Optomechanical entanglement in a whispering-gallery cavity*

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The dynamics of the optomechanical entanglement between optical cavity field modes and a macroscopic mechanical breathing mode in a whispering-gallery cavity as well as the continuous variable entanglement between the phase-quadrature amplitudes of the two whispering-gallery modes have been analysed. Simulated results indicate that under state-of-the-art experimental conditions, optomechanical entanglement is obvious and can occur even at temperatures of above 40 K. Compared with the entanglement of the mechanical oscillator at the ground state temperature, optomechanical entanglement is more intense by several orders of magnitude.

Keywords: optomechanical entanglement, whispering-gallery mode

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1. Introduction

The investigation of the interaction between a macroscopic mechanical resonator and the radiation field from an optical cavity attracts much interest.[1−8] The observation of optomechanical normal-mode splitting provides unambiguous evidence for the strong coupling of cavity photons with the mechanical resonator.[9] Furthermore, many schemes for generating entanglement between optomechanical systems have been proposed by means of radiation pressure.[10−16] Two mirrors of two different cavities illuminated with entangled light beams were considered by Zhang *et al.*[17] Vitali *et al.* found that the stationary entanglement between an optical cavity field mode and a macroscopic vibrating mirror can be generated via radiation pressure and can occur at environmental temperatures of above 20 K.[18−22] More recently, entanglement has been brought to the high-temperature limit.[23] Both dispersive and reactive couplings have been used to generate steady-state room-temperature optomechanical entanglement.[24]

As an important physical phenomenon, entanglement plays a central role in the application of quantum communication and quantum information.[25] Great efforts have been devoted to investigating entanglement.[26−28] Recently, Yuan and Hu have explored continuous variable entanglement in four-wave mixing in a bad cavity case.[29] The model of two qubits separately trapped in two independent Markovian or non-Markovian environments has been investigated.[30] A scheme for the deterministic generation of qutrit entanglement has been proposed for two atoms trapped in an optical cavity.[31]

In our previous work, we theoretically investigated mode–mode coupling in a microdisk system containing a three-level quantum dot.[32] Recently, a practical scheme was proposed to produce Einstein–Podolsky–Rosen (EPR) beams in an optomechanical system, which contains a whispering-gallery mode (WGM) cavity with a movable boundary.[33] In this paper, we analyse the dynamics of the entanglement between optical cavity field modes and a macroscopic mechanical breathing mode in a whispering-gallery cavity, as well as the continuous variable entanglement between the phase-quadrature amplitudes of two WGMs. It is shown that the entanglement can be very strong and insensitive to the bath temperature under current experimental conditions. Increasing the backscattering rate can enhance the entanglement between the two cavity modes but weaken the entanglement between the cavity mode and the mechanical mode.

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2. System and dynamics

We consider an optical whispering-gallery cavity which can create a macroscopic mechanical breathing mode[3] under the effect of a radiation pressure force as seen in Fig. 1. There are two cavity modes a and b with the same frequency $\omega_{\rm c}$ and opposite momenta. They are coupled with the same mechanical oscillation mode and driven by two lasers with the same frequency $\omega_{\rm L}$ (one from the right-hand side and the other from the left-hand side). Neglecting mechanical mode–mode coupling, we only consider a single mechanical mode so that the mechanical breathing mode can be modeled with a harmonic oscillator with effective mass m and frequency $\omega_{\rm m}$. In the adiabatic limit, $\omega_{\rm m} \ll c/2\pi R$ (c is the speed of light in a vacuum and R is the cavity radius). The Hamiltonian H for the system can be written as

$$\begin{aligned} H/\hbar =\ & \omega_{\rm c} a^\dagger a + \omega_{\rm c} b^\dagger b + \frac{\omega_{\rm m}}{2}(q^2+p^2) \\ & - G_0 a^\dagger a q - G_0 b^\dagger b q \\ & + {\rm i}\, E_{\rm a}(a^\dagger\, {\rm e}^{-{\rm i}\,\omega_{\rm L} t} - a\, {\rm e}^{{\rm i}\,\omega_{\rm L} t}) \\ & + {\rm i}\, E_{\rm b}(b^\dagger\, {\rm e}^{-{\rm i}\,\omega_{\rm L} t} - b\, {\rm e}^{{\rm i}\,\omega_{\rm L} t}) + \beta a^\dagger b + \beta^* a b^\dagger, \end{aligned} \tag{1}$$

