Efficient entanglement purification in quantum repeaters*

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(Received 31 October 2011; revised manuscript received 15 November 2011)

We present an efficient entanglement purification protocol (EPP) with controlled-not (CNOT) gates and linear optics. With the CNOT gates, our EPP can reach a higher fidelity than the conventional one. Moreover, it does not require the fidelity of the initial mixed state to satisfy $F > 1/2$. If the initial state is not entangled, it still can be purified. With the linear optics, this protocol can get pure maximally entangled pairs with some probabilities. Meanwhile, it can be used to purify the entanglement between the atomic ensembles in distant locations. This protocol may be useful in long-distance quantum communication.

Keywords: quantum communication, entanglement, entanglement purification, quantum repeater

PACS: 03.67.Dd, 03.67.Hk, 03.65.Ud **DOI:** 10.1088/1674-1056/21/3/030307

1. Introduction

Quantum entanglement is an essential source in quantum communication.[1] For example, in order to achieve a quantum cryptograph,[2−22] the faithful teleportation of unknown quantum states,[23] quantum dense coding,[24,25] quantum secret sharing,[26−29] quantum state sharing,[30−36] and controlled teleportaton,[37,38] people have to set up quantum entanglement channels first. However, with only local operations and the classical communication, the entanglement can only be produced locally. That is, one party in the quantum communication has to prepare one entangled photon pair locally and send a photon from the pair to the other party. Unfortunately, when photons are transmitted in channels, such as a fiber or free space, photon losses and decreases in the quality of entanglement scale exponentially with the length of the communication channel because of optical absorption and channel noise.[39] This will lead to two effects. First, photon transmission has an attenuation length $L_{\rm att}$. The distance L for photon transmission should satisfy $L < L_{\rm att}$. Second, even when the photon arrives at the other party, the fidelity of the transmitted state is degraded, and it may not be suitable for quantum communication processes.

To overcome the above two obstacles, quantum repeaters and quantum entanglement purification (EPP) are proposed. In 1998, Briegel *et al.* first proposed the concept of quantum repeaters.[39,40] In their protocol, the whole long-distance quantum channel is divided into many segments, the length of each segment is comparable to the channel attenuation length $L_{\rm att}$. Entanglement is first generated in each segment, and then the length of entanglement is extended by

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11104159 and 10904074), the Scientific Research Foundation of Nanjing University of Posts and Telecommunications, China (Grant No. NY211008), the University Natural Science Research Foundation of Jiangsu Province, China (Grant No. 11KJA510002), the Open Research Fund of Key Lab of Broadband Wireless Communication and Sensor Network Technology (Nanjing University of Posts and Telecommunications) of Ministry of Education, China, and the Priority Academic Program Development Fund of Jiangsu Higher Education Institutions, China.

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entanglement swapping.[41,42] If entanglement is degraded, the entanglement purification protocol[43] will be used. The entanglement swapping and the purification protocol can be repeated until nearly perfect entangled states are created between two long-distance sites. In 2001, Duan *et al.* developed the above protocol with atomic ensembles and linear optics,[44] which becomes the well-known DLCZ protocol. Recently, the idea has been extended by many groups with the same topological structure in both theory and experiment.[45−59] In particular, in 2007, Zhao *et al.* proposed a protocol for the robust creation of entanglement between remote memory qubits.[54,55]

On the other hand, current entanglement purification protocols are usually used to overcome the other drawback in quantum communication: channel noise.[43] It was first proposed in 1996 by Bennett *et al.* based on the controlled-not (CNOT) gate, called the CNOT protocol.[43] Deustch *et al.* improved the protocol subsequently with the same logical operations.[60] In 2001, Pan *et al.* proposed an entanglement purification protocol based on polarization beam splitters, which is called the PBS protocol.[61−63] There are also some other entanglement purification protocols proposed.[64−80]

In a practical quantum communication process based on quantum repeaters, the channel noise during transmission is not the only source of noise. For example, in the DLCZ protocol, the entanglement connection between the neighbor nodes is probabilistic. That is to say, in order to perform quantum swapping, the neighbor nodes that have been successfully connected should wait until the other nodes are all connected successfully. During the waiting time, the entanglement is usually stored in the atom ensembles, and the entanglement will degrade because of the environment. Generally speaking, for the long-distance quantum communication, the whole communication distance is divided into many segments to make the distance between nodes satisfy $L \ll L_{\rm att}$. In this case, the photon transmission distances are short, the noise coming from the quantum memory becomes the major source of noise.

