Adaptive synchronization of chaos in permanent magnet synchronous motors based on passivity theory*

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An adaptive synchronization control method is proposed for chaotic permanent magnet synchronous motors based on the property of a passive system. We prove that the controller makes the synchronization error system between the driving and the response systems not only passive but also asymptotically stable. The simulation results show that the proposed method is effective and robust against uncertainties in the systemic parameters.

Keywords: passivity theory, synchronization control, adaptive law, permanent magnet synchronous motor

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1. Introduction

The permanent magnet synchronous motor (PMSM) has superior features, such as a compact size, a high torque/inertia ratio, a high torque/weight ratio, no rotor loss, and the advantages of higher efficiency than the induction motors.[1] With the development of the electronic and the control technologies, the PMSMs are widely used in high performance servo drive systems. Over the past few years, the stability of the PMSM, which is an essential requirement for industrial automation manufacturing, has received considerable attention. It is found that with certain values of the system parameters, the PMSM exhibits chaotic behaviour, see Refs. [2]–[4] for examples. Chaos is not desirable in most engineering applications, and many scientists have devoted themselves to finding new ways to efficiently control chaos in the PMSM. [5−8] However, chaos in the PMSM is desirable in certain applications, such as in fluid mixing,[9] and vibratory compactors.[10] Anti-control of chaos in the PMSM has been proposed. To the best of our knowledge, for the analytical investigation of the synchronization of chaos in the PMSM, there has been no result reported in the literature so far. The field remains open and challenging. The main aim of the present paper is to find an appropriate and applicable method to synchronize chaos in the PMSM. The synchronization of chaotic systems has aroused the interest of researchers from various fields since the pioneering work of Pecora and Carroll[11] was published. In the literature, various synchronization schemes, such as the optimize adaptive method,[12] the H-∞ observer-based control,[13] and the impulsive control approach,[14] have been successfully applied to chaos synchronization.

Passivity is part of a broader and general theory of dissipativity, which can be found in Refs. [15] and [16]. The main idea of passivity theory is that the passive properties of a system can keep the internal stability of the system. So, to make a system stable, one can design a feedback controller to render the closed loop system passive. Recently, the concept of passivity has attracted new interest in chaos control and synchronization.[17−19] The references show that the passive controller has many advantages, including less control effort required, ease of implementation, and clear physical interpretation. On the other hand, the adaptive control chaotic system containing undeterministic parameters has attracted many researchers' attention recently.[20−22] One can eliminate the influence of uncertainties in the system parameters by using the adaptive law. In this paper, a passivity-based adaptive synchronization controller for the PMSM is studied, with which the error dynamical PMSM system can be equivalent to a passive sys-

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tem and is globally asymptotically stabilized at zero equilibrium points. The simulation results indicate that the proposed control law is effective and robust against the uncertainties in the system parameters.

2. Description of a permanent magnet synchronous motor

The mathematical model of the PMSM is given by[2,3]

$$\begin{cases} \dfrac{\mathrm{d}\omega}{\mathrm{d}t} = \sigma(i_\mathrm{q} - \omega) - T_\mathrm{L}, \\ \dfrac{\mathrm{d}i_\mathrm{q}}{\mathrm{d}t} = -i_\mathrm{q} - i_\mathrm{d}\omega + \gamma\omega + u_\mathrm{q}, \\ \dfrac{\mathrm{d}i_\mathrm{d}}{\mathrm{d}t} = -i_\mathrm{d} + i_\mathrm{q}\omega + u_\mathrm{d}, \end{cases} \tag{1}$$

where ω is the angle speed; i_q and i_d denote the quadrature- and the direct-axis currents of the motor, respectively; σ and γ are the system parameters, which are positive but unknown; T_L is the load torque; u_q and u_d are the quadrature- and direct-axis stator voltage components, respectively. Here, we only consider the case in which the system is unforced. In this case, the motor is running freely with no loading, and after an operating period, the external inputs are set to zero, i.e., $T_\mathrm{L} = u_\mathrm{q} = u_\mathrm{d} = 0$.[2,3] Then, system (1) becomes

$$\begin{cases} \dfrac{\mathrm{d}\omega}{\mathrm{d}t} = \sigma(i_\mathrm{q} - \omega), \\ \dfrac{\mathrm{d}i_\mathrm{q}}{\mathrm{d}t} = -i_\mathrm{q} - i_\mathrm{d}\omega + \gamma\omega, \\ \dfrac{\mathrm{d}i_\mathrm{d}}{\mathrm{d}t} = -i_\mathrm{d} + i_\mathrm{q}\omega. \end{cases} \tag{2}$$

