Scattering states of modified Pöschl–Teller potential in D-dimension*

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We present a new approximation scheme for the centrifugal term, and apply this new approach to the Schrödinger equation with the modified Pöschl–Teller potential in the D-dimension for arbitrary angular momentum states. The approximate analytical solutions of the scattering states are derived. The normalized wave functions expressed in terms of the hypergeometric functions of the scattering states on the $k/2\pi$ scale and the calculation formula of the phase shifts are given. The numerical results show that our results are in good agreement with those obtained by using the amplitude-phase method (APM).

Keywords: modified Pöschl–Teller potential in D-dimension, Schrödinger equation, scattering states

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1. Introduction

The modified Pöschl–Teller potential given by Flügge[1] can be expressed in the form

$$V(r) = -\frac{\hbar^2\alpha^2}{2M}\frac{\lambda(\lambda+1)}{\cosh^2(\alpha r)}, \tag{1}$$

where λ is a dimensionless parameter for the depth of the well, and α is a scaling parameter of coordinate r. This potential is a short-range model potential and has been used to describe bending molecular vibrations.[2−4] In the one-dimensional (1D) case, bound states and scattering states of the Schrödinger equation with the modified Pöschl–Teller potential have analytical solutions.[1,5−7] For bound states, general calculation formulas and recurrence relations of the coordinate matrix elements have been given in Refs. [7]–[10]. Unfortunately, in the three-dimensional (3D) case, they can be solved analytically only for the states with zero angular momentum.[1,5,6]

The extension of physical problems to higher-dimensional spaces plays an important role in various areas of physics. Many physical systems of interest in quantum mechanics have been thoroughly studied in D-dimensional space,[11] examples include D-dimensional hydrogen atoms[11−17] and harmonic oscillators.[11,13−15,18−20] Recently, the extensions of this study to bound states and scattering states of the D-dimensional Hulthén potential[21,22] and to bound states of the modified Pöschl–Teller potential[23] for the Schrödinger equation were presented by Agboola and Chen *et al.*, respectively. In their studies, the approximate analytical solutions of the bound states and the scattering states have been given. The main idea relied on an approximation of the centrifugal term. In their approximation schemes,

$$\frac{1}{r^2} \approx \frac{\alpha^2\,\mathrm{e}^{-\alpha r}}{(1-\mathrm{e}^{-\alpha r})^2} = \frac{(\alpha/2)^2}{\sinh^2(\alpha r/2)}$$

was used for the centrifugal term. Unfortunately, that approximation is valid only for small values of parameter α and breaks down for large α. Therefore, it is of considerable interest to find a new approximation scheme to deal with the centrifugal term.

Analytically exact solutions of the wave equations with some exponential-type potentials are impossible for $l \neq 0$ states in three dimensions. Therefore, other approximation schemes for the centrifugal term are suggested besides

$$\frac{1}{r^2} \approx \frac{\alpha^2\,\mathrm{e}^{-\alpha r}}{(1-\mathrm{e}^{-\alpha r})^2} = \frac{(\alpha/2)^2}{\sinh^2(\alpha r/2)},$$

such as the Hulthén potential,[24−26] the Eckart potential,[27] and the Manning–Rosen potential.[28−30] In this paper, we develop a new approximation scheme for the centrifugal term and solve the Schrödinger equation of the D-dimensional modified Pöschl–Teller potential for the scattering states. We show that the new approximation scheme is better than the old one.

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The rest of this paper is organized as follows. In Section 2, we present a new approximation scheme for the centrifugal term, solve approximately the Schrödinger equation with the D-dimensional modified Pöschl–Teller potential for the scattering states, and give the normalized wave functions in terms of the hypergeometric functions of scattering states on the $k/2\pi$ scale and the calculation formula of the phase shifts. In order to verify the effectiveness of the new approximation scheme, in Section 3, we calculate numerically the phase shifts for different values of screening parameter α by using the present analytical formula, and compare our results with those obtained by using the amplitude-phase method (APM). The conclusions are given in Section 4.