Though the existing EPPs usually focus on the noise coming from the channel transmission, in fact, they can also deal with other noise in the local operation and classical communication (LOCC). They are universal methods to improve the entanglement quality. Interestingly, if the noise mainly comes from the entanglement memory, and the quantum channel is relatively good, we can propose another EPP, which is more efficient than conventional ones. In this paper, we present a different EPP with both the LOCC and the collective measurements. The basic idea is that, after Alice and Bob perform local operations on the particles, one of the target particles is sent back for a collective measurement. According to the measurement result, they decide whether to remain or discard the source pair.

This paper is organized as follows. In Section 2, we briefly explain our EPP with the CNOT gates. We show that this scheme is more efficient than the original CNOT protocol.[43] Interestingly, different from conventional EPPs, we do not need the initial fidelity of the entangled pair to satisfy $F > \frac{1}{2}$. That is to say, if the initial pairs are not entangled, this scheme can also work well. In Section 3, we show that this protocol combined with the PBSs and the atom ensembles can get a nearly deterministic purification effect, which makes this protocol be more useful in quantum repeaters. Finally, we give the discussion and the conclusion.

2. EPP with CNOT gates

We show our purification scheme by discussing a specific example, which is similar to that in the CNOT protocol.[43] Suppose Alice and Bob would like to share two-photon pairs in a mixed state, i.e., a Werner state

$$\begin{aligned} W_F = {} & F|\Psi^-\rangle\langle\Psi^-| + \frac{1-F}{3}|\Psi^+\rangle\langle\Psi^+| \\ & + \frac{1-F}{3}|\Phi^+\rangle\langle\Phi^+| + \frac{1-F}{3}|\Phi^-\rangle\langle\Phi^-|, \quad (1) \end{aligned}$$

where

$$\begin{aligned} |\Phi^\pm\rangle &= \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \\ |\Psi^\pm\rangle &= \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle). \quad (2) \end{aligned}$$

Equation (1) has another form

$$\begin{aligned} W_F' = {} & F|\Phi^+\rangle\langle\Phi^+| + \frac{1-F}{3}|\Psi^+\rangle\langle\Psi^+| \\ & + \frac{1-F}{3}|\Phi^-\rangle\langle\Phi^-| + \frac{1-F}{3}|\Phi^-\rangle\langle\Phi^-|, \quad (3) \end{aligned}$$

which can be obtained from Eq. (1) through unilateral Pauli rotations σ_y. In fact, any mixed state can be transformed into a Werner state with random bilateral rotations.[43]

The principle of our EPP is shown in Fig. 1. First, one photon from each pair is sent to Alice, and

the other one is sent to the party at the neighboring node of the quantum communication, say Bob. Then, both Alice and Bob pick up two photons and perform CNOT operations on their photons independently. Subsequently, Alice sends her target particle to Bob and lets Bob make a collective measurement to pick up $|\Phi^+\rangle$. If the measurement result of Bob is $|\Phi^+\rangle$, Bob asks Alice through the classical communication to keep the source pair, which will have a higher degree of entanglement than the original two less-entangled pairs. Otherwise, they both discard the two photon pairs. If Alice and Bob start with the pairs produced in the previous entanglement purification step, they can obtain arbitrarily high entangled pairs by repeating the above procedure.

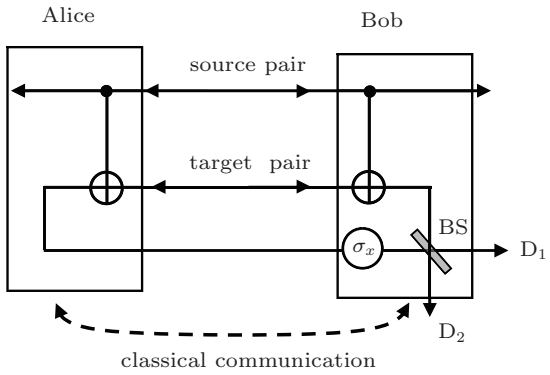


Fig. 1. Schematic diagram showing the principle of our entanglement purification protocol. A photon pair with a higher fidelity is created starting from two less-entangled pairs. BS (50:50) presents a beam splitter, D_1 and D_2 represent two detectors.

