Controlling quantum discord dynamics in cavity QED systems by applying a classical driving field with phase decoherence*

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We investigate a two-level atom interacting with a quantized cavity field and a classical driving field in the presence of phase decoherence and find that a stationary quantum discord can arise in the interaction of the atom and cavity field as the time turns to infinity. We also find that the stationary quantum discord can be increased by applying a classical driving field. Furthermore, we explore the quantum discord dynamics of two identical non-interacting two-level atoms independently interacting with a quantized cavity field and a classical driving field in the presence of phase decoherence. Results show that the quantum discord between two atoms is more robust than entanglement under phase decoherence and the classical driving field can help to improve the amount of quantum discord of the two atoms.

Keywords: stationary quantum discord, classical driving field, phase decoherence

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1. Introduction

It is well known that entanglement[1] is a special quantum correlation that has a non classical interpretation and has been recognized as an essential resource in quantum information processing.[2] The experimental demonstrations of two-particle entanglement and multi-particle entanglement in cavity quantum electrodynamics (QED) have been reported.[3,4] Some applications of entanglement or nonclassical correlations have also been realized in recent experiments.[5−7] However, some recent studies show that there are other kinds of quantum correlations that can offer support for the quantum tasks. It has been found both theoretically[8,9] and experimentally[10,11] that another nonclassical correlation, namely, quantum discord,[12] can be responsible for the computational speedup of certain quantum tasks.[13−16] Quantum discord, defined as the difference between quantum mutual information and classical correlation as introduced by Ollivier and Zurek,[12] is a general measure of quantum correlation. The main difference between quantum discord and entanglement is the fact that the quantum discord is nonzero even for separate mixed states. Merali has pointed out that discord could be like sunlight, which is plentiful but has to be harnessed in certain way to be useful.[17] Therefore, quantum discord may be regarded as a more general and fundamental resource for quantum information processing.

On the other hand, a real quantum system will unavoidably be influenced by its environment. The interaction between the quantum system and its environment leads to a rapid destruction of quantum coherence, which is the main problem for the realization of quantum information processing. Therefore, understanding the dynamics of quantum correlations can help us find their potential applications in quantum information theory. Recently, the dynamics of quantum discord for some open systems has attracted much attention.[18−20] It has been shown that discord is more robust than entanglement under Markovian environments[18] and it vanishes only at some time points under a non-Markovian environment.[19] Interestingly, quantum discord can be completely unaffected by a certain decoherence environment during an initial time interval[20] and this phenomenon has already been observed in a recent experiment.[21]

In this paper, we study quantum discord dynamics in cavity QED systems driven by a classical driving field with phase decoherence. The rest of the paper is organized as follows: In Section 2, we investigated quantum discord dynamics between a two-level atom and the cavity QED field driven by a classical driving field in the presence of phase decoherence. In Sec-

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tion 3, we explored the quantum discord dynamics of two identical non-interacting two-level atoms independently interacting with cavity QED and a classical driving field with phase decoherence. A conclusion is given in Section 4.

2. Quantum discord dynamics between atom and cavity driven by classical fields with phase decoherence

In this section, we consider one atom interacting with a quantized cavity field driven by a classical field in the presence of phase decoherence. The Hamiltonian of the system can be described as[22]

$$\begin{aligned} H =& \omega a^\dagger a + \frac{\omega_0}{2}\sigma_z + g(\sigma_+ a + \sigma_- a^\dagger) \\ &+ \lambda(\mathrm{e}^{-\mathrm{i}\,\omega_c t}\sigma_+ + \mathrm{e}^{\mathrm{i}\,\omega_c t}\sigma_-), \end{aligned} \tag{1}$$

where ω, ω_0, and ω_c are the frequency of the cavity, atom and classical field, respectively. The operators σ_z and $\sigma_\pm$ are defined by $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ and $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$, where $|e\rangle$ and $|g\rangle$ are the excited and ground states of the atom. Here, a and $a^\dagger$ are annihilation and creation operators of the cavity; g and λ are the coupling constants of the interactions of the atom with the cavity and with the classical driving field, respectively. Note that we have set $\hbar = 1$ throughout this paper for simplicity.

