Energy, momentum and angular momentum in the dyadosphere of a charged spacetime in teleparallel equivalent of general relativity

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We apply the energy momentum and angular momentum tensor to a tetrad field, with two unknown functions of radial coordinate, in the framework of a teleparallel equivalent of general relativity (TEGR). The definition of the gravitational energy is used to investigate the energy within the external event horizon of the dyadosphere region for the Reissner–Nordström black hole. We also calculate the spatial momentum and angular momentum.

Keywords: Teleparallel equivalent of general relativity, energy-momentum tensor, angular momentum tensor, angular momentum

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1. Introduction

Energy momentum, angular momentum and electric charge play central roles in modern physics. The conservation of the first two is related to the homogeneity and isotropy of spacetime respectively while charge conservation is related to the invariance of the action integral under internal U(1) transformations. Local quantities such as energy momentum, angular momentum and charge densities are well defined if gravitational fields are not present in the system. However, in general relativity theory a well-behaved energy momentum and angular momentum densities have not yet been defined, although total energy momentum and total angular momentum can be defined for an asymptotically flat spacetime surrounding an isolated finite system. The equality of the gravitational mass and the inertial mass holds within the framework of general relativity.[1−3] However, such equality is not satisfied for the Schwarzschild metric when it is expressed in a certain coordinate system.[4]

At present, teleparallel theory seems to be popular again, and there is a trend of analysing the basic solutions of general relativity with teleparallel theory and comparing the results. It is considered to be an essential part of generalized non-Riemannian theories such as the Poincaré gauge theory[5−12] or metric-affine gravity.[13] as well as a possible physical relevant geometry by itself-teleparallel description of gravity.[14−19] A teleparallel approach is used for positive-gravitational-energy proof.[20] A relation between spinor Lagrangian and teleparallel theory is established.[21]

For a satisfactory description of the total energy of an isolated system, it is necessary that the energy density of the gravitational field is given in terms of the first- and/or second-order derivatives of the gravitational field variables. It is well-known that there exists no covariant, nontrivial expression constructed out of the metric tensor. However, covariant expressions that contain a quadratic form of first-order derivatives of the tetrad field are feasible. Thus, it is legitimate to conjecture that the difficulties regarding the problem of defining the gravitational energy momentum are related to the geometrical description of the gravitational field rather than being an intrinsic drawback of the theory.[22,23] Møller has shown that the problem of an energy-momentum complex has no solution in the framework of gravitational field theories based on Riemannian spacetime.[24] In a series of papers,[24−27] Møller was able to obtain a general expression for a satisfactory energy-momentum complex in the teleparallel spacetime.

We investigate the irreducible mass $M_{\rm irr}$ of the

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charged dyadosphere. It is the total mass of the black hole at the final stage of Penrose's process of energy extraction, considering that the maximum possible energy is extracted. It is also related to the energy contained within the external event horizon $E(r_+)$ of the black hole (the surface of the constant radius $r = r_+$ defines the external event horizon). Every expression for local or quasi-local gravitational energy must necessarily yield the value of $E(r_+)$ in good agreement with $2M_{\rm irr}$, since we know beforehand that the value of the latter is a function of the initial angular momentum of the black hole.[28] The evolution of $2M_{\rm irr}$ is a crucial test for any expression for the gravitational energy. $E(r_+)$ has been obtained by means of different energy expressions in Ref. [29]. The gravitational energy used in this article is the only one that yields a satisfactory value for $E(r_+)$ and that arises in the framework of the Hamiltonian formulation of the gravitational field.

The event horizon of a charged black hole is surrounded by a special region called the dyadosphere where the electromagnetic field exceeds the Eular–Heisenberg critical value for electron–positron pair production.[30,31] It is the aim of this study to reformulate the correction of the Reissner–Nordström black hole from the first contribution of the Eular–Heisenber Lagrangian[32] in the teleparallel equivalent of general relativity (TEGR).[23] We also use the energy-momentum tensor and angular momentum tensor which is independent of the coordinate of the gravitational field established in the Hamiltonian structure of the TEGR to calculate energy, momentum and angular momentum.

In Section 2, we briefly review the derivation of the field equations for the coupled gravitational and electromagnetic fields. The tetrad field with two unknown functions of the radial coordinate in spherical polar coordinates is applied to the field equations and a form of the correction of Reissner–Nordström metric is obtained in Section 3. In Section 4, we give a brief review of the derivation of the energy momentum and angular momentum. In Section 5, we calculate the energy momentum and angular momentum. The final section is devoted to discussion and conclusion. Computer algebra system Maple 13 is used in some calculations.