With the EPP in Ref. [43], Alice and Bob also pick up two pairs of photons in the state shown in Eq. (3) in each round of purification. After performing CNOT operations on their photons independently, they will measure the target pair and judge whether their results are the same or different. If they have the same result, from Table 1, they will obtain a new mixed state with fidelity $F' > F$, where[43]

$$F' = \frac{F^2 + \frac{1}{9}(1-F)^2}{F^2 + \frac{2}{3}F(1-F) + \frac{5}{9}(1-F)^2}. \tag{4}$$

Our EPP is quite different from that in Ref. [43]. After they both perform the CNOT operations, Alice and Bob do not measure the target pair locally, and Alice sends her target particle to Bob to make a collective measurement. In Fig. 1, we take the optical system as an example for describing the principle of our EPP. The beam splitter (BS 50:50) can divide a particle into two spatial modes. Suppose the particles are photons, the BS will scatter two particles into different arms if they are in the singlet state $|\Psi^-\rangle$. If both detectors D_1 and D_2 click, the target pair is $|\Phi^+\rangle$, then Bob should ask Alice to keep the source pair, and they will get a new mixed state with $F'' > F$, where

$$F'' = \frac{F^2}{F^2 + \frac{1}{3}F(1-F) + \frac{2}{9}(1-F)^2}. \tag{5}$$

Otherwise, Bob should ask Alice to discard the source pair. Certainly, with the help of the CNOT gates described above, the four Bell states can be distinguished deterministically. After performing the Bell-state measurement, if the state is $|\Phi^-\rangle$ or $|\Psi^\pm\rangle$, Bob and Alice both need to discard both pairs. Therefore, a complete Bell-state measurement is not necessary, and they only need to pick up the Bell state $|\Phi^+\rangle$. The success probability is given by $P = F^2 + \frac{1}{3}F(1-F) + \frac{2}{9}(1-F)^2$.

Table 1. Relations of the states before and after Alice and Bob performing CNOT operations.

Initial state		Final state	
Source	Target	Source	Target
$\|\Phi^\pm\rangle$	$\|\Phi^+\rangle$	$\|\Phi^\pm\rangle$	$\|\Phi^+\rangle$
$\|\Phi^\pm\rangle$	$\|\Phi^-\rangle$	$\|\Phi^\mp\rangle$	$\|\Phi^-\rangle$
$\|\Psi^\pm\rangle$	$\|\Psi^+\rangle$	$\|\Psi^\pm\rangle$	$\|\Phi^+\rangle$
$\|\Psi^\pm\rangle$	$\|\Psi^-\rangle$	$\|\Psi^\mp\rangle$	$\|\Phi^-\rangle$
$\|\Phi^\pm\rangle$	$\|\Psi^+\rangle$	$\|\Phi^\pm\rangle$	$\|\Psi^+\rangle$
$\|\Phi^\pm\rangle$	$\|\Psi^-\rangle$	$\|\Phi^\mp\rangle$	$\|\Psi^-\rangle$
$\|\Psi^\pm\rangle$	$\|\Phi^+\rangle$	$\|\Psi^\pm\rangle$	$\|\Psi^+\rangle$
$\|\Psi^\pm\rangle$	$\|\Phi^-\rangle$	$\|\Psi^\mp\rangle$	$\|\Psi^-\rangle$

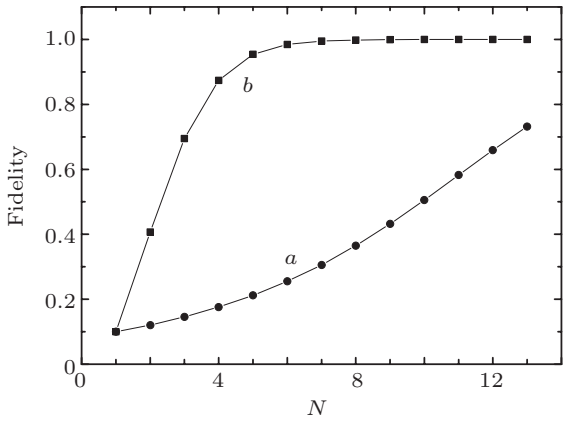


Fig. 2. Fidelity changes with iteration number N of EPPs. Curves a and b show the relations obtained in Ref. [43] and this paper, respectively. The initial fidelity is 0.55.