where $a(b)$ and $a^\dagger(b^\dagger)$ are the annihilation and creation operators of the cavity mode, respectively. The first two terms on the right hand side describe the energy of the cavity mode. The third term describes the energy of the mechanical mode, q and p ($[q,p]={\rm i}$) are the dimensionless position and momentum operators for the mechanical breathing mode. The fourth and fifth terms are the radiation-pressure coupling of the rate $G_0 = (\omega_{\rm c}/R)\sqrt{\hbar/m\omega_{\rm m}}$. The sixth and seventh terms describe the coupling between the input laser and the cavity mode. $|E_{\rm a}| = \sqrt{2\kappa P_{\rm a}/\hbar\omega_L}$ is related to the input laser power $P_{\rm a}$ and $|E_{\rm b}| = \sqrt{2\kappa P_{\rm b}/\hbar\omega_L}$ is related to the input laser power $P_{\rm b}$. κ is the cavity decay rate. The last two terms are the coupling between the whispering-gallery cavity modes a and b with a backscattering coupling rate β. The total Hamiltonian for the system in a frame rotating at a laser frequency ω_L can be written as

$$\begin{aligned} H/\hbar =\ & (\omega_{\rm c}-\omega_{\rm L}) a^\dagger a + (\omega_{\rm c}-\omega_{\rm L}) b^\dagger b + \frac{\omega_{\rm m}}{2}(q^2+p^2) \\ & - G_0 a^\dagger a q + {\rm i}\, E_{\rm a}(a^\dagger - a) \\ & + {\rm i}\, E_{\rm b}(b^\dagger - b) + \beta a^\dagger b + \beta^* a b^\dagger \end{aligned} \tag{2}$$

The dynamics is derived by the following set of nonlinear quantum Langevin equations with the above Hamiltonian:

$$\dot{q} = \omega_{\rm m} p, \tag{3a}$$

$$\dot{p} = -\omega_{\rm m} q - \gamma_{\rm m} p + G_0 a^\dagger a + G_0 b^\dagger b + \xi, \tag{3b}$$

$$\dot{a} = -(\kappa + {\rm i}\,\Delta_0) a + {\rm i}\, G_0 a q + E_{\rm a} + \sqrt{2\kappa} a^{\rm in} - {\rm i}\,\beta b, \tag{3c}$$

$$\dot{b} = -(\kappa + {\rm i}\,\Delta_0) b + {\rm i}\, G_0 b q + E_{\rm b} + \sqrt{2\kappa} b^{\rm in} - {\rm i}\,\beta^* a, \tag{3d}$$

where $\Delta_0 = \omega_{\rm c} - \omega_{\rm L}$, and $\gamma_{\rm m}$ is the mechanical damping rate. We assume that the cavity decay rate $\kappa_{\rm a} = \kappa_{\rm b} = \kappa$. $a^{\rm in}$ and $b^{\rm in}$ are the vacuum radiation input noises whose nonzero correlation functions are[34]

$$\langle a^{\rm in}(t) a^{{\rm in},\dagger}(t') \rangle = \delta(t-t'), \tag{4a}$$

$$\langle b^{\rm in}(t) b^{{\rm in},\dagger}(t') \rangle = \delta(t-t'). \tag{4b}$$

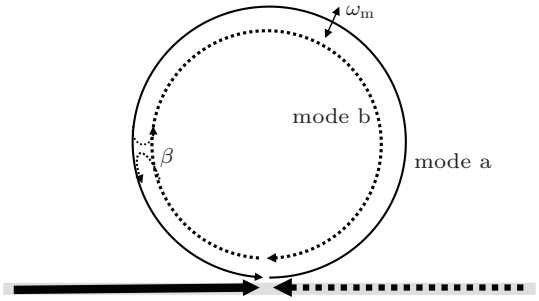


Fig. 1. Sketch of the whispering-gallery cavity system.