2. Approximate analytical solutions of scattering states

In D-dimensional spherical coordinates, the Schrödinger equation is[11,15]

$$\left[-\frac{\hbar^2}{2M}\boldsymbol{\nabla}_D^2 + V(r)\right]\Psi(x) = E\,\Psi(x), \tag{2}$$

with

$$\begin{aligned}\Psi(x) =\ & \mathrm{Y}_{J_{D-2}J_{D-3}\cdots J_1J_0}(\theta_{D-2},\theta_{D-3},\ldots,\theta_1,\theta_0)\\ & \times\frac{u(r)}{r^{(D-1)/2}}\,,\end{aligned} \tag{3}$$

where $\mathrm{Y}_{J_{D-2}J_{D-3}\ldots J_1J_0}(\theta_{D-2},\theta_{D-3},\ldots,\theta_1,\theta_0)$ is the spherical function in D-dimensional space, and J_{D-2} is the quantum number of the Casimir operator of the SO_D group, which is a positive integer or zero. Thus by separating the variables of the Schrödinger equation, we can obtain the radial equation of the D-dimensional modified Pöschl–Teller potential as[23]

$$\begin{aligned}&\frac{\mathrm{d}^2u(r)}{\mathrm{d}r^2} + \left\{\frac{2M}{\hbar^2}E + \frac{\alpha^2\lambda\,(\lambda+1)}{\cosh^2(\alpha r)}\right.\\ &\quad -\frac{1}{r^2}[J_{D-2}+(D-1)/2]\\ &\quad \left.\times\,[J_{D-2}+(D-3)/2]\right\}u(r) = 0\,,\end{aligned} \tag{4}$$

where

$$\frac{1}{r^2}[J_{D-2}+(D-1)/2]\,[J_{D-2}+(D-3)/2]$$

is know as the centrifugal term. Obviously, equation (4) cannot be solved exactly due to the centrifugal term. To obtain a quasi-analytical solution to the above equation, we must use an approximation for the centrifugal term. Agboola[23] has used the following approximation to take into account the centrifugal term:

$$\frac{\alpha^2}{\sinh^2(\alpha r)} = \frac{\alpha^2}{[(\mathrm{e}^{\alpha r}-\mathrm{e}^{-\alpha r})/2]^2} \approx \frac{1}{r^2}, \tag{5}$$

which is a good approximation for small values of parameter α. Here, we propose a new approximation scheme to deal with the centrifugal term. We take the following exponential-type potential to approximate the centrifugal term:

$$\begin{aligned}&\alpha^2\left[\frac{s}{\sinh^2(\alpha r)} + \frac{t}{\cosh^2(\alpha r)}\right]\\ &= \alpha^2\left\{\frac{s}{[(\mathrm{e}^{\alpha r}-\mathrm{e}^{-\alpha r})/2]^2}\right.\\ &\quad \left.+\frac{t}{[(\mathrm{e}^{\alpha r}+\mathrm{e}^{-\alpha r})/2]^2}\right\} \approx \frac{1}{r^2},\end{aligned} \tag{6}$$

where t and s are adjustable dimensionless parameters. Obviously, the above approximation to the centrifugal term turns into $1/r^2$ when parameter α goes to infinity and s goes to 1,

$$\begin{aligned}&\lim_{\alpha\to0,\,s\to1}\ \alpha^2\left[\frac{s}{\sinh^2(\alpha r)} + \frac{t}{\cosh^2(\alpha r)}\right]\\ &= \lim_{\alpha\to0,\,s\to1}\ \alpha^2\left\{\frac{s}{[(\mathrm{e}^{\alpha r}-\mathrm{e}^{-\alpha r})/2]^2}\right.\\ &\quad \left.+\frac{t}{[(\mathrm{e}^{\alpha r}+\mathrm{e}^{-\alpha r})/2]^2}\right\} = \frac{1}{r^2},\end{aligned} \tag{7}$$

which shows that the old approximation scheme (5) is the limit of the new approximation scheme (6) when $s=1$ and $t=0$. Substituting Eq. (6) into Eq. (4), we have