It is interesting to compare Eq. (4) with Eq. (5). In Fig. 2, we let the initial fidelity of the Werner state be $F = 0.55$, and we repeat the two protocols 12

times each. In the CNOT protocol,[43] the final fidelity is $F' = 0.8659$, while $F'' = 0.999996$ in our protocol. The numerical simulation also shows that for five-times purification, our EPP protocol can reach a fidelity of 0.992, while the fidelity of the CNOT protocol is only 0.62. In the CNOT protocol, the initial fidelity is required to satisfy $F > \frac{1}{2}$, while only $F > \frac{1}{4}$ is needed in the new protocol. That is, if the initial state is not entangled, after performing our protocol, the fidelity can also be increased.

3. EPP with atomic ensembles

So far, we have briefly explained our protocol with the CNOT gates. Compared with the CNOT protocol,[43] the most obvious difference is that in the CNOT protocol, Alice and Bob measure their target particles locally along the z spin axis, which is used to distinguish states $|\Phi^{\pm}\rangle$ from $|\Psi^{\pm}\rangle$. In our protocol, the σ_x operation and the BS are used to distinguish $|\Phi^{+}\rangle$ from the other three Bell states, it is essentially the Bell-state measurement. The EPP described above can greatly improve the fidelity of the initial one. However, it cannot get pure maximally entangled states. In this section, we show that with the help of linear optics, this EPP will become more powerful in quantum repeaters.

Now we take the quantum repeater protocol in Ref. [54] as an example to describe the principle of our EPP. Here we call it the Zhao protocol for simplicity. In Fig. 3, two pairs of maximally entangled states between the neighboring nodes A and B are in the form[54]

$$|\phi^{+}\rangle_{\rm AB} = \frac{1}{\sqrt{2}}(S^{\dagger}_{\rm hu}S^{\dagger}_{\rm hu} + S^{\dagger}_{\rm vd}S^{\dagger}_{\rm vd})|0\rangle_{\rm AB}, \qquad (6)$$

where $S^{\dagger}$ is the photon creation operator, $|0\rangle$ is the vacuum state, subscripts h and v represent horizontal and vertical photon polarizations, respectively, subscripts u and d represent up and down atomic ensembles, respectively, which is similar to that in Ref. [54]. The imperfect atomic ensemble memories will make the state in Eq. (6) possess a bit-flip error with a probability of $1-F$, then state $|\phi^{+}\rangle_{\rm AB}$ becomes

$$|\psi^{+}\rangle_{\rm AB} = \frac{1}{\sqrt{2}}(S^{\dagger}_{\rm hu}S^{\dagger}_{\rm vd} + S^{\dagger}_{\rm vd}S^{\dagger}_{\rm hu})|0\rangle_{\rm AB}. \qquad (7)$$

The mixed state of the whole system can be described as

$$\rho = F|\phi^{+}\rangle_{\rm AB}\langle\phi^{+}| + (1-F)|\psi^{+}\rangle_{\rm AB}\langle\psi^{+}|. \qquad (8)$$

