Word diversity can accelerate consensus in naming game

Zhang Xiao-Dan(张晓丹)[a)b)], Wang Zhen (王 震)[a)†],
Zheng Fei-Fei(郑非非)[a)c)], and Yang Miao(杨 淼)[a)]

[a)] *School of Innovation Experiment, Dalian University of Technology, Dalian* 116024, *China*
[b)] *School of Mathematical Science, Dalian University of Technology, Dalian* 116024, *China*
[c)] *School of Chemical Engineering, Dalian University of Technology, Dalian* 116024, *China*

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In this paper, we introduce word diversity that reflects the inhomogeneity of words in a communication into the naming game. Diversity is realized by assigning a weight factor to each word. The weight is determined by three different distributions (uniform, exponential, and power-law distributions). During the communication, the probability that a word is selected from speaker's memory depends on the introduced word diversity. Interestingly, we find that the word diversity following three different distributions can remarkably promote the final convergency, which is of high importance in the self-organized system. In particular, for all the ranges of amplitude of distribution, the power-law distribution enables the fastest consensus, while uniform distribution gives the slowest consensus. We provide an explanation of this effect based on both the number of different names and the number of total names, and find that a wide spread of names induced by the segregation of words is the main promotion factor. Other quantities, including the evolution of the averaging success rate of negotiation and the scaling behavior of consensus time, are also studied. These results are helpful for better understanding the dynamics of the naming game with word diversity.

Keywords: naming game, word diversity, consensus

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1. Introduction

Over the past few decades, the study of collective behavior has received growing attention in statistical physics, economics, and sociology.[1−10] Language dynamics, as a typical aspect of social dynamics, has been extensively studied with focusing on language evolution[11−13] and competition.[14−17] Recently, a language game, the naming game,[1,2] related to opinion formation[7,18] but with many differences, has been considered as an important approach to understand the evolution of a language or communication behaviors among a population of agents. In the naming game, agents mutually communicate about an unknown object and reach a final consensus on naming the object without global supervision or a prior common knowledge. A prototypic example of the naming game is the artificial intelligence and peer-to-peer communication system, the so-called talking heads experiment,[19,20] in which embodied software robots assign names to objects observed through digital cameras and communicate these names with each other. The naming game has also promised practically applications in the study of newly developed web tools, such as del.icio.us or www.flickr.com, which enable network users to share the classification of information about the tagging items.

Recently, a minimal naming game was proposed by Baronchelli *et al.*[21] on a fully-connected network, reproducing the same experimental phenomena, in which a successful negotiation between two agents leads to the preservation of the name in the negotiation and the cancelation of the other names stored in the memory. Then Baronchelli *et al.*[22] further investigated the effect of a low-dimensional lattice on the naming game and found that a consensus is reached through a coarsening process of clusters. Meanwhile, inspired by the speedy development of various networks,[23−29] the game has been extensively investigated on complex networks.[30−35] Additionally, many modified versions of the model have been proposed to study the evolution and the scaling property of the agreement dynamics, such as the reverse naming game,[31] the neighborhood influence,[36]

†Corresponding author. E-mail: wangz@dlut.edu.cn
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the local broadcast,[37,38] the bounded rationality,[39] the reputation,[40] and the forgetting curve.[41] At the same time, social and heterogeneous influences on words have also been identified as a potent mechanism. Tang *et al.*[42] first introduced connectivity-induced weighted words, where the weight of a word spoken by an agent is assigned by his connectivity, which is a natural reflection of the agent's influence in the population. Then, Zhuang *et al.*[43] considered the preferential words in the naming game, which is close to a realistic case in which people usually preferentially use hot words in communication. Recently, Lei *et al.*[44] studied the co-evolution of weight with name in a structured language game, where the weight of a name is defined as the communication frequency. In their study, the probability that a word is chosen from a speaker's memory is proportional to the word's weight and is tuned by a model parameter. Studies have found that a faster formation of consensus can be obtained by tuning the parameter.

In all the weighted naming games above, the word diversity considered is induced by different agents or random usages. However, in real society, name diversity may come from an internal property of the word. For instance, some words are difficult to pronounce or spell, which results in fewer usages. Furthermore, a similar diversity of internal property has been studied in other research fields. Perc and Szolnoki[45] considered social diversity in a spatial prisoner's dilemma game, and Shi *et al.*[46] introduced group diversity into the spatial public goods game. Those two kinds of diversity, the internal properties of agents and groups, are realized by rescaling the multiplication factor r to follow three distributions (uniform, exponential, and power-law distributions). They found that a simple yet meaningful mechanism maintains the diversity of the strategies. Motivated by those successful research efforts, we introduce word diversity following the above three distributions into the minimal naming game and study how the word diversity affects the convergence of a shared lexicon.