Through a unitary transformation $U = U_2U_1 = \exp\left(\frac{\mathrm{i}\,\omega_c t}{2}(\sigma_+ + \sigma_-)\right)\exp\left(-\frac{\mathrm{i}\,\omega_c t}{2}\sigma_z\right)$ and using a method similar to that in Ref. [23], the Hamiltonian of Eq. (1) is transformed to

$$\begin{aligned} H' =& U^\dagger H U - \mathrm{i}\,U^\dagger\frac{\partial U}{\partial t} \\ =& \omega a^\dagger a + \frac{\omega'\sin\theta}{2}(\sigma_+ + \sigma_-) + g'\Big[\Big(-\frac{\sin\theta}{2}\sigma_z \\ &+ \cos^2\frac{\theta}{2}\sigma_+ - \sin^2\frac{\theta}{2}\sigma_-\Big)a + \mathrm{h.c}\Big], \end{aligned} \tag{2}$$

where h.c stands for Hermitian conjugate, $\omega' = \omega_c + \sqrt{(\omega_0-\omega_c)^2 + 4\lambda^2}$, $g' = g\cos^2\frac{\theta}{2}$, and $\theta = \arctan(\frac{2\lambda}{\omega_0-\omega_c})$. We consider the pure phase decoherence mechanism only.[24] In this situation, the master equation governing the time evolution for the quantum system with phase decoherence is given by[24]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\,\mathrm{i}[H',\rho] - \frac{\gamma}{2}[H',[H',\rho]], \tag{3}$$

where γ is the phase decoherence coefficient. Equation (5) reduces to the ordinary von Neuman equation when γ becomes 0. The formal solution of the master equation can be expressed as[24]

$$\rho(t) = \sum_{k=0}^{\infty}\frac{(\gamma t)^k}{k!}M^k(t)\rho(0)M^{\dagger k}(t), \tag{4}$$

where $\rho(0)$ is the initial density operator of the system and $M^k(t)$ is defined by

$$M^k(t) = H'^k\exp(-\,\mathrm{i}\,H't)\exp(-\frac{\gamma t}{2}H'^2). \tag{5}$$

With the help of an $SU(2)$ dynamical algebraic structure,[25] we can rewrite H' as

$$H' = \Delta S_0 + \omega\Big(K - \frac{1}{2}\Big) + g'\sqrt{K}(S_+ + S_-), \tag{6}$$

with $\Delta = \omega' - \omega$, where

$$K = a^\dagger a + \frac{1}{2} + \frac{1}{2}\cos\theta\sigma_z + \frac{1}{4}\sin\theta(\sigma_+ + \sigma_-) \tag{7}$$

is a constant of motion which commutes with the Hamiltonian H' in Eq. (2) and the operators $S_i (i = 0, \pm)$ are defined as follows:

$$\begin{aligned} S_+ &= \frac{a}{\sqrt{K}}\Big(-\frac{\sin\theta}{2}\sigma_z + \cos^2\frac{\theta}{2}\sigma_+ - \sin^2\frac{\theta}{2}\sigma_-\Big), \\ S_- &= \frac{a^\dagger}{\sqrt{K}}\Big(-\frac{\sin\theta}{2}\sigma_z + \cos^2\frac{\theta}{2}\sigma_- - \sin^2\frac{\theta}{2}\sigma_+\Big), \\ S_0 &= \frac{1}{2}\cos\theta\sigma_z + \frac{1}{4}\sin\theta(\sigma_+ + \sigma_-). \end{aligned} \tag{8}$$

It can be shown that the operators $S_i (i = 0, \pm)$ are the generators of the $SU(2)$ algebra, which satisfy the following commutation relations:

$$[S_0, S_\pm] = \pm S_\pm, [S_+, S_-] = 2S_0. \tag{9}$$

With the help of the $SU(2)$ dynamical algebraic structure, we can diagonalize the Hamiltonian in Eq. (6) and obtain the explicit expression for the operator $M^k(t)$ in Eq. (5) as

$$\begin{aligned} M^k(t) =& \frac{1}{2}\Big\{f_+(K)^k\exp[-\,\mathrm{i}\,f_+(K)t]\exp\Big[-\frac{\gamma f_+(K)^2 t}{2}\Big] + f_-(K)^k\exp[-\,\mathrm{i}\,f_-(K)t]\exp\Big[-\frac{\gamma f_-(K)^2 t}{2}\Big]\Big\} \\ &+ \Big\{f_+(K)^k\exp[-\,\mathrm{i}\,f_+(K)t]\exp\Big[-\frac{\gamma f_+(K)^2 t}{2}\Big] - f_-(K)^k\exp[-\,\mathrm{i}\,f_-(K)t]\exp\Big[-\frac{\gamma f_-(K)^2 t}{2}\Big]\Big\} \\ &\otimes\Big\{\frac{\Delta}{\Omega_K}\Big[\frac{1}{2}\cos\theta\sigma_z + \frac{1}{4}\sin\theta(\sigma_+ + \sigma_-)\Big] + \frac{g'}{\Omega_K}\Big[a\Big(-\frac{\sin\theta}{2}\sigma_z + \cos^2\frac{\theta}{2}\sigma_+ - \sin^2\frac{\theta}{2}\sigma_-\Big) \\ &+ a^\dagger\Big(-\frac{\sin\theta}{2}\sigma_z + \cos^2\frac{\theta}{2}\sigma_- - \sin^2\frac{\theta}{2}\sigma_+\Big)\Big]\Big\}, \end{aligned} \tag{10}$$

