Generation of a new bipartite coherent-entangled state and its applications*

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A new bipartite coherent-entangled state is introduced in the two-mode Fock space, which exhibits the properties of both a coherent state and an entangled state. The set of coherent-entangled states makes up a complete and partly nonorthogonal representation. A simple experimental scheme to produce the coherent-entangled state using an asymmetric beamsplitter is proposed. Some applications of the coherent-entangled state in quantum optics are also presented.

Keywords: bipartite coherent-entangled state, asymmetric beamsplitter, Einstein–Podolsky–Rosen entangled state, squeezed state

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1. Introduction

In the last decades, entanglement has aroused a great deal of interest from physicists, since it is useful to quantum communication and quantum information. In history, it was first proposed by Einstein, Podolsky, and Rosen (EPR)[1] for challenging the incompleteness of quantum mechanics. Later, they noticed that two particles' relative coordinate $X_1 - X_2$ as x_0 and total momentum $P_1 + P_2$ are commutable and can be simultaneously measured, where $X_i = (a_i + a_i^\dagger)/\sqrt{2}$, $P_i = (a_i - a_i^\dagger)/(\sqrt{2}\,\mathrm{i})$, $(i = 1, 2)$. So the wave function of the two particles can be explicitly expressed as $\psi(x_1, x_2) = \int_{-\infty}^{\infty} \mathrm{d}p \exp[\mathrm{i}p(x_1 - x_2 + x_0)]/2$. Since then the EPR state has been the subject of many discussions on the non-locality of quantum theory. As Dirac guided in Ref. [2]: "when one has a particular problem to work out in quantum mechanics, one can minimize the labor by using a representation in which the representatives of the more important abstract quantities occurring in that problem are as simple as possible", we believe that establishing various entangled state representations will be inevitable, since many entangled problems can only be clearly explained with them. Thus, searching for new physical applicable quantum entangled state representations is one of the important tasks of theoretical physicists. In this work, we propose a new quantum mechanics representation in the two-mode Fock space by noticing the commutator $[\mu X_2 + \nu P_1, \mu a_1 - \mathrm{i}\nu a_2] = 0$, which is called the coherent-entangled state (CES) $|\alpha, x\rangle$ representation, since the corresponding quantum state $|\alpha, x\rangle$ possesses partly the property of a coherent state and partly the property of an entangled state. Subsequently, we want to investigate its main properties, generating scheme, and important applications.

The work is arranged as follows. In Section 2, we derive an explicit state vector for a new bipartite CES $|\alpha, x\rangle$ using the technique of integral within the ordered product (IWOP) of operators,[3] and then we reveal its partly orthogonal property and entanglement. In Section 3, we discuss how to use an asymmetric beamsplitter (BS)[4] to produce such a state. Sections 4 and 5 are devoted to deriving the conjugate state of $|\alpha, x\rangle$ and discussing some possible superpositions of $|\alpha, x\rangle$, respectively. Finally, we draw some conclusions in Section 6.

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2. New bipartite CES and its main properties

Let us first construct the normally ordered Gaussian operator

$$
\begin{aligned}
&: \exp\left\{-\left(x-\frac{\mu X_2+\nu P_1}{\lambda}\right)^2\right.\\
&-\frac{1}{2}\left[\alpha^*-\frac{\sqrt{2}}{\lambda}\left(\mu a_1^\dagger+\mathrm{i}\nu a_2^\dagger\right)\right]\\
&\left.\times\left[\alpha-\frac{\sqrt{2}}{\lambda}\left(\mu a_1-\mathrm{i}\nu a_2\right)\right]\right\}: \equiv O(\alpha,x), \qquad (1)
\end{aligned}
$$

where μ,ν are two real parameters, $\alpha=\alpha_1+\mathrm{i}\alpha_2$ is a complex number, and symbol : : denotes the normal ordering.[3,5] Using the IWOP technique, we obtain the following unity of the Gaussian operator integration within the normal ordering:

$$
\begin{aligned}
&\int_{-\infty}^{\infty}\frac{\mathrm{d}x}{\sqrt{\pi}}\int\frac{\mathrm{d}^2\alpha}{2\pi}O(\alpha,x)\\
&=\int_{-\infty}^{\infty}\frac{\mathrm{d}x}{\sqrt{\pi}}:\exp\left[-\left(x-\frac{\mu X_2+\nu P_1}{\lambda}\right)^2\right]:\\
&=\int\frac{\mathrm{d}^2\alpha}{2\pi}:\exp\left\{-\frac{1}{2}\left[\alpha^*-\frac{\sqrt{2}}{\lambda}\left(\mu a_1^\dagger+\mathrm{i}\nu a_2^\dagger\right)\right]\right.\\
&\left.\times\left[\alpha-\frac{\sqrt{2}}{\lambda}\left(\mu a_1-\mathrm{i}\nu a_2\right)\right]\right\}:\\
&=1. \qquad (2)
\end{aligned}
$$