2. The TEGR theory and regular solutions

In a spacetime with absolute parallelism the parallel vector fields $e_a{}^\mu$ define the nonsymmetric affine connection

$$\Gamma^\lambda{}_{\mu\nu} \stackrel{\rm def.}{=} e_a{}^\lambda e^a{}_{\mu,\nu}, \tag{1}$$

where $e_{a\mu,\nu} = \partial_\nu e_{a\mu}$. The curvature tensor defined by $\Gamma^\lambda{}_{\mu\nu}$, given in Eq. (1), is identically vanishing, however. The metric tensor $g_{\mu\nu}$ is given by

$$g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu, \tag{2}$$

with the Minkowski metric $\eta_{ab} = {\rm diag}(-1, +1, +1, +1)$. Latin indices are rising and lowering with the aid of η_{ab} and η^{ab}.

The Lagrangian density for the gravitational field in the TEGR, in the presence of matter fields, is given by (Throughout this paper, we use the relativistic units, $c = G = 1$ and $\kappa = 8\pi$)[22,33]

$$\begin{aligned}\mathscr{L}_{\rm G} &= eL_{\rm G}\\ &= -\frac{e}{16\pi}\left(\frac{T^{abc}T_{abc}}{4} + \frac{T^{abc}T_{bzc}}{2} - T^aT_a\right) - L_{\rm m}\\ &= -\frac{e}{16\pi}\Sigma^{abc}T_{abc} - L_{\rm m},\end{aligned} \tag{3}$$

where $e = \det(e^a{}_\mu)$, and $L_{\rm m}$ represents the Lagrangian density for matter fields. The tensor Σ^{abc} is defined by

$$\begin{aligned}\Sigma^{abc} \stackrel{\rm def.}{=}& \frac{1}{4}\left(T^{abc} + T^{bac} - T^{cab}\right)\\ &+ \frac{1}{2}\left(\eta^{ac}T^b - \eta^{ab}T^c\right),\end{aligned} \tag{4}$$

where T^{abc} and T^a are the torsion tensor and the basic vector field defined by

$$\begin{aligned}&T^a{}_{\mu\nu} \stackrel{\rm def.}{=} e^a{}_\lambda T^\lambda{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu,\\ &T^a \stackrel{\rm def.}{=} T^b{}_b{}^a.\end{aligned} \tag{5}$$

It can be shown that the quadratic combination $\Sigma^{abc}T_{abc}$ is proportional to the scalar curvature $R(e)$, except for a total divergence term.[22]

The electromagnetic Lagrangian density $L_{\rm em}$ is[34]

$$\mathscr{L}_{\rm em} = e\ L_{\rm em} = e\ g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}, \tag{6}$$

with $F_{\mu\nu}$ being the Maxwell field associated with a $U(1)$ subgroup of $E_8 \times E_8$ and is defined by $F_{\mu\nu} \stackrel{\rm def.}{=} \partial_\mu A_\nu - \partial_\nu A_\mu$. Heaviside-Lorentz rationalized units will be used throughout this paper.

The gravitational and electromagnetic field equations for the system described by $L_{\rm G} + L_{\rm em}$ are as follows:

$$\begin{aligned} &e_{a\lambda}e_{b\mu}\partial_\nu\left(e\Sigma^{b\lambda\nu}\right)\\ &-e\left(\Sigma^{b\nu}{}_a T_{b\nu\mu} - \frac{1}{4}e_{a\mu}T_{bcd}\Sigma^{bcd}\right) = \frac{1}{2}\kappa e T_{a\mu},\\ &\boldsymbol{\nabla}_\mu\left(\sqrt{-g}F^{\mu\nu}\right) = 0, \end{aligned} \tag{7}$$

where

$$T_{\mu\nu} = g_{\nu\sigma}F_{\mu\rho}F^{\sigma\rho} - \frac{1}{4}g_{\mu\nu}F^2.$$