The dynamics of system (2) has been fully investigated in Refs. [2] and [3], and it is found that when the values of parameters σ and γ fall into a certain range, the PMSM experiences chaotic behaviors. To view the dynamical behavior of the PMSM over a parameter range, the bifurcation diagram of direct-axis current i_d versus parameter γ is plotted in Fig. 1, where the inset shows the typical chaotic attractor of the PMSM with parameters $\sigma = 5$ and $\gamma = 21$.

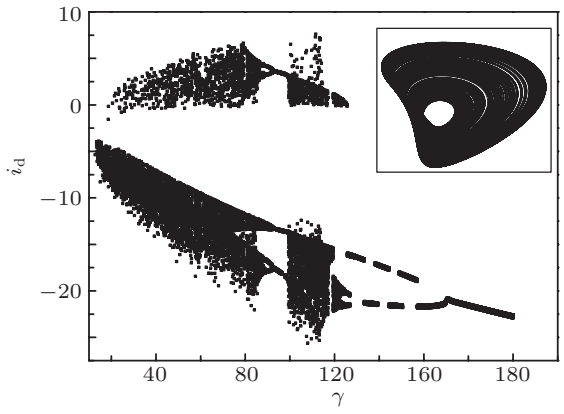


Fig. 1. Bifurcation structure of i_d versus parameter γ at $\sigma = 5.0$, where the inset shows the typical chaotic attractor of the PMSM.

3. Basic conception of passivity of a nonlinear system and passive control method

Consider the following differential equation:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}, \\ \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}), \end{cases} \tag{3}$$

where $\boldsymbol{x} \in \Re^n$ is the state variable; $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ are smooth vector fields, $\boldsymbol{f}(\boldsymbol{0}) = \boldsymbol{0}$; $\boldsymbol{u}(t)$ is the external input; and $\boldsymbol{h}(\boldsymbol{x})$ is a smooth mapping. The notion of passivity can be captured as follows.

Definition 1[15,16] System (3) is passive if there exists a real constant β satisfying

$$\int_0^t \boldsymbol{u}^\mathrm{T}\boldsymbol{y}(\tau)\mathrm{d}\tau \geq \beta \tag{4}$$

for $\forall t \geq 0$. Or there exist real constant β and $\rho \geq 0$ satisfying

$$\int_0^t \boldsymbol{u}^\mathrm{T}\boldsymbol{y}(\tau)\mathrm{d}\tau + \beta \geq \int_0^t \rho\boldsymbol{y}^\mathrm{T}(\tau)\boldsymbol{y}(\tau)\mathrm{d}\tau. \tag{5}$$

Let $\boldsymbol{z} = \boldsymbol{\vartheta}(\boldsymbol{x})$, then system (3) will be changed into the following generalized form:

$$\begin{aligned} \dot{\boldsymbol{z}} &= \boldsymbol{f}_0(\boldsymbol{z}) + \boldsymbol{a}(\boldsymbol{z},\boldsymbol{y})\boldsymbol{y}, \\ \dot{\boldsymbol{y}} &= \boldsymbol{b}(\boldsymbol{z},\boldsymbol{y}) + \boldsymbol{c}(\boldsymbol{z},\boldsymbol{y})\boldsymbol{u}, \end{aligned} \tag{6}$$

where $\boldsymbol{a}(\boldsymbol{z},\boldsymbol{y})$ is nonsingular for any $(\boldsymbol{z},\boldsymbol{y})$. With the conception of passivity of a nonlinear system, a well-known lemma is introduced as follows.