So using the normal ordered product of the two-mode vacuum projector[3,5,6]

$$
|00\rangle\langle 00| =: \exp\left[-a_1^\dagger a_1-a_2^\dagger a_2\right]:
$$

and taking $\lambda=\sqrt{\mu^2+\nu^2}$, we can further decompose $O(\alpha,x)$ as the following projection operator:

$$
O(\alpha,x)=|\alpha,x\rangle\langle\alpha,x|, \qquad (3)
$$

where the new state $|\alpha,x\rangle$ are given as

$$
\begin{aligned}
&|\alpha,x\rangle\\
&=\exp\left[-\frac{|\alpha|^2}{4}-\frac{x^2}{2}+\frac{1}{\sqrt{2}\lambda}(\mu\alpha+\mathrm{i}2\nu x)a_1^\dagger\right.\\
&\left.+\frac{1}{\sqrt{2}\lambda}(2\mu x+\mathrm{i}\nu\alpha)a_2^\dagger+\frac{1}{2\lambda^2}(\nu a_1^\dagger-\mathrm{i}\mu a_2^\dagger)^2\right]|00\rangle. \qquad (4)
\end{aligned}
$$

Thus

$$
\int\frac{\mathrm{d}^2\alpha}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}x}{\sqrt{\pi}}|\alpha,x\rangle\langle\alpha,x|=1, \qquad (5)
$$

so the set of $|\alpha,x\rangle$ can be composed as a new quantum mechanics representation. Using the bosonic communication relation $[a_i,f(a_i,a_i^\dagger)]=(\partial/\partial a_i^\dagger)f(a_i,a_i^\dagger)$, we derive

$$
\begin{aligned}
a_1|\alpha,x\rangle &= \left[\frac{1}{\sqrt{2}\lambda}(\mu\alpha+\mathrm{i}2\nu x)\right.\\
&\left.+\frac{\nu}{\lambda^2}(\nu a_1^\dagger-\mathrm{i}\mu a_2^\dagger)\right]|\alpha,x\rangle, \qquad (6)
\end{aligned}
$$

and

$$
\begin{aligned}
a_2|\alpha,x\rangle &= \left[\frac{1}{\sqrt{2}\lambda}(2\mu x+\mathrm{i}\nu\alpha)\right.\\
&\left.-\frac{\mathrm{i}\mu}{\lambda^2}(\nu a_1^\dagger-\mathrm{i}\mu a_2^\dagger)\right]|\alpha,x\rangle. \qquad (7)
\end{aligned}
$$

Combining Eqs. (6) and (7), we obtain the eigenvalue equations

$$
\begin{aligned}
(\mu X_2+\nu P_1)|\alpha,x\rangle &= \lambda x|\alpha,x\rangle,\\
(\mu a_1-\mathrm{i}\nu a_2)|\alpha,x\rangle &= \frac{\lambda\alpha}{\sqrt{2}}|\alpha,x\rangle, \qquad (8)
\end{aligned}
$$

which show that state $|\alpha,x\rangle$ possesses partly the property of a coherent state and partly the property of an entangled state. So we name state $|\alpha,x\rangle$ the CES. In particular, when $\mu=\nu$, state $|\alpha,x\rangle$ reduces to

$$
\begin{aligned}
&|\alpha,x\rangle_{\mu=\nu}\\
&=\exp\left[-\frac{|\alpha|^2}{4}-\frac{x^2}{2}+\frac{1}{2}(\alpha+\mathrm{i}2x)a_1^\dagger\right.\\
&\left.+\frac{1}{2}(2x+\mathrm{i}\alpha)a_2^\dagger+\frac{1}{4}(a_1^\dagger-\mathrm{i}a_2^\dagger)^2\right]|00\rangle, \qquad (9)
\end{aligned}
$$

which is consistent with the result of Ref. [7] as expected.