It is possible to prove by explicit calculations that the left hand side of the symmetric field equation (7) is exactly given by[22]

$$\frac{e}{2}\left[R_{a\mu}(e) - \frac{1}{2}e_{a\mu}R(e)\right].$$

3. Spherically symmetric solutions and correction from the first contribution of the Eular–Heisenberg

Let us begin with the tetrad having a spherical symmetry[35]

$$(e_l{}^\mu) = \begin{pmatrix} A & 0 & 0 & 0\\ 0 & B\sin\theta\cos\phi & \dfrac{B}{r}\cos\theta\cos\phi & -\dfrac{B\sin\phi}{r\sin\theta}\\ 0 & B\sin\theta\sin\phi & \dfrac{B}{r}\cos\theta\sin\phi & \dfrac{B\cos\phi}{r\sin\theta}\\ 0 & B\cos\theta & -\dfrac{B}{r}\sin\theta & 0 \end{pmatrix}, \tag{8}$$

where A and B are unknown functions of $r = (x^\alpha x^\alpha)^{1/2}$. Applying Eq. (8) to the field equation (7), we obtain a system of nonlinear differential equations.[36]

Now we are interested in solving the following differential equations.[36]

A non-trivial solution can be obtained by solving for $A(r)$ and $B(r)$, then we obtain[36]

$$A(R) = \frac{1}{B(R)} = \frac{1}{\sqrt{1 - 2M/R + Q^2/R^2}}, \tag{9}$$

where R is a new radial coordinate defined by $R = r/B$. Using Eq. (9) in Eq. (8), we find that the associated Riemannian metric has the form

$$\begin{aligned} {\rm d}s^2 =& -\left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right){\rm d}t^2\\ &+ \frac{{\rm d}R^2}{(1 - 2M/R + Q^2/R^2)} + R^2{\rm d}\Omega^2, \end{aligned} \tag{10}$$

and ${\rm d}\Omega^2 = {\rm d}\theta^2 + \sin^2\theta{\rm d}\phi^2$, which is the Reissner–Nordström black hole.[37]

Ruffini[30] introduced a new concept of dyadosphere of an electromagnetic black hole to explain gamma ray bursts. The dyadosphere is defined as the region in which the outside of the horizon of a charged black hole whose electromagnetic field strength is larger than the well-known Heisenberg–Eular critical value $\varepsilon_{\rm crit} = \frac{{m_{\rm e}}^2c^3}{e\hbar}$ for electron–positron pair production, where $m_{\rm e}$ and e denote the mass and charge of an electron respectively. The dyadosphere of Reissner–Nordström spacetime is described by the radial interval $r_+ \le r \le r_{\rm ds}$, where the horizon $r_+ = M\{1 + \sqrt{1 - q^2/M^2}\}$ forms the inner radius of the dyadosphere, while its outer radius is given by

$$r_{\rm ds} = \sqrt{\left(\frac{\hbar}{m_{\rm e}}\right)\left(\frac{GM}{c^2}\right)\left(\frac{m_{\rm p}}{m_{\rm e}}\right)\left(\frac{e}{q_{\rm p}}\right)\left(\frac{Q}{\sqrt{G}M}\right)},$$

with M and Q being the mass and charge parameter, $m_{\rm p} = \sqrt{\hbar c/G}$ and $q_{\rm c} = \sqrt{\hbar c}$ being the Planck mass and Planck charge respectively.[30,31] de Lorenci *et al.*[32] calculated the electron correction for the Reissner–Nordström metric from the first contribution of the Eular–Heisenberg Lagrangian and obtained

$${\rm d}s^2 = -\eta_1^2(r){\rm d}t^2 + \frac{1}{\eta_1^2(r)}{\rm d}r^2 + r^2{\rm d}\Omega^2,$$

where

$$\eta_1(r) = \sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\sigma Q^4}{5r^6}}. \tag{11}$$

de Lorenci *et al.*[32] showed that the correction term $\sigma Q^4/5r^6$ is of the same order of magnitude as the Reissner–Nordström term Q^2/r^2. We are going to find a correction from the first contribution of the Eular–Heesenberg to the solution of Eq. (9) which has the form

$$(e_i{}^{\mu}) = \begin{pmatrix} \dfrac{1}{\eta_1(R)} & 0 & 0 & 0 \\ 0 & \eta_1(R)\sin\theta\cos\phi & \dfrac{\cos\theta\cos\phi}{R} & -\dfrac{\sin\phi}{R\sin\theta} \\ 0 & \eta_1(R)\sin\theta\sin\phi & \dfrac{\cos\theta\sin\phi}{R} & \dfrac{\cos\phi}{R\sin\theta} \\ 0 & \eta_1(R)\cos\theta & -\dfrac{\sin\theta}{R} & 0 \end{pmatrix}, \tag{12}$$

where $\eta_1(R)$ is defined by Eq. (11) by replacing $r = R$.

