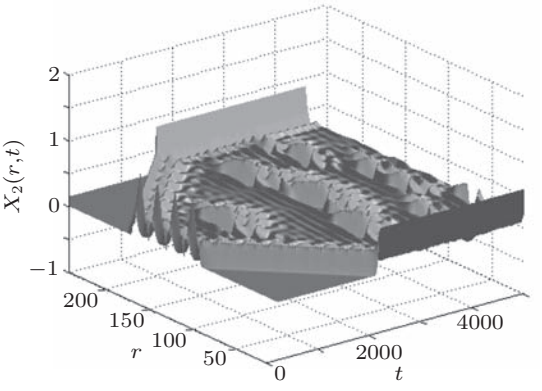


Fig. 2. Spatiotemporal evolution of $X_2(r,t)$.

In order to briefly demonstrate the effectiveness of the synchronization approach to a chain network with N nodes, we first consider a chain network with three nodes. The state equation for each

node is given by the Gray–Scott model described in Eq. (31), and the state variables of the three nodes are $(X_{11}(r,t),\ X_{12}(r,t))$, $(X_{21}(r,t),\ X_{22}(r,t))$, and $(X_{31}(r,t),\ X_{32}(r,t))$, respectively.

The coupling matrix $\boldsymbol{P}$ is set as

$$\boldsymbol{P} = \begin{bmatrix} -1 & 1 & \cdots & 0 & 0 \\ 0 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -1 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{32}$$

The coupling is introduced by using the single state variable $\boldsymbol{X}_i(r,t)$ in the Gray–Scott model according to Eq. (2)

$$\begin{cases} \dot{X}_{11}(r,t) \\ = -X_{11}(r,t)X_{12}(r,t)^2 + a(1 - X_{11}(r,t)) \\ \quad + d_1\boldsymbol{\nabla}^2 X_{11}(r,t) + (X_{21}(r,t) - X_{11}(r,t)), \\ \dot{X}_{12}(r,t) \\ = X_{11}(r,t)X_{12}(r,t)^2 - (a+b)X_{12}(r,t) \\ \quad + d_2\boldsymbol{\nabla}^2 X_{12}(r,t) + (X_{22}(r,t) - X_{12}(r,t)), \end{cases} \tag{33}$$

$$\begin{cases} \dot{X}_{21}(r,t) \\ = -X_{21}(r,t)X_{22}(r,t)^2 + a(1 - X_{21}(r,t)) \\ \quad + d_1\boldsymbol{\nabla}^2 X_{21}(r,t) + (X_{31}(r,t) - X_{21}(r,t)), \\ \dot{X}_{22}(r,t) \\ = X_{21}(r,t)X_{22}(r,t)^2 - (a+b)X_{22}(r,t) \\ \quad + d_2\boldsymbol{\nabla}^2 X_{22}(r,t) + (X_{32}(r,t) - X_{22}(r,t)), \end{cases} \tag{34}$$

$$\begin{cases} \dot{X}_{31}(r,t) \\ = -X_{31}(r,t)X_{32}(r,t)^2 + a(1 - X_{31}(r,t)) \\ \quad + d_1\boldsymbol{\nabla}^2 X_{31}(r,t) + U_1(r,t), \\ \dot{X}_{32}(r,t) \\ = X_{31}(r,t)X_{32}(r,t)^2 - (a+b)X_{32}(r,t) \\ \quad + d_2\boldsymbol{\nabla}^2 X_{32}(r,t) + U_2(r,t). \end{cases} \tag{35}$$

The iteration of the simulation is initiated at $t = 2800$, and the simulation results are shown in Figs. 3–10.

It can be seen from Figs. 3–10 that all the error signals, $\boldsymbol{e}_1 = (e_{11}, e_{12})^{\mathrm{T}}$ and $\boldsymbol{e}_2 = (e_{21}, e_{22})^{\mathrm{T}}$, in the network approach zero smoothly and rapidly once the network coupling is added, which means that the chain network synchronization is realized.

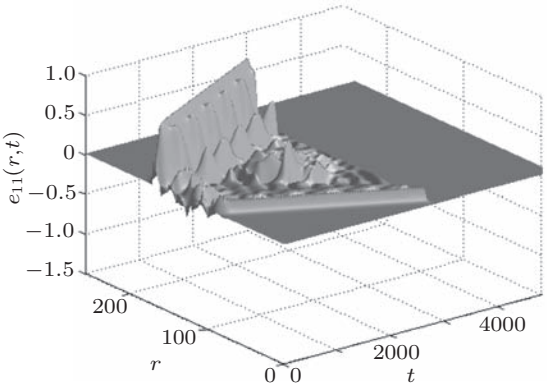


Fig. 3. Spatiotemporal evolution of error $e_{11}(r,t)$.