Next we investigate the main properties of state $|\alpha,x\rangle$. From Eq. (8), we arrive at the following matrix elements:

$$
\begin{aligned}
&\langle\alpha',x'|(\mu X_2+\nu P_1)|\alpha,x\rangle\\
&=\lambda x\langle\alpha',x'|\alpha,x\rangle\\
&=\lambda x'\langle\alpha',x'|\alpha,x\rangle, \qquad (10)
\end{aligned}
$$

which lead to

$$
\langle\alpha',x'|\alpha,x\rangle(x-x')=0. \qquad (11)
$$

To obtain the exact expression of $\langle\alpha',x'|\alpha,x\rangle$, we perform differentiation $\partial/\partial\alpha$ on $|\alpha,x\rangle$ and then obtain the following equation:

$$
\begin{aligned}
&\sqrt{2}\lambda\left(\frac{\partial}{\partial\alpha}+\frac{\alpha^*}{4}\right)|\alpha,x\rangle\\
&=(\mu a_1^\dagger+\mathrm{i}\nu a_2^\dagger)|\alpha,x\rangle. \qquad (12)
\end{aligned}
$$

It then follows from Eq. (12) that

$$\begin{aligned}&\langle\alpha', x'|\left(\mu a_1 - \mathrm{i}\nu a_2\right)|\alpha, x\rangle \\ =\ & \frac{\lambda\alpha}{\sqrt{2}}\langle\alpha', x'\ |\alpha, x\rangle \\ =\ & \sqrt{2}\lambda\left(\frac{\partial}{\partial\alpha'^*}+\frac{\alpha'}{4}\right)\langle\alpha', x'\ |\alpha, x\rangle\,. \end{aligned} \tag{13}$$

Taking the complex conjugate of Eq. (13) and interchanging α', x' with α, x, we have

$$\frac{\partial}{\partial\alpha}\langle\alpha', x'\ |\alpha, x\rangle = \left(\frac{\alpha'^*}{2}-\frac{\alpha^*}{4}\right)\langle\alpha', x'\ |\alpha, x\rangle\,. \tag{14}$$

Further, performing differentiation $\partial/\partial\alpha^*$ on $|\alpha, x\rangle$, we have

$$\frac{\partial}{\partial\alpha^*}|\alpha, x\rangle = -\frac{\alpha}{4}|\alpha, x\rangle\,, \tag{15}$$

so

$$\frac{\partial}{\partial\alpha'}\langle\alpha', x'\ |\alpha, x\rangle = -\frac{\alpha'^*}{4}\langle\alpha', x'\ |\alpha, x\rangle\,. \tag{16}$$

Through solving the partial differential Eqs. (14) and (16) and considering Eqs. (5) and (11), we finally obtain

$$\begin{aligned}&\langle\alpha', x'\ |\alpha, x\rangle \\ =\ & \sqrt{\pi}\exp\left(-\frac{|\alpha|^2}{4}-\frac{|\alpha'|^2}{4}+\frac{\alpha\alpha'^*}{2}\right)\delta(x-x'),\end{aligned} \tag{17}$$

where $\alpha\alpha'^*$ reflects the non-orthogonal property as that of the usual coherent states, while $\delta(x-x')$ reflects the orthogonal property of the entangled state. So we conclude that state $|\alpha, x\rangle$ exhibits both coherent state and entangled state properties. In particular, when $\alpha=\alpha'$,

$$\langle\alpha', x'\ |\alpha, x\rangle = \sqrt{\pi}\delta(x-x'). \tag{18}$$

On the other hand, by making the Fourier transformation of $|\alpha=\alpha_1+\mathrm{i}\alpha_2, x\rangle$ with respect to α_1, we have

$$\begin{aligned}&\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\alpha_1\,|\alpha=\alpha_1+\mathrm{i}\alpha_2, x\rangle\ \mathrm{e}^{\mathrm{i}u\alpha_1} \\ =\ & \exp\left[-\frac{1}{8}(2u+\alpha_2)^2\right] \\ & \times\left|p=\frac{1}{\lambda}\left(\nu x+\mu u+\mu\frac{\alpha_2}{2}\right)\right\rangle_1 \\ & \otimes\left|y=\frac{1}{\lambda}\left(\mu x-u\nu-\nu\frac{\alpha_1}{2}\right)\right\rangle_2,\end{aligned} \tag{19}$$