Lemma 1[16] If system (3) is a minimum phase system, system (6) will be equivalent to a passive system and asymptotically stabilized at an equilibrium point by using a state feedback in the form of

$$\boldsymbol{u} = \boldsymbol{\alpha}(\boldsymbol{x}) + \beta(\boldsymbol{x})\boldsymbol{v}. \tag{7}$$

Lemma 1 implies that a non-passive nonlinear system can be equivalent to a passive system by designing a system controller and then be stabilized at the equilibrium point, which is essential for the proof of the main result in this paper.

4. Synchronization of chaos in permanent magnet synchronous motors based on passivity theory

The response system, which has a controller added into the second formula of system (2), is given by

$$\begin{cases} \dfrac{\mathrm{d}\omega_2}{\mathrm{d}t} = \sigma(i_{\mathrm{q}_2} - \omega_2), \\ \dfrac{\mathrm{d}i_{\mathrm{q}_2}}{\mathrm{d}t} = -i_{\mathrm{q}_2} - i_{\mathrm{d}_2}\omega_2 + \gamma\omega_2 + u, \\ \dfrac{\mathrm{d}i_{\mathrm{d}_2}}{\mathrm{d}t} = -i_{\mathrm{d}_2} + i_{\mathrm{q}_2}\omega_2, \end{cases} \tag{8}$$

where u is the control law.

Let $e_1 = \omega_2 - \omega$, $e_2 = i_{\mathrm{q}_2} - i_{\mathrm{q}}$, and $e_3 = i_{\mathrm{d}_2} - i_{\mathrm{d}}$, then the error dynamical system between the driving system (2) and the response system (8) can be expressed as

$$\begin{cases} \dot{e}_1 = \sigma(e_2 - e_1), \\ \dot{e}_2 = -(e_1e_3 + \omega e_3 + i_{\mathrm{d}}e_1) - e_2 + \gamma e_1 + u, \\ \dot{e}_3 = e_1e_2 + \omega e_2 + i_{\mathrm{q}}e_1 - e_3. \end{cases} \tag{9}$$

It follows that if system (9) can be stabilized by controller u, then error e will converge to a zero equilibrium point. This implies that the trajectories of the controlled response system asymptotically synchronize the trajectories of the driving system.

Let $z_1 = e_1$, $z_2 = e_3$, and $y = e_2$, then system (9) can be rewritten in the normal form of Eq. (6)

$$\begin{cases} \dot{z}_1 = \sigma(y - z_1), \\ \dot{z}_2 = z_1y + \omega y + i_{\mathrm{q}}z_1 - z_2, \\ \dot{y} = -(z_1z_2 + \omega z_2 + i_{\mathrm{d}}z_1) - y + \gamma z_1 + u. \end{cases} \tag{10}$$

We have

$$\begin{aligned} \dot{\boldsymbol{z}} &= \boldsymbol{f}_0(\boldsymbol{z}) + \boldsymbol{a}(\boldsymbol{z}, y)y, \\ \dot{y} &= b(\boldsymbol{z}, y) + c(\boldsymbol{z}, y)u, \end{aligned}$$

where

$$\begin{aligned} &\boldsymbol{z} = [z_1, z_2]^{\mathrm{T}}, \\ &\boldsymbol{f}_0(\boldsymbol{z}) = [-\sigma z_1, i_{\mathrm{q}}z_1 - z_2]^{\mathrm{T}}, \\ &\boldsymbol{a}(\boldsymbol{z}, y) = [\sigma, z_1 + \omega]^{\mathrm{T}}, \\ &b(\boldsymbol{z}, y) = -(z_1z_2 + \omega z_2 + i_{\mathrm{d}}z_1) - y + \gamma z_1, \\ &c(\boldsymbol{z}, y) = 1. \end{aligned} \tag{11}$$

With the above knowledge, we give our main results.

Theorem 1 The controlled error dynamical system (10) is a minimum phase system.