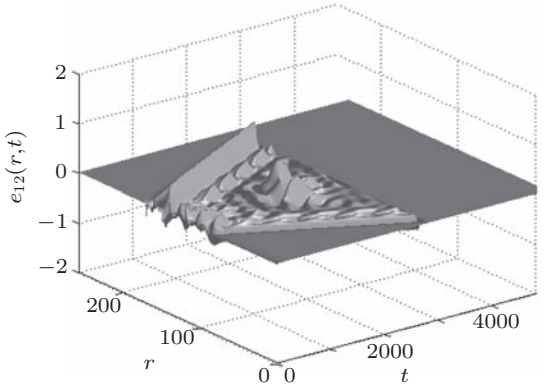


Fig. 4. Spatiotemporal evolution of error $e_{12}(r,t)$.

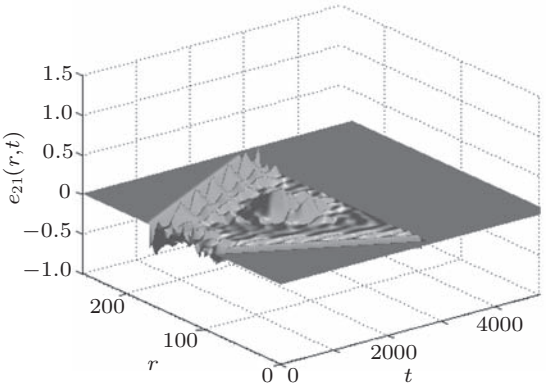


Fig. 5. Spatiotemporal evolution of error $e_{21}(r,t)$.

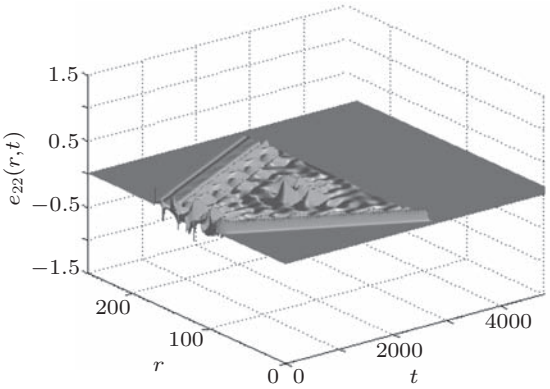


Fig. 6. Spatiotemporal evolution of error $e_{22}(r,t)$.

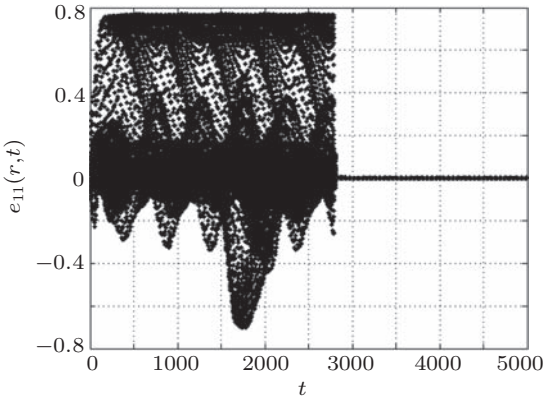


Fig. 7. State trajectories of error $e_{11}(r,t)$.

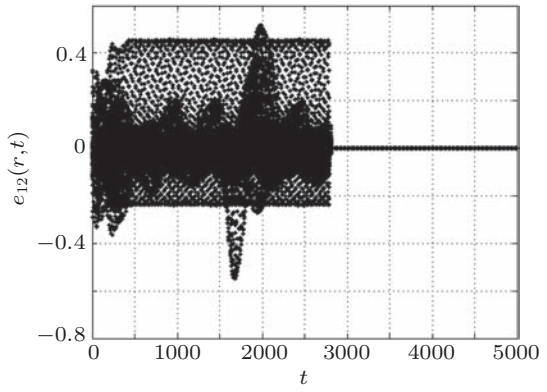


Fig. 8. State trajectories of error $e_{12}(r,t)$.

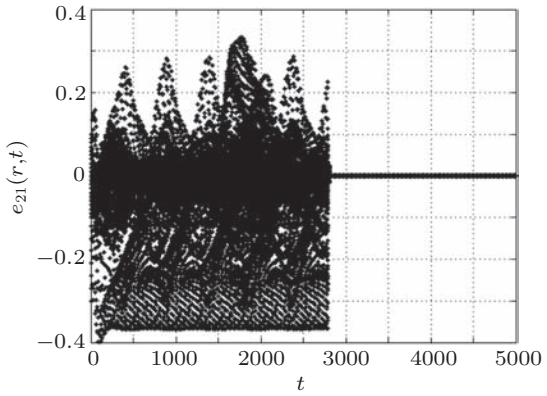


Fig. 9. State trajectories of error $e_{21}(r,t)$.