The Brownian noise operator ξ comes from the coupling of the movable boundary to the thermal bath, whose mean value is zero. The correlation function[35] at a certain temperature T can be expressed as follows:

$$\begin{aligned} \langle \xi(t)\xi(t') \rangle =\ & \frac{\gamma_{\rm m}}{2\pi\omega_{\rm m}} \int \omega\, {\rm e}^{-{\rm i}\,\omega(t-t')} \\ & \times \left[1 + \coth\left(\frac{\hbar\omega}{2k_{\rm B}T}\right)\right] {\rm d}\omega, \end{aligned} \tag{5}$$

where $k_{\rm B}$ is the Boltzmann constant, and $\bar{n} = [\exp(\hbar\omega_{\rm m}/k_{\rm B}T - 1)]^{-1}$ is the mean thermal excitation number. In the limit that the mechanical quality factor $Q_{\rm m} = \omega_{\rm m}/\gamma_{\rm m} \gg 1$, $\xi(t)$ becomes delta-correlated,

$$\langle \xi(t)\xi(t') + \langle \xi(t')\xi(t) \rangle /2 = \gamma_{\rm m}(2\bar{n}+1)\delta(t-t'). \tag{6}$$

We can always consider the dynamics of small fluctuations around the steady state of the system by separating each operator into two parts, i.e., a c-number steady-state value and an additional fluctuation operator with zero-mean value:[36] $q = q_{\rm s} + \delta q$, $p = p_{\rm s} + \delta p$, $a = a_{\rm s} + \delta a$, and $b = b_{\rm s} + \delta b$. From Eqs. (3a)–3(d), one can obtain the steady-state values: $p_{\rm s} = 0$, $q_{\rm s} = G_0(|a_{\rm s}|^2 + |b_{\rm s}|^2)/\omega_{\rm m}$. $a_{\rm s}$ and $b_{\rm s}$ are the solutions of the following nonlinear equations:

$$(k + {\rm i}\,\Delta_{\rm a}) a_{\rm s} + {\rm i}\,\beta b_{\rm s} = E_{\rm a},$$

$$(k+\mathrm{i}\,\Delta_{\mathrm{b}})b_{\mathrm{s}}+\mathrm{i}\,\beta^{*}a_{\mathrm{s}}=E_{\mathrm{b}},$$

where the effective cavity detuning

$$\Delta_{\mathrm{a}}=\Delta_{\mathrm{b}}=\Delta_0-G_0^2(|a_{\mathrm{s}}|^2+|b_{\mathrm{s}}|^2)/\omega).$$

Considering the very large input power P, i.e., $a_{\mathrm{s}}\gg 1$, one can safely neglect the nonlinear terms $\delta a^{\dagger}\delta a$ and $\delta a\delta q$. Choosing such a phase reference of the cavity field that a_{s} and b_{s} are real and defining the cavity field quadratures as

$$\begin{aligned}
\delta X_{\mathrm{a}}&=(\delta a^{\dagger}+\delta a)/\sqrt{2},\\
\delta Y_{\mathrm{a}}&=\mathrm{i}(\delta a^{\dagger}-\delta a)/\sqrt{2},\\
\delta X_{\mathrm{b}}&=(\delta b^{\dagger}+\delta b)/\sqrt{2},\\
\delta Y_{\mathrm{b}}&=\mathrm{i}(\delta b^{\dagger}-\delta b)/\sqrt{2},\\
X_{\mathrm{a}}^{\mathrm{in}}&=(\delta a^{\mathrm{in},\dagger}+\delta a^{\mathrm{in}})/\sqrt{2},\\
Y_{\mathrm{a}}^{\mathrm{in}}&=\mathrm{i}(\delta a^{\mathrm{in},\dagger}-\delta a^{\mathrm{in}})/\sqrt{2},\\
X_{\mathrm{b}}^{\mathrm{in}}&=(\delta b^{\mathrm{in},\dagger}+ba^{\mathrm{in}})/\sqrt{2},\\
Y_{\mathrm{b}}^{\mathrm{in}}&=\mathrm{i}(\delta b^{\mathrm{in},\dagger}-\delta b^{\mathrm{in}})/\sqrt{2},
\end{aligned}$$