Proof Each state variable of the chaotic system has its bound. Suppose that the bound of i_{q} is Ω, namely, $|i_{\mathrm{q}}| \leq i_{\mathrm{q}_{\max}} = \Omega$. We choose a Lyapunov function $W(\boldsymbol{z}) = \frac{\Omega}{2}z_1^2 + \frac{1}{2}z_2^2$ with $W(0) = 0$. The zero dynamics of system (6) describes the internal dynamics, which is consistent with external constraint $y = 0$, i.e.,

$$\dot{\boldsymbol{z}} = \boldsymbol{f}_0(\boldsymbol{z}). \tag{12}$$

Considering Eq. (12) and the parameter σ being a positive constant, we have

$$\begin{aligned} \frac{\mathrm{d}W(\boldsymbol{z})}{\mathrm{d}t} &= \frac{\partial W(\boldsymbol{z})}{\partial \boldsymbol{z}}\boldsymbol{f}_0(\boldsymbol{z}) \\ &= [\Omega z_1, z_2][-\sigma z_1, i_{\mathrm{q}}z_1 - z_2]^{\mathrm{T}} \\ &= -\Omega\sigma z_1^2 + i_{\mathrm{q}}z_1z_2 - z_2^2 \\ &= -\Omega\sigma\left(z_1 - \frac{i_{\mathrm{q}}z_2}{2\Omega}\right)^2 + \frac{i_{\mathrm{q}}^2z_2^2}{4\Omega\sigma} - z_2^2 \\ &\leq -\Omega\sigma\left(z_1 - \frac{i_{\mathrm{q}}z_2}{2\Omega}\right)^2 + \frac{\Omega^2z_2^2}{4\Omega\sigma} - z_2^2. \end{aligned} \tag{13}$$

According to Ref. [23], we have $\Omega < 4\sigma$. So

$$\frac{\mathrm{d}W(\boldsymbol{z})}{\mathrm{d}t} \leq 0. \tag{14}$$

Since $W(\boldsymbol{z})$ is the Lyapunov function of $\boldsymbol{f}_0(\boldsymbol{z})$, the zero dynamics of the error system (10) is stable, namely, the controlled error dynamical system is a minimum phase system.

Theorem 2 For the controlled error dynamical system (10), if there is a loop control

$$\begin{aligned} &u = (i_{\mathrm{d}} - \Omega\hat{\sigma} - \hat{\gamma})e_1 + (1 - \alpha y)e_2 + v, \\ &\dot{\hat{\sigma}} = k_1\Omega z_1y, \\ &\dot{\hat{\gamma}} = k_2z_1y, \end{aligned} \tag{15}$$

where v is the external input signal; α is a positive constant, which can be selected arbitrarily; $\hat{\sigma}$ and $\hat{\gamma}$ are the estimate values of undeterministic parameters σ and γ, respectively; $\dot{\hat{\sigma}}$ and $\dot{\hat{\gamma}}$ are the updating laws; and k_1 and k_2 are arbitrary positive scalars to adjust the performance of the adaptation algorithm, then the controlled system can be equivalent to a passive system and globally asymptotically stabilized at the zero equilibrium point. Moreover, the control properties can avoid the influence of the undeterministic systemic parameters.

Proof We choose

$$V = W(\boldsymbol{z}) + \frac{1}{2}y^2 + \frac{1}{2k_1}(\hat{\sigma} - \sigma)^2 + \frac{1}{2k_2}(\hat{\gamma} - \gamma)^2. \tag{16}$$

Taking the time derivative of V along the trajectory of the controlled error system (10), we can obtain

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} &= \frac{\partial W(\boldsymbol{z})}{\partial \boldsymbol{z}}\dot{\boldsymbol{z}} + y\dot{y} + \frac{1}{k_1}(\hat{\sigma}-\sigma)\dot{\hat{\sigma}} + \frac{1}{k_2}(\hat{\gamma}-\gamma)\dot{\hat{\gamma}} \\ &= \frac{\partial W(\boldsymbol{z})}{\partial \boldsymbol{z}}\boldsymbol{f}_0(\boldsymbol{z}) + \frac{\partial W(\boldsymbol{z})}{\partial \boldsymbol{z}}\boldsymbol{a}(\boldsymbol{z},y)y \\ &\quad + [b(\boldsymbol{z},y) + c(\boldsymbol{z},y)u]y \\ &\quad + \frac{1}{k_1}(\hat{\sigma}-\sigma)\dot{\hat{\sigma}} + \frac{1}{k_2}(\hat{\gamma}-\gamma)\dot{\hat{\gamma}}. \end{aligned} \tag{17}$$