4. Energy, momentum and angular momentum

The Hamiltonian formulation of TEGR is obtained by establishing phase space variables. The Lagrangian density does not contain the time derivative of the tetrad component e_{a0}. Therefore, this quantity will arise as a Lagrange multiplier.[38] The momentum canonically conjugated to e_{ai} is given by $\Pi^{ai} = \delta L/\delta\dot{e}_{ai}$. The Hamiltonian formulation is obtained by rewriting the Lagrangian density in the form $L = p\,\dot{q} - H$, in terms of e_{ai}, Π^{ai} and the Lagrange multipliers. The Legendre transformation can be successfully carried out and the final form of the Hamiltonian density has the form[39]

$$H = e_{a0}C^a + \alpha_{ik}\Gamma^{ik} + \beta_k\Gamma^k, \tag{13}$$

plus a surface term. Here, α_{ik} and β_k are Lagrange multipliers that are identified as

$$\alpha_{ik} = \frac{1}{2}(T_{i0k} + T_{k0i}), \quad \beta_k = T_{00k}, \tag{14}$$

and C^a, Γ^{ik} and Γ^k are first class constraints. The Poisson brackets between any two field quantities F and G is given by

$$\{F, G\} = \int \mathrm{d}^3x \left(\frac{\delta F}{\delta e_{ai}(x)} \frac{\delta G}{\delta \Pi^{ai}(x)} - \frac{\delta F}{\delta \Pi^{ai}(x)} \frac{\delta G}{\delta e_{ai}(x)} \right). \tag{15}$$

We recall that the Poisson brackets $\{\Gamma^{ij}(x), \Gamma^{kl}(x)\}$ reproduce the angular momentum algebra.[22]

The constraint C^a is written as $C^a = -\partial_i\Pi^{ai} + h^a$, where h^a is an intricate expression of the field variables. The integral form of the constraint equation $C^a = 0$ motivates the definition of the gravitational energy momentum P^a four-vector[22]

$$P^a = -\int_V \mathrm{d}^3x \partial_i\Pi^{ai}, \tag{16}$$

where V is an arbitrary volume of the three-dimensional space. In the configuration space we have

$$\Pi^{ai} = -4\kappa\sqrt{-g}\Sigma^{a0i},$$
$$\partial_\nu(\sqrt{-g}\Sigma^{a\lambda\nu}) = \frac{1}{4\kappa}\sqrt{-g}e^a{}_\mu(t^{\lambda\mu} + T^{\lambda\mu}), \tag{17}$$

where

$$t^{\lambda\mu} = \kappa\left(4\Sigma^{bc\lambda}T_{bc}{}^{\mu} - g^{\lambda\mu}\Sigma^{bcd}T_{bcd}\right).$$

The emergence of total divergences in the form of scalar or vector densities is possible in the framework of theories constructed out of the torsion tensor. Metric theories of gravity do not share this feature. By making $\lambda = 0$ in Eq. (17) and identifying Π^{ai} in the left side of the latter, the integral form of Eq. (17) is written as

$$P^a = \int_V \mathrm{d}^3x\sqrt{-g}e^a{}_\mu\left(t^{0\mu} + T^{0\mu}\right). \tag{18}$$

Equation (18) suggests that P^a is now understood as the gravitational energy momentum.[22] The spatial component $P^{(i)}$ forms a total three-momentum, while the temporal component $P^{(0)}$ is the total energy.[40]

It is possible to rewrite the Hamiltonian density of Eq. (13) in the equivalent form[41]

$$H = e_{a0}C^a + \frac{1}{2}\lambda_{ab}\Gamma^{ab}, \tag{19}$$

where $\lambda_{ab} = -\lambda_{ba}$ are the Lagrangian multipliers that are identified as $\lambda_{ik} = \alpha_{ik}$ and $\lambda_{0k} = -\lambda_{k0} = \beta_k$. The constraint $\Gamma^{ab} = -\Gamma^{ba}$[39] embodies both constraints Γ^{ik} and Γ^k by means of the relation

$$\Gamma^{ik} = e_a{}^i e_b{}^k\Gamma^{ab}, \quad \Gamma^k \equiv \Gamma^{0k} = e_a{}^0 e_b{}^k\Gamma^{ab}. \tag{20}$$