$$\begin{aligned}&\frac{\mathrm{d}^2u(x)}{\mathrm{d}x^2} + \left[\frac{k^2}{\alpha^2} + \frac{\lambda'(\lambda'+1)}{\cosh^2 x}\right.\\ &\quad \left.-\frac{\nu'(\nu'-1)}{\sinh^2 x}\right]u(x) = 0,\end{aligned} \tag{8}$$

where

$$x=\alpha r,\quad k=\sqrt{2ME/\hbar^2},\quad \nu = J_{D-2}+(D-1)/2,$$

$$\begin{aligned}\lambda' &= [-1+\sqrt{1+4\lambda(\lambda+1)-4t\,\nu(\nu-1)}]/2,\\ \nu' &= [1+\sqrt{1+4s\,\nu\,(\nu-1)}]/2.\end{aligned} \tag{9}$$

Rewriting Eq. (8) with a new variable of the form $z=\tanh^2 x$ ($r\in[0,\infty)$, $z\in[0,1]$), we obtain

$$\begin{aligned}&\frac{\mathrm{d}^2u(z)}{\mathrm{d}z^2} + \frac{1-3z}{2z(1-z)}\frac{\mathrm{d}\,u(z)}{\mathrm{d}z}\\ &\quad + \frac{1}{4z^2(1-z)^2}\left\{-\lambda'(\lambda'+1)z^2\right.\\ &\quad \left.+\left[\lambda'(\lambda'+1)+\nu'(\nu'-1)+\frac{k^2}{\alpha^2}\right]z\right.\end{aligned}$$

$$-\nu'(\nu'-1)\Big\}u(z)=0. \tag{10}$$

For the scattering states, $E>0$ and $k>0$. In the 3D case, the boundary conditions of the scattering states for a short-range potential are

$$\begin{aligned} &u(r)\underset{r\to 0}{\longrightarrow}0,\\ &u(r)\underset{r\to\infty}{\longrightarrow}2\sin(kr+\delta_l-l\pi/2). \end{aligned} \tag{11}$$

We know from Refs. [31] and [32] that the radial wave functions of the scattering states are normalized on the $k/2\pi$ scale. As shown in Refs. [17], [22], and [33], to extend to the D-dimensional case, we need to have

$$\begin{aligned} &u(r)\underset{r\to 0}{\longrightarrow}0,\\ &u(r)\underset{r\to\infty}{\longrightarrow}2\sin[kr+\delta_{J_{D-2}}\\ &\qquad-\pi\,(J_{D-2}+(D-3)/2)/2)] \end{aligned} \tag{12}$$

to assure that the radial wave functions of the scattering states for a short-range potential are also normalized on the $k/2\pi$ scale, where $\delta_{J_{D-2}}$ represents the phase shift.

According to the asymptotic behavior of the radial wave functions of the continuous states at $r\to 0$, we take the wave function in the form of

$$u(z)=z^{\nu'/2}(1-z)^{-\mathrm{i}\,k/2\alpha}f(z). \tag{13}$$

Substituting Eq. (13) into Eq. (10), we can obtain the following second-order differential equation:

$$\begin{aligned} z(1-z)\frac{\mathrm{d}^2f(z)}{\mathrm{d}z^2}+\left[\nu'+\frac{1}{2}-\left(\nu'+\frac{3}{2}-\frac{\mathrm{i}\,k}{\alpha}\right)z\right]\frac{\mathrm{d}f(z)}{\mathrm{d}z}\\ +\frac{1}{4}\Big[\lambda'(\lambda'+1)-\nu'(\nu'-1)\\ +\frac{k^2}{\alpha^2}+\frac{\mathrm{i}\,k}{\alpha}(2\nu'+1)\Big]f(z)=0, \end{aligned} \tag{14}$$

which is the hypergeometric differential equation.[34,35] Thus, the analytical solution at $z=0$ ($r\to 0$) is the hypergeometric function

$$f(z)={}_2\mathrm{F}_1(a,\,b\,;\;c\,;z). \tag{15}$$

And the parameters are

$$a=(\nu'-\lambda')/2-\mathrm{i}\,k/2\alpha, \tag{16a}$$

$$b=(\lambda'+\nu'+1)/2-\mathrm{i}\,k/2\alpha, \tag{16b}$$

$$c=\nu'+1/2. \tag{16c}$$

Therefore, the radial wave function of the scattering state is

$$\begin{aligned} u(r)=\;&N_{k\,J_{D-2}}[\tanh^2(\alpha r)]^{\nu'/2}\,[1-\tanh^2(\alpha r)]^{-\mathrm{i}\,k/2\alpha}\\ &\times{}_2\mathrm{F}_1(a,\,b;\,c;\,\tanh^2(\alpha r)). \end{aligned} \tag{17}$$