where the two single-mode states $|p\rangle_1$ and $|y\rangle_2$ are the sets of momentum eigenvectors and coordinate eigenvectors, respectively.[7] Performing the inverse Fourier transformation, we derive the Schmidt decomposition of $|\alpha=\alpha_1+\mathrm{i}\alpha_2, x\rangle$, i.e.,

$$\begin{aligned}&|\alpha=\alpha_1+\mathrm{i}\alpha_2, x\rangle \\ =\ & \int_{-\infty}^{\infty}\mathrm{d}u\exp\left[-\frac{1}{8}(2u+\alpha_2)^2-\mathrm{i}u\alpha_1\right] \\ & \times\left|p=\frac{1}{\lambda}\left(\nu x+\mu u+\mu\frac{\alpha_2}{2}\right)\right\rangle_1 \\ & \otimes\left|y=\frac{1}{\lambda}\left(\mu x-u\nu-\nu\frac{\alpha_2}{2}\right)\right\rangle_2,\end{aligned} \tag{20}$$

which means that $|\alpha=\alpha_1+\mathrm{i}\alpha_2, x\rangle$ is an entangled state.[8]

3. Generation scheme of $|\alpha, x\rangle$

Since the BS plays an important role in producing quantum entanglement, now we want to discuss how to produce $|\alpha, x\rangle$ using an asymmetric BS. The asymmetric BS operator with a free phase can be expressed as[9,10]

$$B=\exp\left[-\mathrm{i}\frac{\theta}{2}(a_1^\dagger a_2+a_1 a_2^\dagger)\right]. \tag{21}$$

Let the ideal single-mode momentum eigenstate expressed by $|p=0\rangle_1\equiv\exp[a_1^{\dagger 2}/2]\,|0\rangle_1$ in mode 1 and the vacuum state $|0\rangle_2$ in mode 2 enter the two input ports of the asymmetric BS respectively and get overlapped, when $\theta=2\cos^{-1}(\nu/\lambda)$, the output state is

$$\begin{aligned}&B\,|p=0\rangle_1\otimes|0\rangle_2 \\ =\ & \exp\left[\frac{1}{2\lambda^2}(\nu a_1^\dagger-\mathrm{i}\mu a_2^\dagger)^2\right]|00\rangle\,.\end{aligned} \tag{22}$$

Then, we operate two local oscillator laser beams denoted by two displacement operators $D_1(\varepsilon_1)=\exp(\varepsilon_1 a_1^\dagger-\varepsilon_1^* a_1)$ and $D_2(\varepsilon_2)=\exp(\varepsilon_2 a_2^\dagger-\varepsilon_2^* a_2)$ on the output state, where $\varepsilon_1=(\mu\alpha+\mathrm{i}\nu x)/(\sqrt{2}\lambda)$ and $\varepsilon_2=(\mu x+\mathrm{i}\nu\alpha)/(\sqrt{2}\lambda)$, we obtain

$$\begin{aligned}&D_1(\varepsilon_1)D_2(\varepsilon_2)\exp\left[\frac{1}{2\lambda^2}(\nu a_1^\dagger-\mathrm{i}\mu a_2^\dagger)^2\right]|00\rangle \\ =\ & |\alpha, x\rangle\,.\end{aligned} \tag{23}$$

Then the ideal CES $|\alpha, x\rangle$ can be produced. We can also realize displacement $D_1(\varepsilon_1)$(or $D_2(\varepsilon_2)$) experimentally by reflecting the light field from a partially reflecting mirror and adding through the mirror a field that has been phase and amplitude modulated based on the values α and x.

4. Conjugate state of $|\alpha, x\rangle$

Through making the following Fourier transform of $|\alpha, x\rangle$:

$$\begin{aligned}|\beta, p\rangle &= \int_{-\infty}^{\infty} \frac{\mathrm{d}x}{\sqrt{2\pi}} \mathrm{e}^{\mathrm{i}px} \int_{-\infty}^{\infty} \frac{\mathrm{d}^2\alpha}{4\pi} |\alpha, x\rangle \\ &\quad \times \exp[(\alpha^*\beta - \alpha\beta^*)/4] \\ &= \exp\left[\frac{1}{\sqrt{2}\lambda}(\mu\beta + 2p\nu)a_1^\dagger + \frac{\mathrm{i}}{\sqrt{2}\lambda}(\nu\beta - 2p\mu)a_2^\dagger \right. \\ &\quad \left. -\frac{1}{2\lambda^2}(\nu a_1^\dagger - \mathrm{i}\mu a_2^\dagger)^2 - \frac{p^2}{2} - \frac{|\beta|^2}{4}\right] |00\rangle\,, \quad (24)\end{aligned}$$