According to Theorem 1, $\frac{\partial W(\boldsymbol{z})}{\partial \boldsymbol{z}}\boldsymbol{f}_0(\boldsymbol{z}) \leq 0$, hence equation (17) becomes

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} &\leq \frac{\partial W(\boldsymbol{z})}{\partial \boldsymbol{z}}\boldsymbol{a}(\boldsymbol{z},y)y + [b(\boldsymbol{z},y) + c(\boldsymbol{z},y)u]y \\ &\quad + \frac{1}{k_1}(\hat{\sigma}-\sigma)\dot{\hat{\sigma}} + \frac{1}{k_2}(\hat{\gamma}-\gamma)\dot{\hat{\gamma}} \\ &\leq \Omega\sigma z_1 y + z_1 z_2 y + \omega z_2 y - \Omega\hat{\sigma} z_1 y \\ &\quad - \hat{\gamma} z_1 y - \alpha y_2 + v y - z_1 z_2 y - \omega z_2 y + \gamma z_1 y \\ &\quad + \frac{1}{k_1}(\hat{\sigma}-\sigma)\dot{\hat{\sigma}} + \frac{1}{k_2}(\hat{\gamma}-\gamma)\dot{\hat{\gamma}}. \end{aligned} \tag{18}$$

Substituting Eq. (15) into inequality (18), we have

$$\frac{\mathrm{d}V}{\mathrm{d}t} \leq vy - \alpha y^2. \tag{19}$$

Taking integration over both sides of inequality (19), we obtain

$$V - V_0 \leq \int_0^t v(\tau)y(\tau)\,\mathrm{d}\tau - \int_0^t \alpha y^2(\tau)\,\mathrm{d}\tau, \tag{20}$$

where V_0 is the initial state of V. As $V \geq 0$, let $\beta = V_0$, then we have

$$\begin{aligned}\int_0^t v(\tau)y(\tau)\,\mathrm{d}\tau + \beta &\geq \int_0^t \alpha y^2(\tau)\,\mathrm{d}\tau + V \\ &\geq \int_0^t \alpha y^2(\tau)\,\mathrm{d}\tau, \end{aligned}$$

which satisfies Definition 1. According to Lemma 1, system (9) is rendered passive and asymptotically stabilized at an equilibrium point by using controller (15). On the other hand, since the adaptation algorithm is employed to estimate the unknown parameters, the control properties can avoid the influence of uncertainties.

The control law u can also be designed by using the Lyapunov stability theory.[12,21] Compared with the original Lyapunov stability theory, the passivity theory, which is considered as an alternative tool for analysing the stability of nonlinear systems and designing controllers, has superior features, such as having a distinct physical meaning and a positive definite storage function.

5. Numerical simulation and analysis

To verify the performance of the proposed controller, some numerical simulations are presented in this section. The fourth-order Runge–Kutta method is employed to solve the systems of differential equations with a time step size of 0.005. The parameters of the motor are selected as those in Section 2 i.e., $\sigma = 5$ and $\gamma = 21$. The initial values of the drive and the response systems are chosen as $(\omega(0), i_{\mathrm{q}}(0), i_{\mathrm{d}}(0)) = (5.0, 5.0, 3.0)$ and $(\omega_2(0), i_{\mathrm{q}_2}(0), i_{\mathrm{d}_2}(0)) = (1.0, 1.0, 2.0)$, respectively. The initial estimated parameters are $(\sigma(0), \gamma(0)) = (0.9, 7.0)$. The initial conditions of the error system are $(e_1(0), e_2(0), e_3(0)) = (1.0, 1.2, 0.2)$. Figures 2 and 3 show the results for the synchronization errors between the driving and the response systems when the control parameters are set as $\alpha = 0.5, k_1 = 1, k_2 = 1$ and $\alpha = 1, k_1 = 2, k_2 = 2$, respectively. It is obvious that the synchronization errors converge asymptotically to zero, and two systems are indeed synchronized. The numerical simulation results also show that with the increasing of control parameters k_1 and k_2, the systematic overshoot of the synchronization errors is decreased, and the transition time is reduced so that it takes a very short time for the system to be synchronized. Figures 4 and 5 display the estimations of the system parameters for different control parameters, from which we can see that the controlled system distinguishes the unknown systematic parameters in a short time. In other words, the designed controller can avoid the influence of the undeterministic parameters.