From Eq. (8), we can see that the whole system can be viewed as a probabilistic mixture of four pure states $|\phi^{+}\rangle_{\rm A_1B_1}|\phi^{+}\rangle_{\rm A_2B_2}$, $|\phi^{+}\rangle_{\rm A_1B_1}|\psi^{+}\rangle_{\rm A_2B_2}$, $|\psi^{+}\rangle_{\rm A_1B_1}|\phi^{+}\rangle_{\rm A_2B_2}$, and $|\psi^{+}\rangle_{\rm A_1B_1}|\psi^{+}\rangle_{\rm A_2B_2}$ with the probabilities of F^2, $F(1-F)$, $F(1-F)$, and F^2, respectively. The first step of our EPP is to convert the entanglement in the atomic ensembles to the optical ensembles. That is, with the help of reading laser pulses,[54] the whole mixed state in Eq. (8) can become an optical mode in the form of

$$\rho = F|\Phi^{+}\rangle_{\rm AB}\langle\Phi^{+}| + (1-F)|\Psi^{+}\rangle_{\rm AB}\langle\Psi^{+}|, \qquad (9)$$

where

$$|\Phi^{+}\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle),$$

and

$$|\Psi^{+}\rangle = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle),$$

with $|H\rangle$ and $|V\rangle$ being the horizontal and vertical polarized photo states, respectively. With this transformation, the whole state can be viewed as a probabilistic mixture of four pure states $|\Phi^{+}\rangle_{\rm A_1B_1}|\Phi^{+}\rangle_{\rm A_2B_2}$, $|\Phi^{+}\rangle_{\rm A_1B_1}|\Psi^{+}\rangle_{\rm A_2B_2}$, $|\Psi^{+}\rangle_{\rm A_1B_1}|\Phi^{+}\rangle_{\rm A_2B_2}$, and $|\Psi^{+}\rangle_{\rm A_1B_1}|\Psi^{+}\rangle_{\rm A_2B_2}$ with the probabilities of F^2, $F(1-F)$, $F(1-F)$, and $(1-F)^2$, respectively. It is essentially similar to the purification protocol in Refs. [61] and [62].

Following the same step as in Refs. [61] and [62] and selecting the four-mode cases, we obtain exactly one photon of the four spatial output modes in $\rm PBS_1$ and $\rm PBS_2$. The cross-combinations of $|\Phi^{+}\rangle_{\rm A_1B_1}|\Psi^{+}\rangle_{\rm A_2B_2}$ and $|\Psi^{+}\rangle_{\rm A_1B_1}|\Phi^{+}\rangle_{\rm A_2B_2}$ cannot lead the four-mode cases, then the remaining system is in the state

$$\begin{aligned}|\phi\rangle &= \frac{1}{\sqrt{2}}(|H\rangle_{\rm c_1}|H\rangle_{\rm d_1}|H\rangle_{\rm c_2}|H\rangle_{\rm d_2} \\ &\quad + |V\rangle_{\rm c_1}|V\rangle_{\rm d_1}|V\rangle_{\rm c_2}|V\rangle_{\rm d_2}) \\ &= \frac{1}{\sqrt{2}}\left[(|H\rangle_{\rm c_1}|H\rangle_{\rm d_1})\frac{|\Phi^{+}\rangle_{\rm c_2d_2} + |\Phi^{-}\rangle_{\rm c_2d_2}}{\sqrt{2}}\right. \\ &\quad \left. + |V\rangle_{\rm c_1}|V\rangle_{\rm d_1}\frac{|\Phi^{+}\rangle_{\rm c_2d_2} - |\Phi^{-}\rangle_{\rm c_2d_2}}{\sqrt{2}}\right] \\ &= \frac{1}{\sqrt{2}}(|\Phi^{+}\rangle_{\rm c_1d_1}|\Phi^{+}\rangle_{\rm c_2d_2} + |\Phi^{-}\rangle_{\rm c_1d_1}|\Phi^{-}\rangle_{\rm c_2d_2}) \qquad (10)\end{aligned}$$