with

$$\Omega_K = \sqrt{\Delta^2 + 4Kg'^2},$$
$$f_\pm(K) = \omega\Big(K - \frac{1}{2}\Big) \pm \frac{1}{2}\Omega_K. \tag{11}$$

We assume that the atom is initially prepared in the dressed state $|+\rangle = \cos\frac{\theta}{2}|e\rangle + \sin\frac{\theta}{2}|g\rangle$ with $\theta = \arctan[2\lambda/(\omega_0 - \omega)]$ and the cavity field is initially prepared in the fock state $|n\rangle$. Then, the initial density operator of the system reads

$$\rho(0) = |+\rangle|n\rangle\langle+|\langle n|. \tag{12}$$

By making use of Eqs. (4), and (10)–(12), we obtain the density matrix of the whole system at time t by means of the standard basis $|-\rangle|n\rangle$, $|-\rangle|n+1\rangle$, $|+\rangle|n\rangle$, $|+\rangle|n+1\rangle$ (where $|+\rangle = \cos\frac{\theta}{2}|e\rangle + \sin\frac{\theta}{2}|g\rangle$, $|-\rangle = -\sin\frac{\theta}{2}|e\rangle + \cos\frac{\theta}{2}|g\rangle$), as follows:

$$\rho(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix},$$

where

$$\begin{aligned}
\rho_{11} &= \rho_{44} = \rho_{14} = \rho_{41} = 0, \\
\rho_{22} &= \frac{2g'^2(n+1)}{\Omega_{n+1}^2}\Big[1 - \cos(\Omega_{n+1}t)\exp\Big(-\frac{\gamma\Omega_{n+1}^2}{2}t\Big)\Big], \\
\rho_{33} &= \frac{1}{2}\Big[1 + \frac{\Delta^2}{\Omega_{n+1}^2} + \Big(1 - \frac{\Delta^2}{\Omega_{n+1}^2}\Big)\cos(\Omega_{n+1}t) \\
&\quad \times \exp\Big(-\frac{\gamma\Omega_{n+1}^2}{2}t\Big)\Big], \\
\rho_{32} &= (\rho_{23})^* \\
&= \frac{g'\sqrt{n+1}}{\Omega_{n+1}}\Big\{\frac{\Delta}{\Omega_{n+1}} + \Big[\mathrm{i}\sin(\Omega_{n+1}t) \\
&\quad - \frac{\Delta}{\Omega_{n+1}}\cos(\Omega_{n+1}t)\Big]\exp\Big(-\frac{\gamma\Omega_{n+1}^2}{2}t\Big)\Big\}.
\end{aligned} \tag{13}$$

It is obvious that the density matrix $\rho(t)$ is belongs to the class of X structure states. The advantage of X structure states is that, it is easy to calculate the quantum discord and entanglement, and it keeps the X structure unchanged during time evolution.[26]

Next, we study the quantum discord dynamics of a two-level atom interacting with quantized cavity field and classical driving field in the presence of phase decoherence. The definition of quantum discord is based on quantum mutual information which contains both classical and quantum correlations. For a bipartite system ρ^{AB}, its total correlation can be measured by quantum mutual information[12]

$$I(\rho^{AB}) = S(\rho^A) + S(\rho^B) - S(\rho^{AB}), \tag{14}$$

where $S(\rho) = -\mathrm{Tr}(\rho\log_2\rho)$ is the von Neumann entropy, and ρ^A and ρ^B denote the reduced density matrices of parts A and B, respectively. The quantum discord is defined as the difference between the quantum mutual information and the classical correlation[12]

$$Q(\rho^{AB}) = I(\rho^{AB}) - C(\rho^{AB}), \tag{15}$$

where $C(\rho^{AB})$ is the classical correlation which depends on the maximal information obtained with measurement on one of the subsystems and can be expressed as[26]

$$C(\rho^{AB}) = \max_{\{B_k\}}[S(\rho^A) - S(\rho^{AB}|\{B_k\})], \tag{16}$$

where $\{B_k\}$ is a complete set of projectors preformed on subsystem B locally, $S(\rho^{AB}|\{B_k\}) = \sum_k p_k S(\rho_k)$ is the quantum conditional entropy, $\rho_k = 1/p_k(I\otimes B_k)\rho^{AB}(I\otimes B_k)$ is the conditional density operator and $p_k = \mathrm{tr}_{(AB)}[(I\otimes B_k)\rho^{AB}(I\otimes B_k)]$ is the probability.