one can obtain the following linear Langevin equations:

$$\delta\dot{q}=\omega_{\mathrm{m}}\delta p, \tag{7a}$$

$$\delta\dot{p}=-\omega_{\mathrm{m}}\delta q-\gamma_{\mathrm{m}}\delta P+G_{\mathrm{a}}\delta X_{\mathrm{a}}+G_{\mathrm{b}}\delta X_{\mathrm{b}}+\xi, \tag{7b}$$

$$\delta\dot{X}_{\mathrm{a}}=-\kappa\delta X_{\mathrm{a}}+\Delta_{\mathrm{a}}\delta Y_{\mathrm{a}}+\sqrt{2\kappa}X_{\mathrm{a}}^{\mathrm{in}}+\mathrm{i}\,\beta\delta Y_{\mathrm{b}}, \tag{7c}$$

$$\delta\dot{Y}_{\mathrm{a}}=G_{\mathrm{a}}\delta q-\Delta_{\mathrm{a}}\delta X_{\mathrm{a}}-\kappa\delta Y_{\mathrm{a}}+\sqrt{2\kappa}Y_{\mathrm{a}}^{\mathrm{in}}-\beta\delta X_{\mathrm{b}}, \tag{7d}$$

$$\delta\dot{X}_{\mathrm{b}}=-\kappa\delta X_{\mathrm{b}}+\Delta_{\mathrm{b}}\delta Y_{\mathrm{b}}+\sqrt{2\kappa}X_{\mathrm{b}}^{\mathrm{in}}+\mathrm{i}\,\beta\delta Y_{\mathrm{a}}, \tag{7e}$$

$$\delta\dot{Y}_{\mathrm{b}}=G_{\mathrm{b}}\delta q-\Delta_{\mathrm{b}}\delta X_{\mathrm{b}}-\kappa\delta Y_{\mathrm{b}}+\sqrt{2\kappa}Y_{\mathrm{b}}^{\mathrm{in}}-\beta\delta X_{\mathrm{a}}, \tag{7f}$$

where β is real. Equations (7a)–(7f) can be written in the matrix form

$$\dot{u}(t)=Au(t)+n(t), \tag{8}$$

where

$$u(t)=(\delta q,\delta p,\delta X_{\mathrm{a}},\delta Y_{\mathrm{a}},\delta X_{\mathrm{b}},\delta Y_{\mathrm{b}})^{\mathrm{T}}$$

is the column vector of the continuous variable fluctuation, and $n(t)=(0,\xi(t),\sqrt{2\kappa}X_{\mathrm{a}}^{\mathrm{in}}(t),\sqrt{2\kappa}Y_{\mathrm{a}}^{\mathrm{in}}(t),\sqrt{2\kappa}X_{\mathrm{b}}^{\mathrm{in}}(t),\sqrt{2\kappa}Y_{\mathrm{b}}^{\mathrm{in}}(t))^{\mathrm{T}}$ is the column vector of the noise sources. The matrix A is given by

$$A=\begin{pmatrix}
0 & \omega_{\mathrm{m}} & 0 & 0 & 0 & 0\\
-\omega_{\mathrm{m}} & -\gamma_{\mathrm{m}} & G_{\mathrm{a}} & 0 & G_{\mathrm{b}} & 0\\
0 & 0 & -\kappa & \Delta_{\mathrm{a}} & 0 & \beta\\
G_{\mathrm{a}} & 0 & -\Delta_{\mathrm{a}} & -\kappa & -\beta & 0\\
0 & 0 & 0 & \beta & -\kappa & \Delta_{\mathrm{b}}\\
G_{\mathrm{b}} & 0 & -\beta & 0 & -\Delta_{\mathrm{b}} & -\kappa
\end{pmatrix}. \tag{9}$$