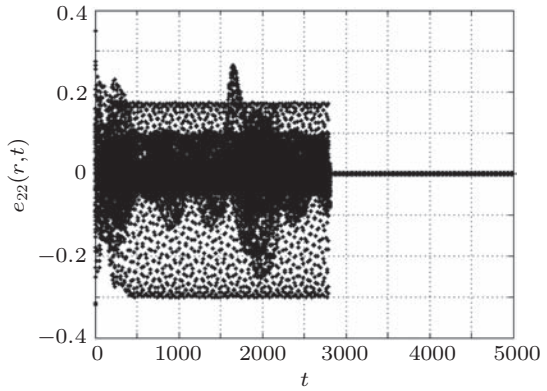


Fig. 10. State trajectories of error $e_{22}(r,t)$.

4. Conclusion

A synchronization controller is designed to study spatiotemporal chaotic systems with identical structures and to realize the synchronization between multi-Gray–Scott systems. The structure of the terminal synchronization controller and the error equations of the state variables between spatiotemporal chaotic systems are presented on the basis of the Lyapunov stability theory. The error signals approach zero smoothly and rapidly and the phase maps are exactly the same, which indicate that the chain network synchronization is achieved. The design of the terminal synchronization controller is simple in principle. Moreover, it can be widely used in synchronization.

References

[1] Ott E, Grebogi C and Yorke J A 1990 *Phys. Rev. Lett.* **64** 1196

[2] Pecora L M and Carroll T L 1990 *Phys. Rev. Lett.* **64** 821

[3] Guo R W 2008 *Phys. Lett.* A **372** 5593

[4] Yang T, Yang C M and Yang L B 1997 *Phys. Lett.* A **232** 356

[5] Nazzal J M and Natsheh A N 2007 *Chaos Soliton. Fract.* **33** 695

[6] Chen L Q 2004 *Chaos Soliton. Fract.* **21** 113

[7] Chen L Q 2001 *Phys. Lett.* A **281** 327

[8] Mascolo S and Grassi G 1997 *Phys. Rev.* E **56** 6166

[9] Njah A N, Ojo K S, Adebayo G A and Obawole A O 2010 *Physica* C **470** 558

[10] Laoye J A, Vincent U E and Kareem S O 2009 *Chaos Soliton. Fract.* **39** 356

[11] Sharma B B and Kar I N 2009 *Chaos Soliton. Fract.* **41** 2437

[12] Pishkenari H N, Jalili N, Mahboobi S H, Alasty A and Meghdari A 2010 *Chaos* **20** 023105

[13] Yu Y G and Li H X 2011 *Nonlinear Anal. Real World Appl.* **12** 388

[14] Watts D J and Strogatz S H 1998 *Nature* **393** 440

[15] Newman M E J and Watts D J 1999 *Phys. Lett.* A **263** 341

[16] Barabási A and Albert R 1999 *Science* **286** 509

[17] Liu H, Song Y R, Fan C X and Jiang G P 2010 *Chin. Phys.* B **19** 070508

[18] Hu A H, Xu Z Y and Guo L X 2011 *Chin. Phys.* B **20** 090511

[19] Silva J A L, Barrionuevo J A and Giordani F T 2010 *Nonlinear Anal. Real World Appl.* **11** 1005

[20] Ahn C K 2010 *Nonlinear Dyn.* **60** 295

[21] Sorrentino F and Ott E 2008 *Phys. Rev. Lett.* **100** 114101

[22] Gorochowski T E, Bernardo M D and Grierson C S 2010 *Phys. Rev.* E **81** 056212

[23] Flunkert V, Yanchuk S, Dahms T and Schóll E 2010 *Phys. Rev. Lett.* **105** 254101

[24] Chen W S, Li W and Miao Q G 2010 *ISA Trans.* **49** 283

[25] Lin D, Wang X Y, Nian F Z and Zhang Y L 2010 *Neurocomputing* **73** 2873

[26] Grigoriev R O, Cross M C and Schuster H G 1997 *Phys. Rev. Lett.* **79** 2795

[27] Zhang Q L, Lü L and Zhang Y 2011 *Chin. Phys.* B **20** 090514

[28] Parekh N, Parthasarathy S and Sinha S 1998 *Phys. Rev. Lett.* **81** 1401

[29] Wang X F and Chen G R 2002 *Physica* A **310** 521

[30] Li X, Wang X F and Chen G R 2004 *IEEE Trans. Circ. Syst.* I **51** 2074

[31] Chen T P, Liu X W and Lu W L 2007 *IEEE Trans. Circ. Syst.* I **54** 1317

[32] Pearson J E 1993 *Science* **261** 189

[33] Lü L, Chai Y and Luan L 2010 *Chin. Phys.* B **19** 080506