The constraint Γ^{ab} can be read as

$$\Gamma^{ab} = M^{ab} + 4\kappa\sqrt{-g}e_{(0)}{}^0\left(\Sigma^{a(0)b} - \Sigma^{b(0)a}\right). \tag{21}$$

Similar to the definition of P^a, the integral form of the constraint equation $\Gamma^{ab} = 0$ motivates the new

definition of the spacetime angular momentum. The equation $\Gamma^{ab}=0$ implies

$$M^{ab}=-4\kappa\sqrt{-g}e_c{}^0\left(\Sigma^{acb}-\Sigma^{bca}\right). \tag{22}$$

Maluf *et al.*[22,41] defined

$$L^{ab}=\int_V \mathrm{d}^3x\ e_\mu{}^a e_\nu{}^b M^{\mu\nu} \tag{23}$$

as the 4-angular momentum of the gravitational field for an arbitrary volume V of three-dimensional space. In Einstein–Cartan type theories, there also appear constraints that satisfy the Poisson bracket as given by Eq. (15). However, such constraints arise in the form $\Pi^{[ij]}=0$, and so a definition similar to Eq. (23), i.e., interpreting the constraint equation as an equation for the angular momentum of the field, is not possible. Definition (23) is a three-dimensional integral. The quantities P^a and L^{ab} are separately invariant under general coordinate transformations of three-dimensional space and under time reparametrizations, which is an expected feature since these definitions arise in the Hamiltonian formulation of the theory. Moreover, these quantities transform covariantly under global $SO(3,1)$ transformations.[41]

Now, we are going to calculate the energy, momentum and angular momentum associated with the tetrad field (12). For asymptotically flat space times P^0 yields the ADM energy.[42] In the context of tetrad theories of gravity, asymptotically flat space times may be characterized by the asymptotic boundary condition

$$e_{a\mu}\cong\eta_{a\mu}+\frac{1}{2}h_{a\mu}(1/r), \tag{24}$$

and by the condition $\partial_\mu e^a{}_\mu=O(1/r^2)$ in the asymptotic limit $r\to\infty$. An important property of tetrad fields that satisfy Eq. (24) is that in the flat space time limit one has $e^a{}_\mu(t,x,y,z)=\delta^a{}_\mu$, and therefore the torsion tensor $T^a{}_{\mu\nu}=0$.

Now, we are going to apply Eq. (16) to the tetrad field (12) to calculate the energy content. We perform the calculations in the spherical coordinate. Eq. (12) assumed that the reference space is determined by a set of tetrad fields $e^a{}_\mu$ for the flat space time such that the condition $T^a{}_{\mu\nu}=0$ is satisfied. Using Eq. (5) in Eq. (12), we obtain the non-vanishing components of the torsion tensor as

$$T^{(0)}{}_{01}=\frac{\eta_1'(R)}{\eta_1(R)},\quad T^{(2)}{}_{12}=T^{(3)}{}_{13}=\frac{1-\eta_1(R)}{R\eta_1(R)}, \tag{25}$$

and the non-vanishing component of the tensor $T^{(a)}$ as

$$T^{(1)}=\frac{\eta_1(R)\left(2-2\eta_1(R)-R\eta_1'(R)\right)}{R}, \tag{26}$$

where $\eta_1(R)$ is given by Eq. (11).

Now, we are going to apply Eq. (16) to the tetrad field (12) using Eqs. (25) and (26) to calculate the energy content. We perform the calculations in the spherical coordinate. The only required component of $\Sigma^{\mu\nu\lambda}$ is

$$\Sigma^{(0)01}=\frac{R\sin\theta(1-\eta_1(R))}{4\pi}. \tag{27}$$

Using Eq. (27) in Eq. (16), we obtain

$$\begin{aligned}P^{(0)}&=E=-\oint_{S\to\infty}\mathrm{d}S_k\Pi^{(0)k}\\&=-\frac{1}{4\pi}\oint_{S\to\infty}\mathrm{d}S_k e\Sigma^{(0)0k}=R(1-\eta_1(R))\\&\cong M-\frac{Q^2}{2R}+\frac{\sigma Q^4}{10R^5},\end{aligned} \tag{28}$$

which is a satisfactory result that is obtained when $\sigma=0$.[43]