The quantum fluctuations of the field and the breathing mode are now coupled by larger effective optomechanical couplings $G_{\mathrm{a}}\equiv G_0\sqrt{2}a_{\mathrm{s}}$ and $G_{\mathrm{b}}\equiv G_0\sqrt{2}b_{\mathrm{s}}$, so that the generation of significant optomechanical entanglement becomes possible. Since the quantum noises ξ, a^{in}, and b^{in} are zero-mean quantum Gaussian noises, the dynamics is linear. The quantum steady state for the fluctuations is a zero-mean tripartite Gaussian state, which can be fully characterized by its 6×6 correlation matrix (CM):

$$V_{ij}=[\langle u_i(\infty)u_j(\infty)+u_j(\infty)u_i(\infty)\rangle],$$

where $u(\infty)=(\delta q(\infty),\ \delta p(\infty),\ \delta X_{\mathrm{a}}(\infty),\ \delta Y_{\mathrm{a}}(\infty),\ \delta X_{\mathrm{b}}(\infty),\ \delta Y_{\mathrm{b}}(\infty))^{\mathrm{T}}$ is the vector of continuous variable fluctuation operators in the steady state ($t\to\infty$). Equations (7a)–(7f) can be solved as

$$u(t)=M(t)u(0)+\int_0^{\infty}\mathrm{d}t'M(t')n(t-t'), \tag{10}$$

where $M(t)=\exp(At)$. Our aim is to study the entanglement properties of the steady state. The stability conditions exist when the real parts of the eigenvalues of A are negative ($M\ (\infty)=0$). The stability conditions of the system parameters can be derived by applying the Routh–Hurwitz criterion.[37] Since the five components of $n(t)$ are uncorrelated, one can obtain

$$V_{ij}=\sum_{k,l}\int_0^{\infty}\mathrm{d}t\int_0^{\infty}\mathrm{d}t'M_{ik}(t)M_{jl}(t')\Phi_{kl}(t-t'), \tag{11}$$

where $\Phi_{kl}(t-t')=(\langle n_k(t)n_l(t')+n_l(t')u_k(t)\rangle)/2$ is the matrix of the stationary noise correlation functions. Furthermore, $\Phi_{kl}(t-t')=D_{kl}(t-t')$, where $D=\mathrm{Diag}[0,\gamma_{\mathrm{m}}(2\bar{n}+1),\kappa,\kappa,\kappa,\kappa]$ is a diagonal matrix, and Eq. (11) can be written as

$$V=\int_0^{\infty}\mathrm{d}tM(t)DM(t)^{\mathrm{T}}. \tag{12}$$

Under the stability conditions, one can obtain the following equation for the steady state CM:

$$AV+VA=-D. \tag{13}$$

The exact solution of this equation can be derived, which is not reported here because of its cumbersome expression. The 3×3 block form of the CM can be written as follows:

$$V=\begin{pmatrix}
B_{\mathrm{m}} & C_{\mathrm{am}} & C_{\mathrm{bm}}\\
C_{\mathrm{am}}^{\mathrm{T}} & B_{\mathrm{a}} & C_{\mathrm{ab}}\\
C_{\mathrm{bm}}^{\mathrm{T}} & C_{\mathrm{ab}}^{\mathrm{T}} & B_{\mathrm{b}}
\end{pmatrix}, \tag{14}$$

where B_j is a 2×2 matrix that describes the local properties of mode j, C_{ij} $(i, j = \mathrm{a}, \mathrm{b}, \mathrm{m})$ describes the intermode correlations. The symplectic spectrum of the matrix $V_{ij}^S = SV_{ij}S$ associates with the partially transposed density matrix of the i–j subsystem, and V_{ij} is a 4×4 matrix extracted from V by taking the blocks in Eq. (14) relative to modes i and j.[38] We will present the numerical results of the logarithmic negativity E_{N}^{ij} to establish the conditions, under which the optical modes and the mechanical breathing mode are entangled as well as the optical modes a and b. E_{N}^{ij} can be written as[18]