We now study the asymptotic form of the above expression for large r, and calculate the normalization constant $N_{k\,J_{D-2}}$ of the radial wave function and phase shift $\delta_{J_{D-2}}$. From Eq. (16), we have

$$c-a-b=\mathrm{i}\,k/\alpha=(a+b-c)^*, \tag{18a}$$

$$c-a=(\nu'+\lambda'+1)/2+\mathrm{i}\,k/2\alpha=b^*, \tag{18b}$$

$$c-b=(\nu'-\lambda')/2+\mathrm{i}\,k/2\alpha=a^*. \tag{18c}$$

Applying the transformation formulas to the hypergeometric function[34,35]

$$\begin{aligned} {}_2\mathrm{F}_1(a,\,b\,;\;c\,;z)=\;&\frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\,{}_2\mathrm{F}_1(a,b;a+b-c+1;\,1-z)\\ &+(1-z)^{c-a-b}\frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}\,{}_2\mathrm{F}_1(c-a,c-b;c-a-b+1;\,1-z), \end{aligned} \tag{19}$$

we have

$$\begin{aligned} {}_2\mathrm{F}_1(a,\,b\,;\;c\,;\tanh^2(\alpha r))=\;&\frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\,{}_2\mathrm{F}_1(a,b;a+b-c+1;\,1-\tanh^2(\alpha r))+[1-\tanh^2(\alpha r)]^{c-a-b}\\ &\times\frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}\,{}_2\mathrm{F}_1(c-a,c-b;c-a-b+1;\,1-\tanh^2(\alpha r)). \end{aligned} \tag{20}$$

From

$$1-\tanh^2(\alpha r)=\frac{1}{\cosh^2(\alpha r)}=\frac{4}{(\,\mathrm{e}^{\alpha r}+\mathrm{e}^{-\alpha r})^2}\underset{r\to\infty}{\longrightarrow}4\,\mathrm{e}^{-2\alpha r}, \tag{21}$$

we have

$$\left[1-\tanh^2(\alpha r)\right]^{-\mathrm{i}\,k/2\alpha}\underset{r\to\infty}{=\!\!=}2^{-\mathrm{i}\,k/\alpha}\,\mathrm{e}^{\mathrm{i}\,kr},\quad\text{and}\quad\left[1-\tanh^2(\alpha r)\right]^{c-a-b}\underset{r\to\infty}{=\!\!=}2^{2\,\mathrm{i}\,k/\alpha}\,\mathrm{e}^{-2\,\mathrm{i}\,kr}. \tag{22}$$

Substituting Eqs. (20), (21), and (22) into Eq. (17) and considering ${}_2\mathrm{F}_1(a,\,b;\,c;\,0)=1$, we have

$$
\begin{aligned}
u(r) &= N_{k\,J_{D-2}}[\tanh^2(\alpha r)]^{\nu'/2}\,[1-\tanh^2(\alpha r)]^{-\mathrm{i}\,k/2\alpha}\,{}_2\mathrm{F}_1(a,\,b;\,c;\,\tanh^2(\alpha r)) \\
&\underset{r\to\infty}{\longrightarrow} N_{k\,J_{D-2}}2^{-\mathrm{i}\,k/\alpha}\,\mathrm{e}^{\mathrm{i}\,kr}\Gamma(c)\left[\frac{\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}+2^{2ik/\alpha}\,\mathrm{e}^{-2ikr}\frac{\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}\right] \\
&\underset{r\to\infty}{\longrightarrow} N_{k\,J_{D-2}}\Gamma(c)\left[\frac{\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}2^{-\mathrm{i}\,k/\alpha}\,\mathrm{e}^{\mathrm{i}\,kr}+2^{\mathrm{i}\,k/\alpha}\,\mathrm{e}^{-\mathrm{i}\,kr}\left(\frac{\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\right)^*\right]. \qquad (23)
\end{aligned}
$$