we find that state $|\beta, p\rangle$ is the common eigenstate of $\nu X_1 - \mu P_2$ and $\mu a_1 - \mathrm{i}\nu a_2$, i.e.,

$$\begin{aligned}(\nu X_1 - \mu P_2)\,|\beta, p\rangle &= \lambda p\,|\beta, p\rangle\,, \\ (\mu a_1 - \mathrm{i}\nu a_2)\,|\beta, p\rangle &= \frac{\lambda}{\sqrt{2}}\beta\,|\beta, p\rangle\,. \quad (25)\end{aligned}$$

Employing the IWOP technique, we can prove the following completeness relation:

$$\int \frac{\mathrm{d}^2\beta}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{d}p}{\sqrt{\pi}} |\beta, p\rangle \langle\beta, p| = 1, \quad (26)$$

which shows that the set of state $|\beta, p\rangle$, as the conjugate state of $|\alpha, x\rangle$, also forms a CES representation.

5. Possible superpositions of $|\alpha, x\rangle$

In the past few decades, the superposition of quantum states in quantum optics has drawn the attention of many theoretical physicists, because it can produce nonclassical states exhibiting some remarkable nonclassical properties. For example, quantum superpositions of coherent states[11,12] exhibit second-order squeezing, higher-order squeezing, and sub-Poissonian photon statistics. Here we examine the possible superpositions of $|\alpha, x\rangle$. Using Eq. (4) and performing the integral over $\mathrm{d}\alpha_1$ for $|\alpha, x\rangle$, we obtain

$$\begin{aligned}&\frac{1}{2\sqrt{\pi}} \int \mathrm{d}\alpha_1\, |\alpha = \alpha_1 + \mathrm{i}\alpha_2, x\rangle \\ &= \exp\left\{-\frac{\alpha_2^2}{4} - \frac{x^2}{2} + \frac{1}{\sqrt{2}\lambda}\left[(\mu\alpha_2 + \mathrm{i}2\nu x)a_1^\dagger \right.\right. \\ &\quad + (2\mu x + \mathrm{i}\nu\alpha_2)a_2^\dagger\Big] + \frac{1}{2\lambda^2}\left[(\nu^2 - \mu^2)\right. \\ &\quad \left.\left. \times (a_1^{\dagger 2} + a_2^{\dagger 2}) - \mathrm{i}4\mu\nu a_1^\dagger a_2^\dagger\right]\right\} |00\rangle \\ &= |\alpha_2, x\rangle\,, \quad (27)\end{aligned}$$

which is the common eigenstate of two commuting operators $\mu X_2 + \nu P_1$ and $\mu X_1 + \nu P_2$, i.e.,

$$\begin{aligned}(\mu X_1 + \nu P_2)\,|\alpha_2, x\rangle &= \frac{\lambda\alpha_2}{2}\,|\alpha_2, x\rangle\,, \\ (\mu X_2 + \nu P_1)\,|\alpha_2, x\rangle &= \lambda x\,|\alpha_2, x\rangle\,. \quad (28)\end{aligned}$$

So the superposition of $|\alpha = \alpha_1 + \mathrm{i}\alpha_2, x\rangle$ along the α_1 axis yields a new EPR entangled state.[13]

Next, starting from the superpositions of $|\alpha, x\rangle$, we derive some new squeezing operators and new squeezed states. Through constructing the asymmetric ket–bra integration

$$S = \int_{-\infty}^{\infty} \frac{\mathrm{d}x}{\sqrt{k\pi}} \int \frac{\mathrm{d}^2\alpha}{2\pi} |\alpha, x/k\rangle \langle\alpha, x|\,, \quad (29)$$

and using formula : $\exp\left[(\mathrm{e}^f - 1)a^\dagger a\right] : = \mathrm{e}^{fa^\dagger a}$, we directly obtain

$$\begin{aligned}S &= \mathrm{sech}^{1/2}\gamma \exp\left(\frac{\tanh\gamma}{2}R^{\dagger 2}\right) \exp\left(R^\dagger R \ln\mathrm{sech}\,\gamma\right) \\ &\quad \times \exp\left(-\frac{\tanh\gamma}{2}R^2\right), \quad (30)\end{aligned}$$