The eigenvalues of the density matrix $\rho(t)$ can be derived as

$$\begin{aligned}
\lambda_{1,2} &= \frac{1}{2}\Big[\rho_{11} + \rho_{44} \pm \sqrt{(\rho_{11} - \rho_{44})^2 + 4|\rho_{14}|^2}\Big], \\
\lambda_{3,4} &= \frac{1}{2}\Big[\rho_{22} + \rho_{33} \pm \sqrt{(\rho_{22} - \rho_{33})^2 + 4|\rho_{23}|^2}\Big].
\end{aligned} \tag{17}$$

The quantum mutual information between the atom and cavity can be calculated as follows:

$$I(\rho(t)) = S(\rho^{(1)}(t)) + S(\rho^{(2)}(t)) + \sum_{i=1}^{4}\lambda_i\log_2\lambda_i, \tag{18}$$

where

$$\begin{aligned}
(\rho^{(1)}(t)) = &-(\rho_{11} + \rho_{22})\log_2(\rho_{11} + \rho_{22}) \\
&-(\rho_{33} + \rho_{44})\log_2(\rho_{33} + \rho_{44}), \\
(\rho^{(2)}(t)) = &-(\rho_{11} + \rho_{33})\log_2(\rho_{11} + \rho_{33}) \\
&-(\rho_{22} + \rho_{44})\log_2(\rho_{22} + \rho_{44}),
\end{aligned}$$

where $\rho^i(t)$ is the reduced density matrix of the ith partite. By substituting $\rho_{ij}(i,j = 1,2,3,4)$ of Eq. (13) into Eq. (14)–(18), the quantum discord $D_1(\rho(t))$ and the quantum mutual information $I(\rho(t))$ for the system can be obtained.[26] Here, we do not list the concrete expression of the quantum discord, because it is rather long and complex.

In Fig. 1, the quantum discord between the atom and cavity is plotted as a function of the evolution time t for two different values of ω_0: $\omega_0 = 1$ (dashed line), $\omega_0 = 2$ (solid line) with $g = 1$, $\omega = 1$, $\omega_c = \lambda = 0$, $\gamma = 0.2$, $n = 0$. We can see from Fig. 1 that the quantum discord between the atom and cavity oscillates and eventually becomes zero when $\omega = \omega_0$ in the presence of phase decoherence. However, the quantum discord between the atom and cavity is not completely destroyed and remains at a stationary non-zero value

in the case of $\omega \neq \omega_0$ when the time approaches infinity.

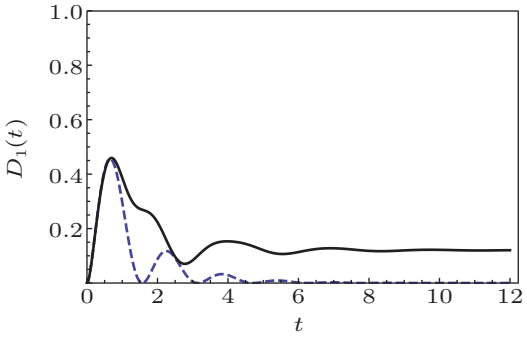


Fig. 1. The quantum discord between the atom and cavity plotted as a function of evolution time t for different values of $\omega_0 : \omega_0 = 1$ (dashed line) and $\omega_0 = 2$ (solid line) with $g = 1$, $\omega = 1$, $\omega_c = \lambda = 0$, $\gamma = 0.2$, $n = 0$.

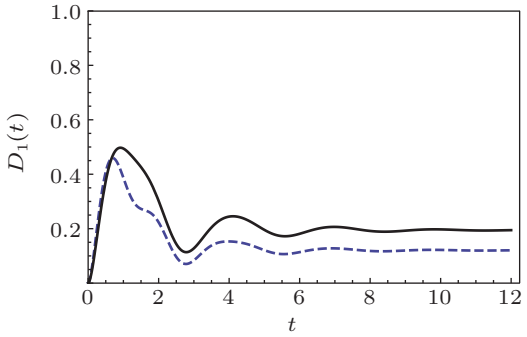


Fig. 2. The quantum discord between the atom and cavity plotted as a function of evolution time t for different values of ω_c and $\lambda : \omega_c = \lambda = 0$ (dashed line) and $\omega_c = 1, \lambda = 0.5$ (solid line) with $g = 1$, $\omega = 1$, $\omega_0 = 2$, $\gamma = 0.2$, $n = 0$.