Let

$$
\frac{\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}=\left|\frac{\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\right|\,\mathrm{e}^{\mathrm{i}\,\delta}, \qquad (24a)
$$

then

$$
\left(\frac{\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\right)^*=\left|\frac{\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\right|\,\mathrm{e}^{-\mathrm{i}\,\delta}, \qquad (24b)
$$

where δ is a real number. Equation (23) then becomes

$$
\begin{aligned}
u(r) &\underset{r\to\infty}{\longrightarrow} N_{k\,J_{D-2}}\Gamma(c)\left|\frac{\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\right|\left[\mathrm{e}^{\mathrm{i}(kr+\delta-k\ln 2/\alpha)}+\mathrm{e}^{-\mathrm{i}(kr+\delta-k\ln 2/\alpha)}\right] \\
&\underset{r\to\infty}{\longrightarrow} 2N_{k\,J_{D-2}}\Gamma(c)\left|\frac{\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\right|\cos[kr+\delta-k\ln 2/\alpha] \\
&\underset{r\to\infty}{\longrightarrow} 2N_{k\,J_{D-2}}\Gamma(c)\left|\frac{\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\right| \\
&\quad\times\sin\{kr+\delta-k\ln 2/\alpha-\pi[J_{D-2}+(D-3)/2]/2+\pi(J_{D-2}+(D-1)/2)/2\}. \qquad (25)
\end{aligned}
$$

Comparing Eq. (12) with Eq. (25), we obtain the normalization constant of the scattering state as

$$
\begin{aligned}
N_{k\,l} &= \frac{1}{\Gamma(c)}\left|\frac{\Gamma(c-a)\Gamma(c-b)}{\Gamma(c-a-b)}\right| \\
&= \frac{1}{\Gamma(\nu'+1/2)}\left|\frac{\Gamma((\nu'+\lambda'+1)/2+\mathrm{i}\,k/2\alpha)\Gamma((\nu'-\lambda')/2+\mathrm{i}\,k/2\alpha))}{\Gamma(\mathrm{i}\,k/\alpha)}\right|, \qquad (26)
\end{aligned}
$$

and phase shifts $\delta_{J_{D-2}}$ as

$$
\begin{aligned}
\delta_{J_{D-2}} &= \pi[J_{D-2}+(D-1)/2]/2-k\ln 2/\alpha+\arg\Gamma(c-a-b)-\arg\Gamma(c-a)-\arg\Gamma(c-b) \\
&= \pi[J_{D-2}+(D-1)/2]/2-k\ln 2/\alpha+\arg\Gamma(c-a-b)+\arg\Gamma(b)+\arg\Gamma(a) \\
&= \pi[J_{D-2}+(D-1)/2]/2-k\ln 2/\alpha+\arg\Gamma(\mathrm{i}\,k/\alpha)+\arg\Gamma((\nu'+\lambda'+1)/2-\mathrm{i}\,k/2\alpha) \\
&\quad+\arg\Gamma((\nu'-\lambda')/2-\mathrm{i}\,k/2\alpha). \qquad (27)
\end{aligned}
$$

Substituting Eq. (26) into Eq. (17), we obtain the normalized wave functions of the continuous states with the modified Pöschl–Teller potential in the D–dimensional on the $k/2\pi$ scale as

$$
\begin{aligned}
u_{kJ_{D-2}}(r) &= \frac{1}{\Gamma(\nu'+1/2)}\left|\frac{\Gamma((\nu'+\lambda'+1)/2+\mathrm{i}\,k/2\alpha)\Gamma((\nu'-\lambda')/2+\mathrm{i}\,k/2\alpha)}{\Gamma(\mathrm{i}\,k/\alpha)}\right| \\
&\quad\times[\tanh^2(\alpha r)]^{\nu'/2}[1-\tanh^2(\alpha r)]^{-\mathrm{i}\,k/2\alpha}\,{}_2\mathrm{F}_1(a,b;c;\tanh^2(\alpha r)), \qquad (28)
\end{aligned}
$$

where k and ν' are given in formula (9), a, b, and c are given in formula (16).