where $k = \mathrm{e}^\gamma$ is the squeezing parameter, and $R^\dagger = (\nu a_1^\dagger - \mathrm{i}\mu a_2^\dagger)/\lambda$. Since $R^{\dagger 2}$, R^2, and $R^\dagger R$ form an SU(1,1) Lie algebra, $\left[R, R^\dagger\right] = 1$, and $\left[R^2/2, R^{\dagger 2}/2\right] = R^\dagger R + 1/2$, S is a new squeezing operator. From Eqs. (5) and (29), we know that S squeezes $|\alpha, x\rangle$ in a natural way,

$$S\,|\alpha, x\rangle = \frac{1}{\sqrt{k}}\,|\alpha, x/k\rangle\,. \quad (31)$$

It then follows that

$$S\,|00\rangle = \mathrm{sech}^{1/2}\gamma \exp\left[\frac{\tanh\gamma}{2\lambda^2}(\nu a_1^\dagger - \mathrm{i}\mu a_2^\dagger)^2\right] |00\rangle\,, \quad (32)$$

which is a generalized two-mode squeezed state, since with squeezing parameter γ both $a_i^{\dagger 2}$ and $a_1^\dagger a_2^\dagger$ are involved, and are completely different from the usual two-mode squeezing operators in Ref. [14].

Carrying out the integral over $\mathrm{d}\alpha_1$ for projection operator $|\alpha = \alpha_1 + \mathrm{i}\alpha_2, x\rangle \langle\alpha = \alpha_1 + \mathrm{i}\alpha_2, x|$ yields

$$\begin{aligned}&\frac{1}{\sqrt{2\pi}} \int \mathrm{d}\alpha_1\, |\alpha = \alpha_1 + \mathrm{i}\alpha_2, x\rangle \\ &\quad \times \langle\alpha = \alpha_1 + \mathrm{i}\alpha_2, x|\, 00\rangle \\ &= \exp\left\{-x^2 - \frac{\alpha_2^2}{2} + \frac{1}{4}\left[a_1^{\dagger 2} - a_2^{\dagger 2}\right.\right. \\ &\quad \left.\left. + \frac{1}{\lambda^2}(\nu a_1^\dagger - \mathrm{i}\mu a_2^\dagger)^2\right] + \frac{\mathrm{i}}{\sqrt{2}\lambda}(\mu\alpha_2 + 2\nu x)a_1^\dagger \right. \\ &\quad \left. + \frac{1}{\sqrt{2}\lambda}(2\mu x - \nu\alpha_2)a_2^\dagger\right\} |00\rangle \\ &\equiv |\mathrm{squ.}\rangle\,. \quad (33)\end{aligned}$$

In particular, when $\alpha_2 = x = 0$,

$$|\text{squ.}\rangle = \exp\left\{\frac{1}{4}\left[a_1^{\dagger 2} - a_2^{\dagger 2} + \frac{1}{\lambda^2}(\nu a_1^\dagger - \mathrm{i}\mu a_2^\dagger)^2\right]\right\}|00\rangle\,, \tag{34}$$

which is also a generalized two-mode squeezed state. So from Eq. (34), we conclude that the superposition of projection operator

$$|\alpha = \alpha_1 + \mathrm{i}\alpha_2, x\rangle\langle\alpha = \alpha_1 + \mathrm{i}\alpha_2, x|_{\alpha_2 = x = 0}$$

along the α_1 axis leads to a generalized squeezing operator. In particular, if $\mu = \nu$, equation (34) reduces to

$$|\text{squ.}\rangle = \exp\left[\frac{3}{8}(a_1^{\dagger 2} - a_2^{\dagger 2}) - \frac{\mathrm{i}}{4}a_1^\dagger a_2^\dagger\right]|00\rangle\,. \tag{35}$$

6. Conclusion

In summary, by making full use of the IWOP technique, we have introduced a new bipartite CES, which can be produced by using an asymmetric BS in a feasible experimental set-up. We investigate the completeness relation and the part orthogonality of the CES. It is found that CES $|\beta, p\rangle$ is the conjugate state of $|\alpha, x\rangle$, and some possible superpositions of $|\alpha, x\rangle$ lead to new squeezed states. So we think that state $|\alpha, x\rangle$ can provide a direct way to construct some new quantum states in quantum optics.

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