In order to study the influence of a classical driving field on the dynamics of quantum discord between the atom and cavity, we plot the quantum discord in Fig. 2 as a function of the evolution time t for different values of ω_c and $\lambda : \omega_c = \lambda = 0$ (dashed line), $\omega_c = 1, \lambda = 0.5$ (solid line) with $g = 1$, $\omega = 1$, $\omega_0 = 2$, $\gamma = 0.2$, $n = 0$. By comparing the solid line and dashed line of Fig. 2, we can see clearly that the amount of stationary quantum discord between the atom and cavity can be increased by adjusting the frequency ω_c and strength λ of the classical driving field in the presence of phase decoherence.

3. Quantum discord dynamics between two atoms driven by classical fields with phase decoherence

In this section, we investigate the quantum discord dynamics of two identical non-interacting atoms independently interacting with a cavity field and a classical driving field in the presence of phase decoherence. Suppose the cavity fields are both initially prepared in the Fock state $|n\rangle$ and the atom-pair is initially prepared in the Extend-Wenner-state with its density operator written as $p|B\rangle\langle B| + (1-p)I/4$, where $|B\rangle = \frac{1}{\sqrt{2}}(|-\rangle|-\rangle + |+\rangle|+\rangle)$, I is a unit 4×4 matrix and $0 \leq p \leq 1$ is the purity of the Extend-Wenner-state. Then, the initial density operator of the system reads

$$\varrho(0) = \Big(p|B\rangle\langle B| + \frac{1-p}{4} I\Big) \otimes |n\rangle\langle n| \otimes |n\rangle\langle n|. \quad (19)$$

By making use of Eqs. (4), and (10)–(21) and tracing over the mode of the cavities, we obtain the reduced density matrix of two atoms at time t (formed by the standard basis $|-\rangle|-\rangle$, $|-\rangle|+\rangle$, $|+\rangle|-\rangle$, $|+\rangle|+\rangle$) as

$$\varrho(t) = \begin{pmatrix} \varrho_{11} & 0 & 0 & \varrho_{14} \\ 0 & \varrho_{22} & \varrho_{23} & 0 \\ 0 & \varrho_{32} & \varrho_{33} & 0 \\ \varrho_{41} & 0 & 0 & \varrho_{44} \end{pmatrix},$$

where

$$\begin{aligned}
\varrho_{11} =& p\Big\{\frac{1}{8}\Big[1 + \frac{\Delta^2}{\Omega_n^2} + \Big(1 - \frac{\Delta^2}{\Omega_n^2}\Big)\cos(\Omega_n t)\exp\Big(-\frac{\gamma\Omega_n^2 t}{2}\Big)\Big]^2 + \frac{2g'^4(n+1)^2}{\Omega_{n+1}^4}\Big[1 - \cos(\Omega_{n+1}t) \\
&\times \exp\Big(-\frac{\gamma\Omega_{n+1}^2 t}{2}\Big)\Big]^2\Big\} + \frac{1-p}{4}\Big\{\frac{1}{2}\Big[1 + \frac{\Delta^2}{\Omega_n^2} + \Big(1 - \frac{\Delta^2}{\Omega_n^2}\Big)\cos(\Omega_n t)\exp\Big(-\frac{\gamma\Omega_n^2 t}{2}\Big)\Big] + \frac{2g'^2(n+1)}{\Omega_{n+1}^2} \\
&\times \Big[1 - \cos(\Omega_{n+1}t)\exp\Big(-\frac{\gamma\Omega_{n+1}^2 t}{2}\Big)\Big]\Big\}^2, \\
\varrho_{22} =& \varrho_{33} = p\Big\{\frac{g'^2 n}{2\Omega_n^2}\Big[1 + \frac{\Delta^2}{\Omega_n^2} + \Big(1 - \frac{\Delta^2}{\Omega_n^2}\Big)\cos(\Omega_n t)\exp\Big(-\frac{\gamma\Omega_n^2 t}{2}\Big)\Big]\Big[1 - \cos(\Omega_n t)\exp\Big(-\frac{\gamma\Omega_n^2 t}{2}\Big)\Big] \\
&+ \frac{g'^2(n+1)}{2\Omega_{n+1}^2}\Big[1 + \frac{\Delta^2}{\Omega_{n+1}^2} + \Big(1 - \frac{\Delta^2}{\Omega_{n+1}^2}\Big)\cos(\Omega_{n+1}t)\exp\Big(-\frac{\gamma\Omega_{n+1}^2 t}{2}\Big)\Big]\Big[1 - \cos(\Omega_{n+1}t)\exp\Big(-\frac{\gamma\Omega_{n+1}^2 t}{2}\Big)\Big]\Big\} \\
&+ \frac{1-p}{4}\Big\{\frac{1}{2}\Big[1 + \frac{\Delta^2}{\Omega_n^2} + \Big(1 - \frac{\Delta^2}{\Omega_n^2}\Big)\cos(\Omega_n t)\exp\Big(-\frac{\gamma\Omega_n^2 t}{2}\Big)\Big] + \frac{2g'^2(n+1)}{\Omega_{n+1}^2}\Big[1 - \cos(\Omega_{n+1}t)
\end{aligned}$$