with a probability of F^2 and in the state

$$\begin{aligned}|\psi\rangle &= \frac{1}{\sqrt{2}}(|H\rangle_{\rm c_1}|V\rangle_{\rm d_1}|H\rangle_{\rm c_2}|V\rangle_{\rm d_2} \\ &\quad + |V\rangle_{\rm c_1}|H\rangle_{\rm d_1}|V\rangle_{\rm c_2}|H\rangle_{\rm d_2}) \\ &= \frac{1}{\sqrt{2}}\left[(|H\rangle_{\rm c_1}|V\rangle_{\rm d_1})\frac{|\Psi^{+}\rangle_{\rm c_2d_2} + |\Psi^{-}\rangle_{\rm c_2d_2}}{\sqrt{2}}\right. \\ &\quad \left. + |V\rangle_{\rm c_1}|H\rangle_{\rm d_1}\frac{|\Psi^{+}\rangle_{\rm c_2d_2} - |\Psi^{-}\rangle_{\rm c_2d_2}}{\sqrt{2}}\right] \\ &= \frac{1}{\sqrt{2}}(|\Psi^{+}\rangle_{\rm c_1d_1}|\Psi^{+}\rangle_{\rm c_2d_2} + |\Psi^{-}\rangle_{\rm c_1d_1}|\Psi^{-}\rangle_{\rm c_2d_2}) \qquad (11)\end{aligned}$$

with a probability of $(1-F)^2$.

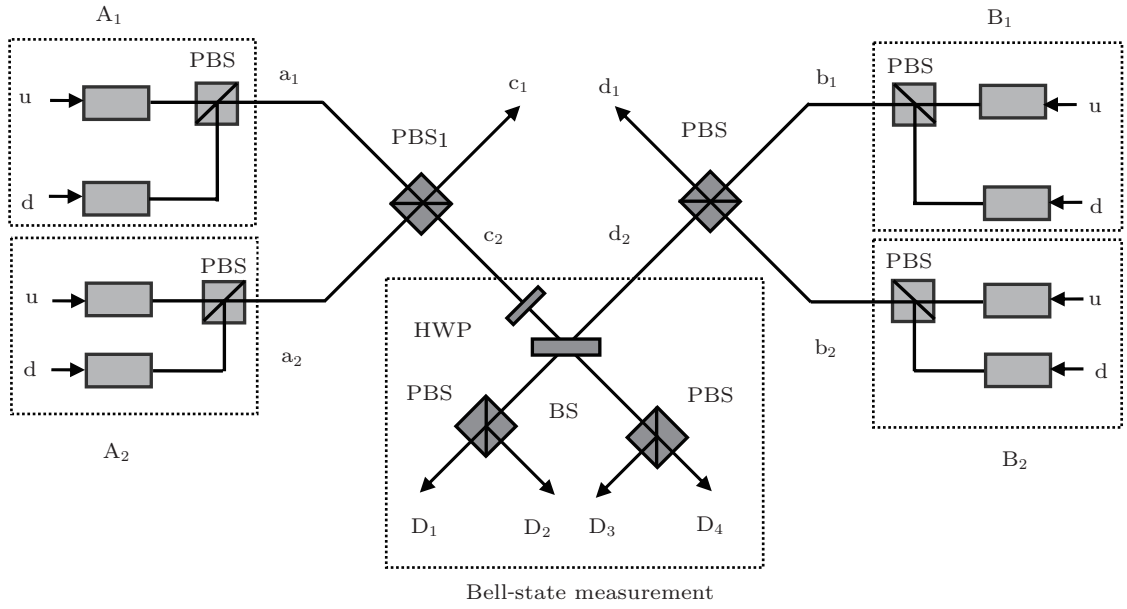


Fig. 3. Setup for entanglement purification in nodes A and B. The effectively maximally entangled state may be imperfect in the memory. The mixed state is first converted to a linear-optics entangled state by reading laser pulses. Then the two pairs are subjected to PBSs respectively. After that, the two lower modes are sent to the middle to make a Bell-state measurement. Finally, the photons in up modes are projected into an effectively maximally entangled state. The maximally entangled pair can be restored into the memory qubit for further manipulation.

From Eqs. (10) and (11), it is obvious that if a complete Bell-state measurement can be performed on c_2d_2, we can obtain a maximally entangled pair deterministically. However, a complete Bell-state analysis cannot be fulfilled by linear optics alone, the optimal probability of success obtained is only $\frac{1}{2}$.[81,82] Fortunately, it does not affect the fidelity of the remaining pairs, it only affects the success probability. In Fig. 3, the setup of the Bell-state measurement is located at the middle of nodes A and B. It is used to discriminate $|\Phi^{\pm}\rangle_{c_2d_2}$ from the other three Bell states. If D_1, D_2 or D_3, D_4 fires, the state of c_2d_2 collapses to $|\Phi^{+}\rangle$. If D_1, D_3 or D_2, D_4 fire, the state collapses to $|\Phi^{-}\rangle$. Finally, we can judge the state in mode a_1b_1 according to the relationship between the two pairs. Here we can only discriminate $|\Phi^{\pm}\rangle$, so the whole success probability is $p = F^2/2$.