$$E_{\mathrm{N}}^{ij} = \max[0, -\ln 2\eta^-], \tag{15}$$

and

$$\eta^- \equiv \sqrt{\frac{\sum(V_{ij}) - \sqrt{\sum(V_{ij})^2 - 4\det V_{ij})}}{2}}, \tag{16}$$

where $\sum(V_{ij}) \equiv \det B_i + \det B_j - 2\det C_{ij}$. B_i, B_j, and C_{ij} are the components of the following 2×2 block form of the CM:

$$V_{ij} = \begin{pmatrix} B_i & C_{ij} \\ C_{ij}^{\mathrm{T}} & B_j \end{pmatrix}. \tag{17}$$

From Eq. (15), Gaussian states are entangled only if $\eta^- < 1/2$, which is equivalent to Simon's necessary and sufficient entanglement nonpositive partial transpose criterion for Gaussian states.[38]

3. Numerical results and discussions

In our scheme, we set $\Delta_{\mathrm{a}} = \Delta_{\mathrm{b}} = \Delta$ as shown in Section 2. Assuming $P_{\mathrm{a}} = P_{\mathrm{b}}$ so that $G_{\mathrm{a}} = G_{\mathrm{b}}$, we find that the entanglement between light mode a and the mechanical breathing mode has the same nature as that between light mode b and the mechanical breathing mode due to the symmetry of the regime. Under recent optomechanical experimental conditions[3] with respect to a silica microtoroidal optical cavity and the stability conditions, we control the temperature $T = 0.4$ K, the microcavity radius $R = 25$ μm, the effective mass of the oscillator $m = 10$ ng, the driven laser wavelength to be 1064 nm with power $P = 35$ mW, and $(k, \omega_{\mathrm{m}}, \gamma_{\mathrm{m}}, \beta)/2\pi = (5.8 \times 10^7, 7.3 \times 10^6, 1.3 \times 10^3, 6 \times 10^6)$ Hz corresponding to $Q_{\mathrm{m}} \simeq 6 \times 10^4$.

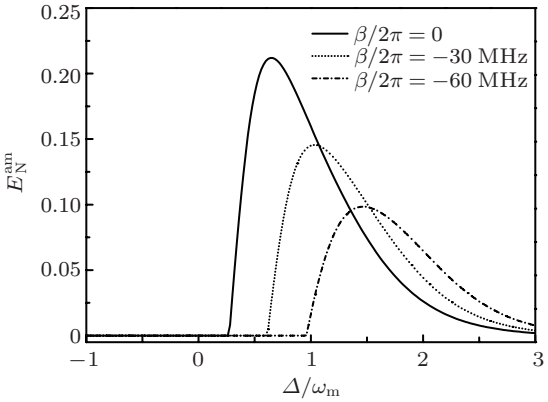


Fig. 2. Logarithmic negativity $E_{\mathrm{N}}^{\mathrm{am}}$ as functions of the normalized detuning $\Delta/\omega_{\mathrm{m}}$.

We calculated the dependence of E_{N} on the cavity normalized detuning $\Delta/\omega_{\mathrm{m}}$ for three different values of the backscattering rate β. Significant entanglement between light mode a and the mechanical breathing mode is achieved, as shown in Fig. 2. The entanglement between light modes a and b are shown in Fig. 3. Optomechanical entanglement is presented within a finite interval of Δ around $\Delta \simeq \omega_{\mathrm{m}}$, which is in agreement with the results Ref. [18]. It can be seen that entanglement between light mode a and the mechanical breathing mode decreases when the backscattering rate $|\beta|$ increases, as shown in Fig. 2. This can be interpreted from Eq. (9). The entanglement between light mode a and the mechanical breathing mode depends on the opto-mechanics coupling strength

$$\begin{aligned} G_{\mathrm{a}} &\equiv G_0\sqrt{2}\alpha_{\mathrm{s}} \\ &= E(k + \mathrm{i}\,\Delta - \mathrm{i}\,\beta)/(k^2 - \Delta^2 - \beta^2 + \mathrm{i}\,2k\Delta). \end{aligned}$$