$$\times \exp\Big(-\frac{\gamma\Omega_{n+1}^2 t}{2}\Big)\Big]\Big\}\Big\{\frac{1}{2}\Big[1+\frac{\Delta^2}{\Omega_{n+1}^2}+\Big(1-\frac{\Delta^2}{\Omega_{n+1}^2}\Big)\cos(\Omega_{n+1}t)\exp\Big(-\frac{\gamma\Omega_{n+1}^2 t}{2}\Big)\Big]$$
$$+\frac{2g'^2 n}{\Omega_n^2}\Big[1-\cos(\Omega_n t)\exp\Big(-\frac{\gamma\Omega_n^2 t}{2}\Big)\Big]\Big\},$$

$$\varrho_{44} = p\Big\{\frac{1}{8}\Big[1+\frac{\Delta^2}{\Omega_{n+1}^2}+\Big(1-\frac{\Delta^2}{\Omega_{n+1}^2}\Big)\cos(\Omega_{n+1}t)\exp\Big(-\frac{\gamma\Omega_{n+1}^2 t}{2}\Big)\Big]^2+\frac{2g'^4 n^2}{\Omega_n^4}\Big[1-\cos(\Omega_n t)\exp\Big(-\frac{\gamma\Omega_n^2 t}{2}\Big)\Big]^2\Big\}$$
$$+\frac{1-p}{4}\Big\{\frac{1}{2}\Big[1+\frac{\Delta^2}{\Omega_{n+1}^2}+\Big(1-\frac{\Delta^2}{\Omega_{n+1}^2}\Big)\cos(\Omega_{n+1}t)\exp\Big(-\frac{\gamma\Omega_{n+1}^2 t}{2}\Big)\Big]$$
$$+\frac{2g'^2 n}{\Omega_n^2}\Big[1-\cos(\Omega_n t)\exp\Big(-\frac{\gamma\Omega_n^2 t}{2}\Big)\Big]\Big\}^2,$$

$$\varrho_{41} = \varrho_{14}^* = \frac{p}{32}\Bigg\{\Big(1+\frac{\Delta}{\Omega_{n+1}}\Big)\Big(1-\frac{\Delta}{\Omega_n}\Big)\exp\Big[-\mathrm{i}\Big(\omega+\frac{\Omega_{n+1}-\Omega_n}{2}\Big)t\Big]\exp\Bigg[-\frac{\gamma\Big(\omega+\frac{\Omega_{n+1}-\Omega_n}{2}\Big)^2 t}{2}\Bigg]$$
$$+\Big(1-\frac{\Delta}{\Omega_{n+1}}\Big)\Big(1-\frac{\Delta}{\Omega_n}\Big)\exp\Big[-\mathrm{i}\Big(\omega-\frac{\Omega_{n+1}+\Omega_n}{2}\Big)t\Big]\exp\Bigg[-\frac{\gamma\Big(\omega-\frac{\Omega_{n+1}+\Omega_n}{2}\Big)^2 t}{2}\Bigg]$$
$$+\Big(1+\frac{\Delta}{\Omega_{n+1}}\Big)\Big(1+\frac{\Delta}{\Omega_n}\Big)\exp\Big[-\mathrm{i}\Big(\omega+\frac{\Omega_{n+1}+\Omega_n}{2}\Big)t\Big]\exp\Bigg[-\frac{\gamma\Big(\omega+\frac{\Omega_{n+1}+\Omega_n}{2}\Big)^2 t}{2}\Bigg]$$
$$+\Big(1-\frac{\Delta}{\Omega_{n+1}}\Big)\Big(1+\frac{\Delta}{\Omega_n}\Big)\exp\Big[-\mathrm{i}\Big(\omega-\frac{\Omega_{n+1}-\Omega_n}{2}\Big)t\Big]\exp\Bigg[-\frac{\gamma\Big(\omega-\frac{\Omega_{n+1}-\Omega_n}{2}\Big)^2 t}{2}\Bigg]\Bigg\}^2,$$

$$\varrho_{23} = \varrho_{32} = 0. \tag{20}$$

By making use of the explicit expression of $\varrho(t)$ in Eq. (20), the quantum discord $D_2(t)$ between two atoms can be calculated similarly as in the atom-cavity system.