2. Model

Below, we describe the modified minimal naming game with word diversity on the fully-connected network. In the game, N identical agents obverse a single object and communicate its name with each other. Each agent is endowed with a memory to store an unlimited number of different names or opinions, and initially the memory is empty. At each time step, a pair of agents are randomly selected to communicate with each other. One acts as a speaker, and the other acts as a hearer. The speaker has to transmit a name to the hearer. So a new name is invented if the speaker's memory is empty. Otherwise, it selects one of the names stored in the memory with probability

$$P_i = \frac{r_i}{\Sigma_{j\in\Omega_{\rm speaker}} r_j}\,, \tag{1}$$

where r_i is the weight of name i. According to Eq. (1), names with higher weights have more opportunities to be selected. This is based on the consideration that in reality, some words are difficult to pronounce or spell, which is the internal property of the words. If $r_i = 1$, all the names are identical, which is consistent with the minimal naming game. Motivated by the work of the prisoner's dilemma game,[45,46] we define factor r as

$$r = 1 + \xi, \tag{2}$$

where ξ is a scaling factor drawn randomly from a given distribution to determine the diversity of the word. Here we mainly study three different distributions, uniform, exponential, and power-law distributions, which are defined by the following functions respectively:

$$\xi = A(-2\chi + 1), \tag{3}$$

$$\xi = A(-\ln\chi - 1), \tag{4}$$

$$\xi = A(\chi^{-0.5} - 2), \tag{5}$$

where χ is a random number uniformly distributed on a unit interval, and $\int_0^1 \xi(\chi)\mathrm{d}\chi = 0$ for the above three distributions, which ensures that the average of ξ over all the words is zero. So we can have $\langle r\rangle = 1$, which is a relation of r_i in the naming game, where $\langle r\rangle$ is the average value of r_i over all the words. The parameter A ($0 \le A \le 1$) determines the amplitude of the undulation of ξ, and a lager value of A induces a larger undulation. When $A = 0$, the system reduces to the original naming game, in which all the words have the same value of r. The $A = 1$ is the largest value ensuring the relation of $(1 + \xi) \ge 0$.

The spatial distributions of the three considered conditions with $A = 1.0$ are shown in Fig. 1. It is obviously seen that the uniform distribution has the gentlest dispersion of ξ, whereas the power-law distribution leads to the largest segregation of word diversity. The exponential distribution yields a dispersion

of the diversity that is in between the uniform and the power-law distributions.

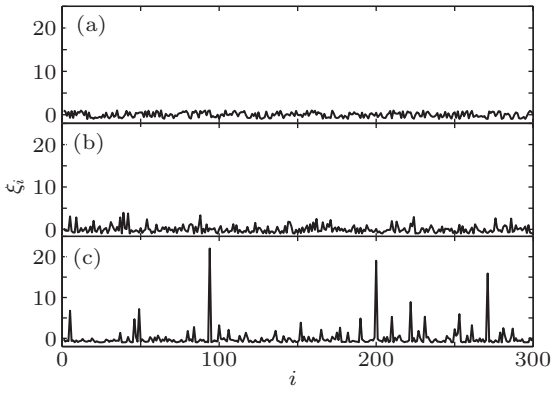


Fig. 1. Spatial distributions of ξ obtained with $A = 1.0$ for the cases of (a) uniform, (b) exponential, and (c) power-law distributions. Lines show numbers of different names, and we take 300 names for example.

We need to point out that the word weight has been considered previously in the naming game,[42−44] and it is totally different from that in our model. In the previous models, the weight is determined by the topological properties of networks,[42] the success rate,[44] or the using frequency,[43] while in our model, the word weight is the internal property of the word itself.

Similar to previous studies,[21] the speaker transmits the invented name or the selected one to the hearer. If the hearer finds the same name existing in his memory, the negotiation is successful, and both agents delete all other names and preserve this common one. Otherwise, the negotiation fails, and the new name is included in the memory of the hearer without any deletion, i.e., the new word is learned.

With respect to the definition of word diversity r_i, which is the weight of a word, we assume that (i) the value of r_i is determined only when the name is invented; (ii) no matter whether the negotiation is successful or not, the hearer endows the retained or the newly added word i with its weight r_i. It is evident that the names with larger weights have more opportunities to be transmitted. The system will go on until it reaches a global consensus.