Now let us apply expression (16) to the evaluation of the irreducible mass by fixing V as the volume within the $R=R_+$ surface, where R_+ is the external horizon, i.e., $=\eta_1(R)=0$. Therefore,

$$\begin{aligned}P^{(0)}=E&=-\int_{S_i}\mathrm{d}S_i\Pi^{(0)i}\\&=-\int_S\mathrm{d}\theta\mathrm{d}\phi\Pi^{(0)1}(r,\theta,\phi),\end{aligned} \tag{29}$$

where the surface S is determined by the condition $R=R_+$. The expression of $\Pi^{(0)1}$ will be obtained by considering Eq. (17) using Eqs. (4) and (5). The expression of $\Pi^{(0)1}(r,\theta,\phi)$ for the tetrad (12) reads

$$\Pi^{(0)1}=\frac{R_+\sin\theta(1-\eta_1(R_+))}{4\pi}. \tag{30}$$

Equation (30) is integrated on the surface of constant radius $R=R_+$, where R_+ is the external horizon of the black hole. On this surface, the second term of Eq. (30) vanishes, i.e., $\eta_1(R_+)$. Therefore, on the surface $R=R_+$, we obtain

$$P^{(0)}=E=R_+, \tag{31}$$

which is a satisfactory result that is obtained before.[22,44]

To calculate the momentum and angular momentum associated with the tetrad field given by Eq. (12), we use Eq. (17) and finally we obtain

$$\Pi^{(1)1}=0. \tag{32}$$

Substituting Eq. (32) into Eq. (16), we obtain

$$P^{(1)} = \int_V \mathrm{d}V\partial_1(\Pi^{(1)1}(r,\theta,\phi)) = \int_S \mathrm{d}S_1\Pi^{(1)1}(r,\theta,\phi) = 0. \quad (33)$$

By the same method, we obtain

$$\Pi^{(2)1} = 0, \quad P^{(2)} = 0, \quad \Pi^{(3)1} = 0, \quad P^{(3)} = 0. \quad (34)$$

The results of Eqs. (33) and (34) are expected since the space time given by Eq. (12) is a spherically symmetric static space time. Therefore, the spatial momentum associated with any static solution is identically vanishing.[1]

The non vanishing components of the tensor $M^{(a)(b)}$ are given by

$$M^{(0)(1)} = \frac{R\sin^2\theta\cos\phi\ \eta_1(R)(1-\eta_1(R))}{4\pi},$$
$$M^{(0)(2)} = \frac{R\sin^2\theta\sin\phi\ \eta_1(R)(1-\eta_1(R))}{4\pi},$$
$$M^{(0)(3)} = \frac{R\sin\theta\cos\theta\ \eta_1(R)(1-\eta_1(R))}{4\pi},$$
$$M^{(1)(2)} = M^{(1)(3)} = M^{(2)(3)} = 0. \quad (35)$$

Using Eq. (35) in (23), we obtain the angular momentum in the form

$$L^{(0)(1)} = \int_0^\pi\int_0^{2\pi}\int_{R_1}^\infty \mathrm{d}\theta\mathrm{d}\phi\mathrm{d}rM^{(0)(1)}(r,\theta,\phi) = \frac{R\sin^2\theta\cos\phi\ \eta_1(R)(1-\eta_1(R))}{4\pi} \times \int_0^\pi\int_0^{2\pi}\int_{R_1}^\infty \mathrm{d}\theta\mathrm{d}\phi\mathrm{d}r = 0, \quad (36a)$$

by the same method we can obtain

$$L^{(0)(2)} = L^{(0)(3)} = L^{(1)(2)} = L^{(1)(3)} = L^{(2)(3)} = 0. \quad (36b)$$

5. Main results and conclusion

The main results of this study are as follows:

1) We have obtained a correction for a charged solution from the first contribution of the Eular–Heesenberg in the TEGR.

2) We have calculated the energy distribution associated with this correction form using the energy momentum tensor which is independent of coordinate. The result of energy is satisfactory[45], since if the correction term, i.e., $\sigma = 0$, the form of energy will be identical to the Reissner–Nordström black hole.

3) We have calculated the spatial momentum and angular momentum associated with the correction form. The results we have obtained are in agreement with previous results related to a spherically symmetric static black hole.[1]

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