3. Numerical results and discussion

In order to show that the approximation scheme of the centrifugal term given by Eq. (6) is better than that given by Eq. (5) for the scattering states, we plot the variations of $1/r^2$, $\alpha^2/\sinh^2(\alpha r)$, and $\alpha^2\left[s/\sinh^2(\alpha r)+t/\cosh^2(\alpha r)\right]$ with αr in Fig. 1. It is obvious that the approximation scheme of the centrifugal term given by Eq. (6) is much closer to $1/r^2$ than that given by Eq. (5) for both small and large values of parameter α.

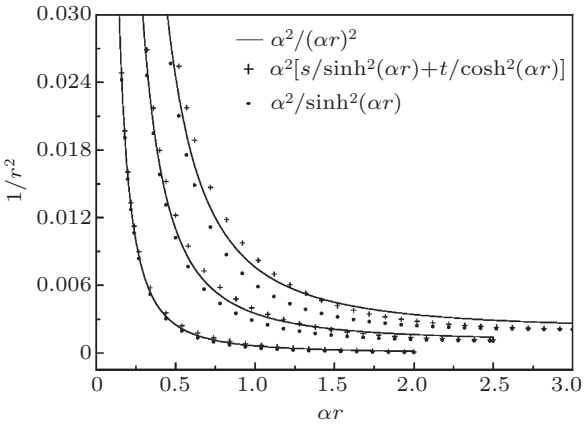


Fig. 1. Variations of $1/r^2$, $\alpha^2/\sinh^2(\alpha r)$ (old scheme), and $\alpha^2\left[s/\sinh^2(\alpha r)+t/\cosh^2(\alpha r)\right]$ (new scheme) with αr. The $s=1$, $t=1.016$, and α changes from 0.025, 0.05 to 0.075 respectively.

We give the values of the scattering phase shifts calculated by Eq. (27) (present) α for $D=3$ and 4 in Tables 1 and 2, respectively. Some values of k (equivalent to energy E) and some typical values of α are considered. To make a comparison, we also list the values of the scattering phase shifts calculated by the amplitude-phase method.[28,36−39] The previous columns in Tables 1 and 2 give the results calculated using the approximation scheme given by Eq. (5). It is obvious that the present results are in better agreement with those obtained by the APM than the previous ones. The data in all tables show that the approximate expression in Eq. (6) is much better than that in Eq. (5).

Table 1. Scattering phase shift as a function of $k=\sqrt{2E}$ for $D=3$, $\lambda=10$, $s=1$ and $t=1.016$. Results are given in atomic units ($\hbar=M=1$).

k	α	$l=1$			$l=2$		
		Present	Previous	APM	Present	Previous	APM
1.0	0.025	1.358231	1.383049	1.358279	1.355487	1.429803	1.355518
	0.050	2.628565	2.675226	2.627007	2.614364	2.753596	2.609044
	0.075	3.761249	3.825562	3.755829	3.726903	3.918328	3.709160
3.0	0.025	0.457577	0.466022	0.457731	0.457233	0.482560	0.457599
	0.050	0.911469	0.928225	0.911693	0.910334	0.960560	0.910780
	0.075	1.358231	1.383049	1.358289	1.355487	1.429803	1.355513
5.0	0.025	0.274785	0.279860	0.274844	0.274608	0.289833	0.274758
	0.050	0.548766	0.558887	0.548870	0.548313	0.578667	0.548672
	0.075	0.821156	0.836267	0.821344	0.820235	0.865534	0.820692
7.0	0.025	0.196322	0.199949	0.196378	0.196202	0.207082	0.196331
	0.050	0.392350	0.399593	0.392329	0.392073	0.413798	0.392401
	0.075	0.587794	0.598632	0.587913	0.587288	0.619790	0.587685
9.0	0.025	0.152710	0.155532	0.152756	0.152618	0.161082	0.152835
	0.050	0.305282	0.310920	0.305359	0.305081	0.321993	0.305414
	0.075	0.457577	0.466022	0.457731	0.457233	0.482560	0.457599
11.0	0.025	0.124951	0.127260	0.124931	0.124877	0.131802	0.125001
	0.050	0.249826	0.254441	0.249894	0.249668	0.263511	0.249894
	0.075	0.374549	0.381464	0.374645	0.374289	0.395031	0.374589
13.0	0.025	0.105731	0.107684	0.105773	0.105668	0.111529	0.105757
	0.050	0.211416	0.215321	0.211493	0.211285	0.223001	0.211506
	0.075	0.317008	0.322862	0.317098	0.316798	0.334358	0.317038
15.0	0.025	0.091635	0.093328	0.091695	0.091581	0.096660	0.091668
	0.050	0.183240	0.186625	0.183296	0.183128	0.193284	0.183268
	0.075	0.274785	0.279860	0.274869	0.274608	0.289833	0.274754