Computer simulations are carried out with a population size of $N = 2000$. The key quality-consensus time T_c is defined as the time to reach the convergence of a shared lexicon in the system. In all the simulations, the final results are averaged over up to 10000 independent runs for each set of parameters in order to assure a suitable accuracy.

3. Numerical results

First, we study the effects of the amplitude of undulation A on consensus time T_c for the different distributions in the modified naming game with name diversity. Figure 2 shows the variations of consensus time T_c with A for different distributions of word diversity r_i. It is obvious that T_c decreases monotonically as parameter A increases irrespective of the distribution of r_i. For small values of A, the influence of A on T_c is marginal, and the system needs a sufficiently long time to reach the final consensus. However, as A increases, the consensus time can be effectively shortened, but the acceleration is not so evident as A approaches 1. Moreover, it should be noted that among the three distributions, the power-law distribution always leads to the shortest consensus time, as shown in Fig. 2. We know that a large value of A will induce a large segregation of the word diversity,[45,46] particularly the power-law distribution leads to the largest segregation, as shown in Fig. 1. So the result above suggests that the inhomogeneity of the word diversity we introduced can facilitate the global consensus, which attests to the influence of word inhomogeneity on the naming game as observed in a similar report.[43]

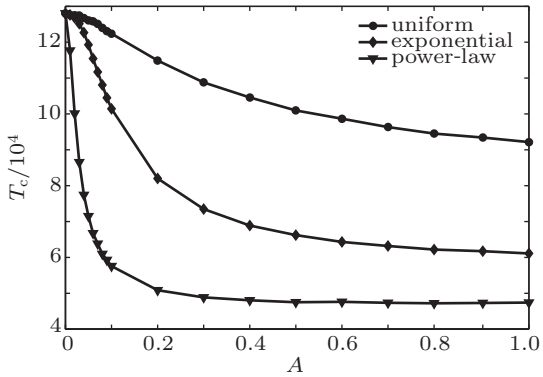


Fig. 2. Dependences of convergence time T_c on the amplitude of undulation A for different distributions of word diversity. Note that T_c decreases monotonically as A increases, and the power-law distribution always leads to the shortest consensus time.

To study the effects of different distributions on the final consensus, we will examine the time evolution of our system by looking at some usual global quantities:[21,22] the numbers of different words (N_d), the total words (N_w) in the population, and the success rate (S) that measures the average rate of success of communications.

The dynamics of the naming game can be regarded as a process of competition among different

names. Via the competition, one of them will invade others until a global consensus is reached. Figure 3 shows the evolutions of different word numbers $N_{\rm d}(t)$ for different distributions with $A = 1.0$. Similar to the case in the original naming game, the total number of different words quickly reaches a maximum value of $N/2$, which is called the plateau. This is because the inventory is assigned to be empty initially and almost all the speakers have to invent new names and directly transmit them to the hearers. So the increase of $N_{\rm d}$ is very rapid, and the different distributions have no effect on the evolution of $N_{\rm d}$ in this period. However, we note that the time for which the system stays in the plateau is different. For the power-law distribution, the decrease of $N_{\rm d}$ happens first and is the fastest, which is due to fewer names with large weights. For the original game, all the names are identical, so the competition between different names is very intense, which leads to the late and slowest decrease of $N_{\rm d}$. Thus the early and quick decrease of the number of different words $N_{\rm d}$ in the population is the main reason to promote the consensus time.

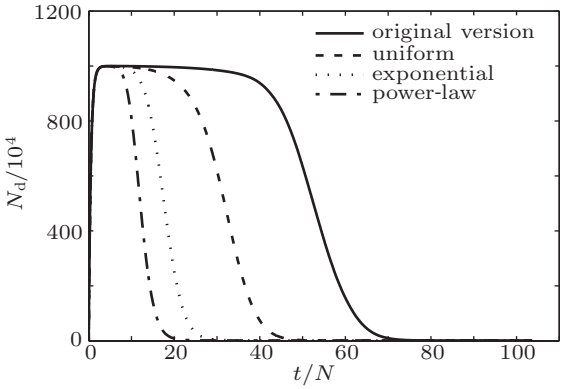


Fig. 3. Time courses depicting evolutions of numbers of different words $N_{\rm d}(t)$ for different distributions of word diversity with $A = 1.0$. It is evident that $N_{\rm d}$ will quickly reduce to 1 for the power-law distribution.