So far, we have described the protocol for bit-flip errors. Phase-flip errors can be transformed into bit-flip errors and can be purified in a subsequent purification step.[61,62] In this way, we can implement a complete purification scheme for a general mixed state.

It is interesting to compare this protocol with the PBS protocol.[61] In the PBS protocol, by selecting the four-mode cases, we can create a new ensemble with the fidelity

$$F' = \frac{F^2}{F^2 + (1-F)^2}. \tag{12}$$

It needs the initial fidelity to satisfy $F > \frac{1}{2}$, because the LOCC cannot increase the entanglement and the initial state should be an entangled one. In our protocol, we send the target pair to one party in the quantum communication to make a Bell-state measurement, which means that the whole process is not a local operation but a collective one. So the process can create and increase the entanglement. If the initial mixed state is not an entangled one, i.e., $F = \frac{1}{2}$, we can also get an entangled pair with the success probability of $\frac{1}{8}$.

4. Discussion and conclusion

So far, we have described our EPP protocol using both CNOT gates and linear optics. Our protocol shows that with CNOT gates, the fidelity of the purified entangled state increases more rapidly than the original one.[43] Moreover, if the initial state is not entangled, the new protocol can also work well. It is known that the LOCC cannot increase the entanglement. Therefore, the original CNOT protocol requires an entangled initial mixed state, that is $F > \frac{1}{2}$. After performing the protocol, at least one of the initial

pairs should be destroyed. The entanglement of the destroyed pair is essentially added into the source pair if the protocol is successful. That is to say, this kind of EPP essentially is an entanglement transfer process. In our EPP, the target pair is sent back to Bob for making a Bell-state measurement after the CNOT operations. So our EPP is not based on the LOCC. The Bell-state measurement is a collective measurement, and it can create entanglement. This is the reason that our EPP does not require the initial state to be entangled. Interestingly, with linear optics, our EPP can obtain a deterministic pure entangled pair with a successful Bell-state measurement. From Eq. (10), if the result of the Bell-state measurement is $|\Phi^+\rangle$, the remaining state is also $|\Phi^+\rangle$. If the result of the Bell-state measurement is $|\Phi^-\rangle$, the remaining state is also $|\Phi^-\rangle$. Therefore, if the bit-flip error can be corrected, the phase-flip error can also be corrected in the next step. In this way, we can implement the EPP for a general mixed state.

In the protocol, we suppose that the noise comes form the quantum memory and the quantum channel is perfect. In a practical quantum communication process, the perfect quantum channel is not available, and the entangled pair cannot be prohibited to interact with the noisy environment, which will limit the application of this protocol in the long-distance quantum communication. Certainly, if the quantum channel is relative good, this protocol can also work well. Interestingly, Reichle *et al.*[80] reported their experimental result about the purification of two-atom entanglement in 2006. In their experiment, four $^9Be^+$ ions in one trapping zone of a linear multi-zone Paul trap were confined. They could form a linear array along the weakest axis of the trapping potential. The general single-qubit rotations and the phase gates were carried out by using Raman laser beams. The positions of the ions relative to the trap center were ~ 7.31 μm. The whole purification principle followed the CNOT protocol.[43] In that case, the noise usually came from the memory of the solid qubit, and our EPP can be good for that task.

In conclusion, we have presented an EPP based on CNOT gates and linear optics. In this protocol, we adopt the collective measurement to distinguish the target being in one of the four Bell states. Compared with the CNOT protocol, this protocol is more efficient and can increase the fidelity rapidly. Combined with the atomic ensembles, this protocol seems to be more powerful. It can get a pure maximally entangled pair, which is difficult to obtain with the conventional protocol. In the practical quantum communication, if the quantum channel is relatively good and the memory noise is the main noise, this protocol can work well. This protocol may be useful in current quantum information processing.

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