Table 2. Scattering phase shift as a function of $k = \sqrt{2E}$ for $D = 4$, $\lambda = 10$, $s = 1$ and $t = 1.016$. Results are given in atomic units ($\hbar = M = 1$).

k	α	$J_{D-2} = 0$			$J_{D-2} = 1$		
		Present	Previous	APM	Present	Previous	APM
1.0	0.025	1.359120	1.368432	1.359151	1.357011	1.403507	1.357096
	0.050	2.633203	2.650730	2.632658	2.622232	2.709517	2.619123
	0.075	3.772443	3.796618	3.770451	3.745944	3.866120	3.735374
3.0	0.025	0.457686	0.460853	0.457731	0.457426	0.473258	0.457621
	0.050	0.911833	0.918119	0.911894	0.910966	0.942373	0.911320
	0.075	1.359120	1.368432	1.359146	1.357011	1.403507	1.357107
5.0	0.025	0.274841	0.276744	0.274861	0.274708	0.284223	0.274882
	0.050	0.548909	0.552705	0.548966	0.548566	0.567541	0.548780
	0.075	0.821451	0.827119	0.821503	0.820748	0.849072	0.821101
7.0	0.025	0.196360	0.197720	0.196403	0.196270	0.203070	0.196307
	0.050	0.392438	0.395154	0.392441	0.392229	0.405808	0.392386
	0.075	0.587955	0.592019	0.588049	0.587571	0.607889	0.587822
9.0	0.025	0.152739	0.153797	0.152756	0.152670	0.157960	0.152747
	0.050	0.305345	0.307459	0.305390	0.305194	0.315764	0.305301
	0.075	0.457686	0.460853	0.457731	0.457426	0.473258	0.457621
11.0	0.025	0.124974	0.125840	0.124986	0.124918	0.129247	0.124987
	0.050	0.249876	0.251606	0.249936	0.249757	0.258409	0.249916
	0.075	0.374632	0.377225	0.374650	0.374435	0.387400	0.374572
13.0	0.025	0.105750	0.106483	0.105754	0.105703	0.109366	0.105729
	0.050	0.211457	0.212921	0.211497	0.211358	0.218681	0.211408
	0.075	0.317075	0.319270	0.317143	0.316916	0.327892	0.317129
15.0	0.025	0.091652	0.092287	0.091631	0.091611	0.094786	0.091669
	0.050	0.183275	0.184544	0.183248	0.183191	0.189538	0.183334
	0.075	0.274841	0.276744	0.274877	0.274708	0.284223	0.274816

4. Conclusion

We propose a new approximation scheme for the centrifugal term, with which we obtain approximate analytical solutions of the scattering states for the D-dimensional modified Pöschl–Teller potential. The phase shift is given in Eq. (27), and the normalized wave function expressed in terms of the hypergeometric functions of the scattering state on the $k/2\pi$ scale is given in Eq. (28). The numerical results show that our results are in good agreement with those obtained by using the APM, and that our new approximation scheme is better than the old one.

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