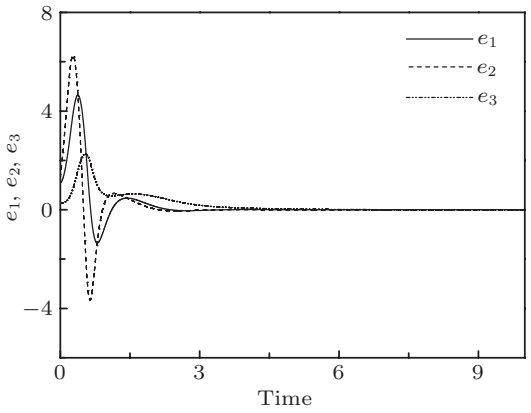


Fig. 2. Synchronization errors between driving and response systems with control parameters $\alpha = 0.5$, $k_1 = 1$, and $k_2 = 1$.

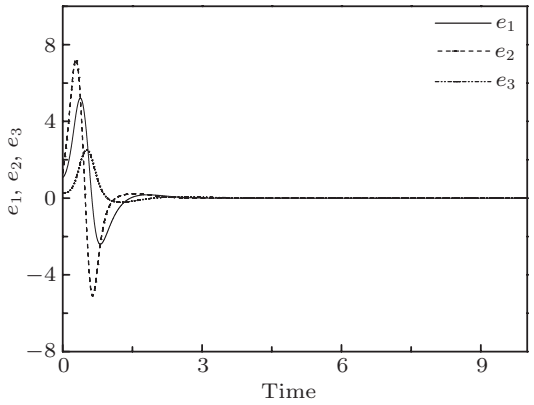


Fig. 3. Synchronization errors between driving and response systems with control parameters $\alpha = 1$, $k_1 = 2$, and $k_2 = 2$.

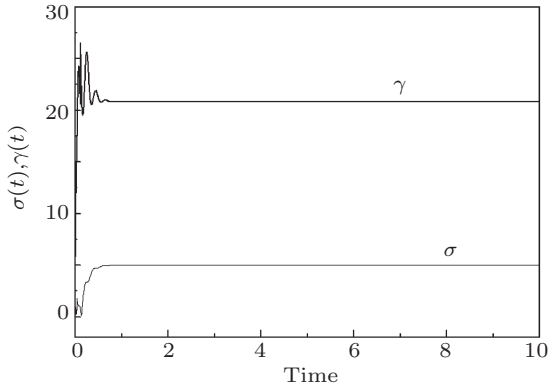


Fig. 4. Curves of the parameter estimate with $\alpha = 1$, $k_1 = 2$, and $k_2 = 2$.

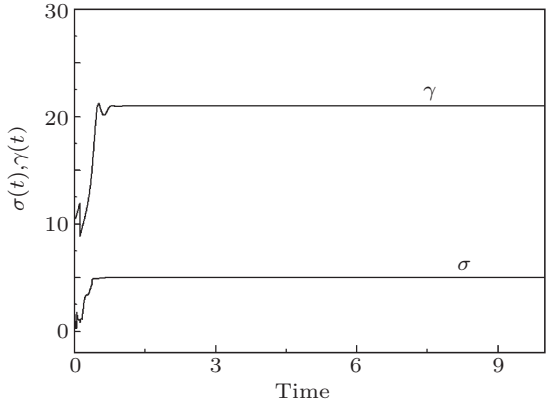


Fig. 5. Curves of parameter estimate with $\alpha = 1$, $k_1 = 4$, and $k_2 = 4$.

6. Conclusion

In this paper, a passivity theory is introduced for designing a feedback controller for the synchronization of chaotic permanent magnet synchronous motors with uncertain parameters. The controller transforms the error system into an equivalent passive system. And then, we prove that the equivalent system can be asymptotically stabilized at the zero equilibrium point, namely, the trajectories of the controlled response system asymptotically synchronize the trajectories of the driving system. The simulation results indicate that the proposed control law is effective and robust against the uncertainties in the system parameters.

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