In order to quantify the entanglement dynamics of the atoms and make a comparison with their quantum discord dynamics, we adopt the measure concurrence which is defined by[27]

$$E = \max\{0, \sqrt{\eta_1}-\sqrt{\eta_2}-\sqrt{\eta_3}-\sqrt{\eta_4}\}, \tag{21}$$

where the $\eta_i (i = 1, 2, 3, 4)$ are the eigenvalues in decreasing order of the magnitude of the "spin-flipped" density matrix operator $R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, with ρ being the density matrix of a 2×2 system and σ_y being the Pauli Y matrix, i.e., $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. Combing the definition of concurrence with the density matrix $\varrho(t)$, we find that the concurrence between two atoms is given by

$$E_2(t) = 2\max(0, |\varrho_{41}| - |\varrho_{22}|). \tag{22}$$

We plot the quantum discord and entanglement between two atoms in Fig. 3 as a function of time t and p with $g = 1, \omega = \omega_0 = 1, \omega_c = \lambda = 0, \gamma = 0, n = 0$. It can be found that quantum discord and entanglement

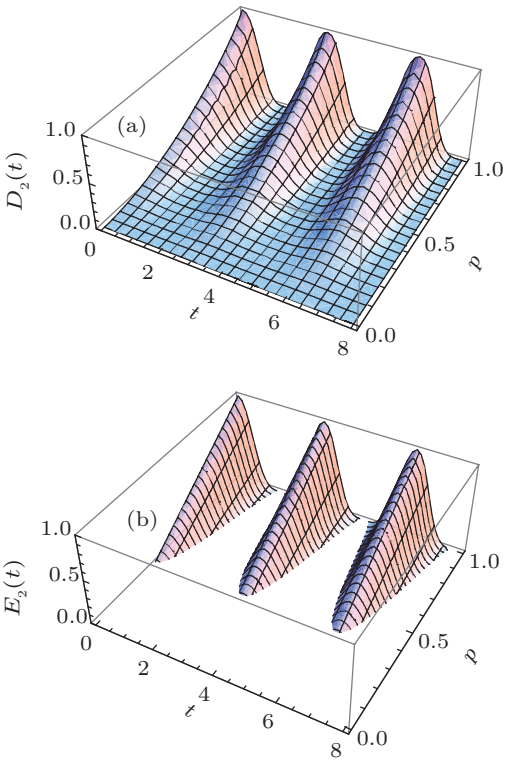


Fig. 3. (colour online) The quantum discord (a) and entanglement (b) between two atoms plotted as a function of t and p with $g = 1$, $\omega = \omega_0 = 1$, $\omega_c = \lambda = 0$, $\gamma = 0$, $n = 0$.

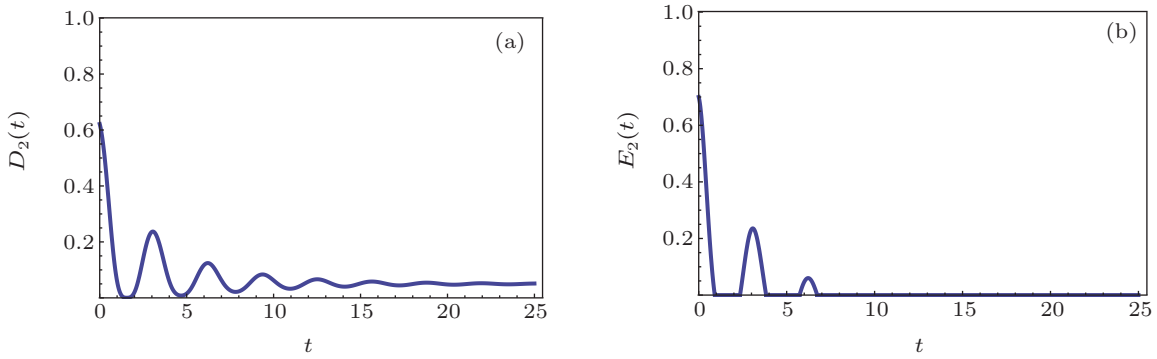


Fig. 4. The quantum discord (a) and entanglement (b) between two atoms plotted as a function of time with $p = 0.8$, $g = 1$, $\omega = \omega_0 = 1$, $\omega_c = \lambda = 0$, $\gamma = 0.1$, $n = 0$.