We now provide a reasonable explanation for the different lengths of the plateaus in Fig. 3 by using the evolution of the total number of names $N_{\rm w}$ in the system. From Fig. 4, we can see that for different distributions, the evolutions of $N_{\rm w}$ share the same trends, while the peak values ($N_{\rm w}^{\rm max}$) of $N_{\rm w}(t)$ are different. It is evident that the power-law corresponds to the smallest $N_{\rm w}^{\rm max}$ and the earliest decrease of $N_{\rm w}(t)$, while the original game corresponds to the largest one and the latest decrease. By comparison with Fig. 3, we find that the time of $N_{\rm w}^{\rm max}$ in Fig. 4 is just the time when $N_{\rm d}$ begins to decrease in Fig. 3 for all the distributions. During the period from the time when $N_{\rm d}$ reaches the plateau to the time when $N_{\rm w}$ reaches the peak, the names with different diversities spread and compete with each other in the population. Along with the spread of the weighted names, the system is dominated by some big name clusters, within which agents have a common name in their memories, and then one cluster will invade another until a global consensus is reached. Naturally, the name with a large weight will be chosen to transmit more frequently, and it will form a big cluster more quickly. For the power-law distribution, there are few names with large weights as shown in Fig. 1, and these names will form big clusters quickly, corresponding to $N_{\rm w}^{\rm max}$. After that, name elimination will happen until one big name cluster dominates the system. Hence, we confirm that a large segregation of words can make the word with a large weight have a better environment for spreading.

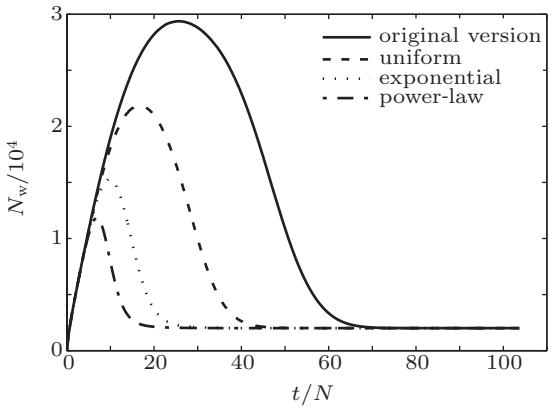


Fig. 4. Numbers of total word number $N_{\rm w}$ for different distributions of word diversity with $A = 1.0$. Note that $N_{\rm w}$ values share the same trends: increasing to a maximum and then reducing until $N = 2000$.

Another relevant quantity in the naming game is the average success rate $S(t)$ of negotiation, and it is shown in Fig. 5. Evidently, for the original version, it needs to spend a sufficiently long time to let the names spread fully before the speaker and the hearer can have a successful negotiation. However, in the case of the three diversity distributions with a segregation of word diversity, the spread will be greatly improved. From Fig. 5, we can see that the larger the segregation of word diversity, the faster $S(t)$ approaches 1, the shorter the convergence time. Moreover, by comparing Figs. 3 and 4, the distribution with the fastest increase of the success rate is found to correspond to the one with the earlist reduction of $N_{\rm d}$ and $N_{\rm w}$.

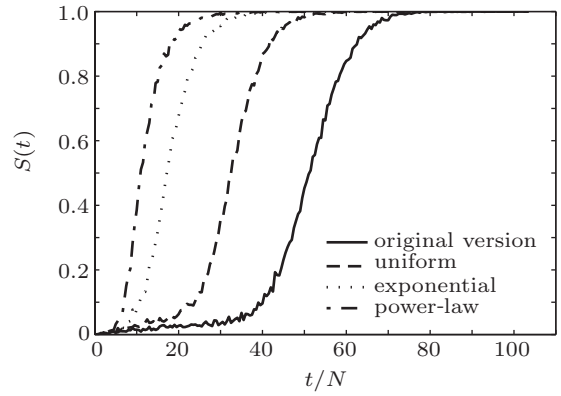


Fig. 5. Evolution of averaging success rate $S(t)$ for different distributions of word diversity with $A = 1.0$. Note that the power-law distribution leads to the fastest consensus.