Moreover, from Eq. (1), we can see that entanglement between light modes a and b is enhanced by increasing the backscattering rate $|\beta|$. For this three-body system composed of the mechanical breathing mode, light modes a and b, entanglement between light mode a and the mechanical breathing mode competes with entanglement between light mode a and light mode b. Entanglement between light modes a and b is stronger than that between light mode a and the mechanical breathing mode. This competition can also be seen from Fig. 3, where it is clearly shown that entanglement between light modes a and b is enhanced by increasing the backscattering rate $|\beta|$. This is mainly because increasing β enhances the coupling between cavity field modes a and b. The mechanical dissipation rate $\gamma_{\mathrm{m}} = \omega_{\mathrm{m}}/Q_{\mathrm{m}}$ governs the interaction of the system with the thermal bath. Quantum entanglement can survive because the interaction between the

light and the oscillator is much stronger than the rate of relaxation to thermal equilibrium.[21] These results suggest that one can reveal optomechanical entanglement by using the field-field correlation. Increasing the backscattering rate β can decrease entanglement between light mode a and the mechanical breathing mode.

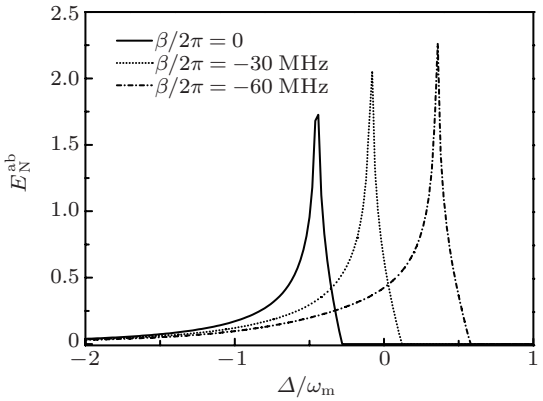


Fig. 3. Logarithmic negativity $E_{\rm N}^{\rm ab}$ as a function of normalized detuning $\Delta/\omega_{\rm m}$.

In Fig. 4, we take the same parameters as those in Fig. 2 and show the plot of logarithmic negativity $E_{\rm N}^{\rm am}$ as functions of temperature T. It is shown that optomechanical entanglement persists even at temperatures above 40 K, and is several orders of magnitude stronger than that for the ground state temperature of the mechanical oscillator.

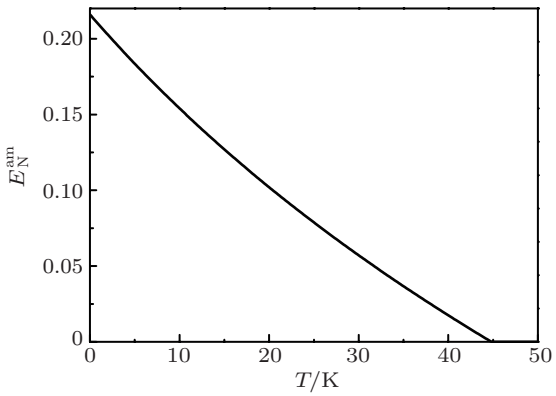


Fig. 4. Plot of logarithmic negativity $E_{\rm N}^{\rm am}$ versus temperature T. $\Delta = 0.64\omega_m$.

4. Conclusion

In conclusion, macroscopic entanglement between optical cavity field modes and a mechanical breathing mode in a whispering-gallery cavity as well as the continuous variable entanglement between the phase-quadrature amplitudes of the two whispering-gallery modes have been created. Under the current experimental conditions, we show that entanglement can be very obvious and occurs even at temperatures above 40 K. Increasing the backscattering rate β can enhance entanglement of the two cavity modes but weakens entanglement between the cavity mode and the mechanical mode. The numerical results of the logarithmic negativity $E_{\rm N}^{ij}$ have been established under the conditions where the optical modes and the mechanical breathing mode are entangled as well as optical modes a and b.

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