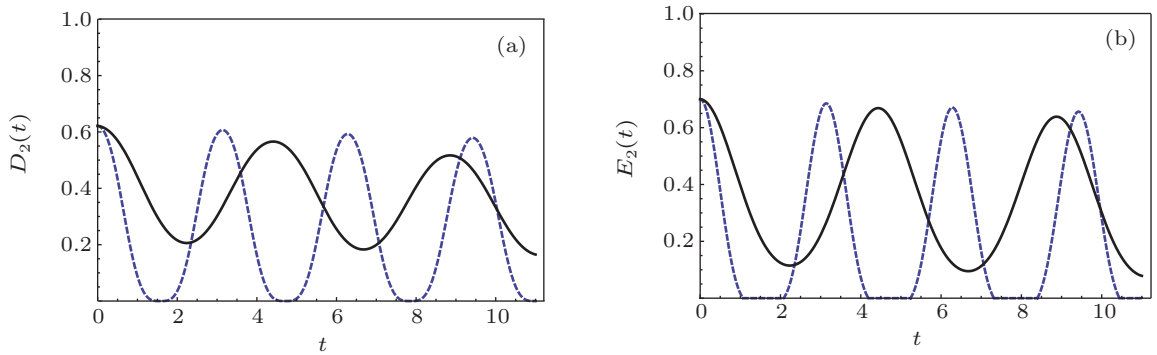


Fig. 5. The quantum discord (a) and entanglement (b) between two atoms plotted as a function of time for different values of ω_c and $\lambda : \omega_c = \lambda = 0$ (dashed line) and $\omega_c = 1$, $\lambda = 0.5$ (solid line) with $p = 0.8$, $g = 1$, $\omega = \omega_0 = 1$, $\gamma = 0.002$, $n = 0$.

between two atoms will recover their initial values without phase decoherence and entanglement disappears and revives suddenly during evolution although quantum discord vanishes and revives asymptotically. Furthermore, in the case that $0 < p \leq \frac{1}{3}$, the atom-pair is at a separable mixed state so entanglement between the atoms becomes zero, but quantum discord between the atoms remains non-zero when $0 < p \leq \frac{1}{3}$.

In Fig. 4, quantum discord Fig. 3 and entanglement between two atoms are plotted as a function of time t with $p = 0.8$, $g = 1$, $\omega = \omega_0 = 1$, $\omega_c = \lambda = 0$, $\gamma = 0.1$, $n = 0$. We found that the long-time evolution behaviours of quantum discord and entanglement between two atoms are rather different in the presence of phase dechorerence. The quantum discord between two atoms oscillates and decays much more slowly than entanglement for a long-time evolution, which means that quantum discord exhibits more robustness than entanglement under the phase decoherence.

Finally, for the sake of studying the influence of the classical driving field on the dynamics of quantum discord and entanglement between two atoms under the phase decoherence, we plot quantum discord and entanglement in Fig. 5 as a function of evolution time t for different values of ω_c and $\lambda : \omega_c = \lambda = 0$ (dashed line) and $\omega_c = 1$, $\lambda = 0.5$ (solid line) with $p = 0.8$, $g = 1$, $\omega = \omega_0 = 1$, $\gamma = 0.002$, $n = 0$. It can be seen clearly that the classical driving field can help to improve quantum discord and entanglement between two atoms under phase decoherence.

4. Conclusions

In this paper, we study quantum discord dynamics in cavity QED systems driven by a classical driving field with phase decoherence. We first investigate a two-level atom interacting with a quantized cavity field and classical driving field in the presence of phase decoherence and find that the amount of quantum discord between the atom and cavity remains at a stationary non-zero value during the long time evolution, which means that stationary quantum discord can arise in the interaction of atom and cavity field as time approaches infinity. It is shown that stationary quantum discord can be increased by applying a classical driving field. We also consider the quantum discord dynamics of two identical non-interacting two-level atoms independently interacting with a quantized cavity field and a classical driving field in the

presence of phase decoherence. It is found that the long-time evolution behaviours of quantum discord and entanglement between two atoms are rather different in the presence of phase decoherence. The quantum discord between two atoms is more robust than entanglement under phase decoherence and the classical driving field can help to improve the amount of quantum discord between two atoms. The approach adopted here may be extended to improve the implementation of quantum tasks based on the quantum correlations.

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