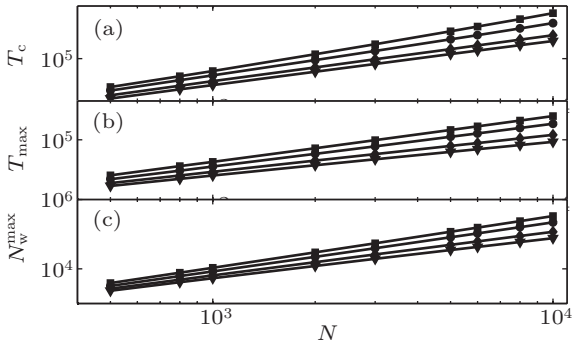


Fig. 6. (a) Convergence time T_c, (b) time of memory peak T_{max}, and (c) maximum total memory N_w^{max} used by agents each as a function of population size N for the original model with $A = 0$ and the different distributions with $A = 1.0$. On each figure, lines from top to bottom are for original, uniform, exponential, and power-law distributions, respectively. It is evident that T_c, T_{max}, and N_w^{max} scale as N^α, N^β, and N^γ, respectively. From top to bottom, $\alpha = 1.47, 1.28, 1.14, 1.09$, $\beta = 1.53, 1.43, 1.24, 1.13$, and $\gamma = 1.49, 1.41, 1.26, 1.16$, respectively.

Then we investigate the scaling behaviors of convergence time T_c, the peak of $N_w(t)$, namely T_{max}, and $N_w^{max} = N_w(T_{max})$, with the population size. From Fig. 6, we can obtain $T_c \propto N^\alpha$, $T_{max} \propto N^\beta$, and $N_w^{max} \propto N^\gamma$, with α, β, and γ being estimated parameters and depending on the distribution of the name diversity. In particular, the values of scaling parameter α are $1.28, 1.14$, and 1.09 for the distributions of uniform, exponential, and power-law, respectively, which are all smaller than that of the original game $\alpha = 1.47$ ($\alpha \approx 1.5$ in Refs. [21] and [31]) shown in the figure with the thick line. Similar scaling behaviors also exist for T_{max} and N_w^{max}.

In the above simulations, we consider the modified naming game on the fully-connected network. Previous research has shown that consensus time T_c is also related to network topology,[22,30,32] so we also study this model on a two-dimensional lattice and on a BA scale-free network. Note that on the BA network, the exponent of the power-law distribution degree $\langle k \rangle$ is fixed to 4. And we assume that on both topologies, the agents can interact only with their nearest neighbors. From Fig. 7, we can observe that convergence time T_c also decreases monotonically as parameter A grows, and the power-law distribution always leads to the shortest consensus time. It implies that word diversity can accelerate the consensus independent of the topologies. From the above analysis, we can conclude that the consensus time is largely dependent on the value of word diversity and its type of distribution.

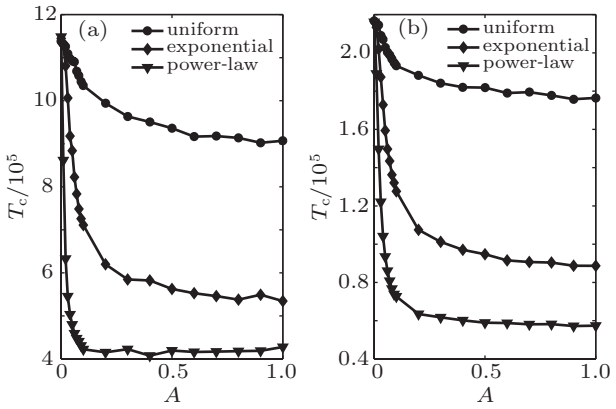


Fig. 7. Variations of time T_c with amplitude of undulation A for different distributions of word diversity on (a) BA network and (b) two-dimensional lattice. Note that T_c also shows a monotonic decrease with A increasing, and the power-law distribution leads to the shortest consensus time.

4. Conclusion

We study a variant of the naming game with respect to word diversity on fully-connected networks. Diversity is realized by assigning a weight, which is dependent on the distribution type, to each word, and the probability of each name being chosen from a speaker's memory is proportional to its weight. Compared with the original case in which all the words have equal weights, the consensus can be largely accelerated due to the introduction of word diversity. In particular, the power-law distribution leads to the shortest consensus time, while uniform distribution induces the longest one. In order to explain the boosting effect of word diversity, we examine the number of different words and the total number of words in the system. For power-law distribution, the total number of names can quickly reach a peak after the maximum number of different names is reached. Moreover,

for the power-law distribution, the peak of the total number of names is the lowest, because a few names with large weights spread so widely that almost all agents quickly have the names in their memories. So we attribute the faster consensus to the quick and wide spread of a few words in the population. Furthermore, we show that the consensus time and the peak of the total number of names show similar scaling behaviors to those of the previous reports. Social diversity exists widely in society, so our work may be of practical significance.

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