Novel stability and stabilization criteria for Takagi–Sugeno fuzzy time-delay systems*

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(Received 28 May 2011; revised manuscript received 2 November 2011)

We study the problems of stability and stabilization for Takagi–Sugeno (T–S) fuzzy time-delay systems. First, by constructing a less-redundant Lyapunov–Krasovskii function and introducing a useful inequality, an innovative stability criterion is obtained, which gives a significant improvement on the performance. Compared with the exiting references, our result can use fewer unknown variables and get better results. Furthermore, based on the derived stability criteria, a new stabilization condition is developed, in which the controller gain and the maximum allowable delay bound can be obtained simultaneously. The conditions are all derived in the form of linear matrix inequality, which are easy to verify. Finally, numerical examples are given to show the effectiveness of the proposed methods.

Keywords: stabilization, free-weighting matrix

PACS: 02.30.Yy, 02.60.Cb, 87.16.A– **DOI:** 10.1088/1674-1056/21/3/030204

1. Introduction

During the last two decades, the problems of stability analysis and stabilization of nonlinear systems in the Takagi and Sugeno (T–S) fuzzy model[1] have been extensively studied. In most of those applications, the fuzzy systems were thought of as universal approximators for nonlinear systems and proved to be a very good representation of a certain class of nonlinear dynamic system, and they were extensively studied in many systems.[2−4] In those systems, a nonlinear plant is first approximated by a T–S fuzzy linear model, and then the analysis will be based on the T–S linear model. Among the existing stabilization methods for the T–S fuzzy model, the most frequently considered setting is the so-called parallel compensation (PDC) scheme.[2] Research has focused on conceiving less conservative quadratic stabilization conditions, such as the ones in Refs. [5]–[13].

On the other hand, time delays are frequently encountered in practical systems, such as nuclear reactors, chemical processes, and biological systems. The authors in Ref. [14] first studied the problems of stability and stabilization for the T–S fuzzy systems with time delays. Based on the Lyapunov–Krasvoskii functional method, the guaranteed cost control has been studied for T–S fuzzy systems with time-delays in Ref. [15]. By using the free weighting matrix method, delay-dependent stabilization conditions for the T–S fuzzy system with state delay were investigated in Refs. [16] and [17].

However, in order to further improve the performance of the criteria for T–S fuzzy systems, the Lyapunov functions in the above mentioned results are always very complicated, and much effort has been devoted to the development of the free-weighting matrix method. In the Lyapunov function, the number of variables is usually huge, and the computing time is very long. Meanwhile, too many free-weighting matrices will complicate the system synthesis and consequently lead to a significant computational demand.[18−24] So how to improve the system performance while removing the redundant variables and reducing the computational time is still an unsolved problem and a challenging task.

Motivated by the above discussion, in this paper, we investigate the stability and the stabilization of T–S fuzzy time-delay systems. We aim to design a state-feedback fuzzy controller such that the resulting closed-loop system is asymptotically stable. By constructing a less-redundant Lyapunov–Krasovskii function and using a new matrix inequality, some further

*Project supported by the National Natural Science Foundation of China (Grant Nos. 50977008, 61034005, and 61104021), the National High Technology Research and Development Program of China (Grant No. 2009AA04Z127), and the National Basic Research Program of China (Grant No. 2009CB320601).

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improved criteria will be employed to analyze the stability and design a controller for the system, which will prove to be a significant improvement on the performance compared with the existing results. Finally. numerical examples will illustrate the achieved improvement.

2. Preliminaries

The following notations are used through out this paper, $\boldsymbol{N}^{\mathrm{T}}$ and $\boldsymbol{N}^{-1}$ represent the transpose and the inverse of matrix $\boldsymbol{N}$, respectively; R^n denotes the n-dimensional Euclidean space; $\boldsymbol{P} > 0$ means that matrix $\boldsymbol{P}$ is positive definite; $\boldsymbol{I}$ is the identity matrix of appropriate dimensions; diag$(\cdots)$ denotes a block-diagonal matrix; and $\begin{bmatrix} \boldsymbol{X} & \boldsymbol{Y} \\ * & \boldsymbol{Z} \end{bmatrix}$ refers to $\begin{bmatrix} \boldsymbol{X} & \boldsymbol{Y} \\ \boldsymbol{Y}^{\mathrm{T}} & \boldsymbol{Z} \end{bmatrix}$. The matrix dimensions, unless otherwise stated, are assumed to be compatible with algebraic operations.

Consider the nonlinear systems described by the following T–S fuzzy rules. Then, the i-th fuzzy is given by rule i: if $\theta_1(t)$ is F_{i1}, $\theta_2(t)$ is F_{i2}, …, $\theta_g(t)$ is F_{ig}, then

$$\begin{aligned} &\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{A}_{\mathrm{d}i}\boldsymbol{x}(t-h) + \boldsymbol{B}_i\boldsymbol{u}(t), \\ &\boldsymbol{x}(t) = \boldsymbol{\phi}(t),\ t \in [-h, 0],\ i = 1, 2, \ldots, r, \end{aligned} \tag{1}$$

where F_{ij} $(i = 1, \ldots, r;\ j = 1, \ldots, g)$ are the fuzzy sets; $\boldsymbol{x}(t) \in \mathrm{R}^n$ is the state vector; $\boldsymbol{u}(t) \in \mathrm{R}^m$ is the control input; r is the number of if-then rules; θ_i $(i = 1, \ldots, g)$ are the premise variables; $\boldsymbol{\phi}(t)$ is a real-valued initial function on $\mathrm{L}^2_{\mathcal{F}_0}([-h, 0];\ \mathrm{R}^n)$; $\boldsymbol{A}_i$, $\boldsymbol{A}_{\mathrm{d}i}, \boldsymbol{B}_i$ are known constant matrices with appropriate dimensions; and h is a time-delay scalar.

By using the standard fuzzy inference method, fuzzy system (1) can be expressed by the following global model:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) = &\sum_{i=1}^{r} \mu_i(\boldsymbol{\theta}(t))[\boldsymbol{A}_i\boldsymbol{x}(t) \\ &+ \boldsymbol{A}_{\mathrm{d}i}\boldsymbol{x}(t-h) + \boldsymbol{B}_i\boldsymbol{u}(t)], \end{aligned} \tag{2}$$

where $\boldsymbol{\theta}(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_g(t)]$, $\mu_i(\boldsymbol{\theta}(t)) = \omega_i(\boldsymbol{\theta}(t))/\sum_{i=1}^{r} \omega_i(\boldsymbol{\theta}(t))$, and the weighted and averaged weight of each fuzzy rule is denoted by $w_i(\boldsymbol{\theta}(t)) = \prod_{j=1}^{g} F_{ij}(\theta_j(t))$, in which $F_{ij}(\theta_j(t))$ is the grade of membership of $\theta_j(t)$ in F_{ij}. It is obvious that the fuzzy weighting functions $\mu_i(\boldsymbol{\theta}(t))$ satisfy

$$\mu_i(\boldsymbol{\theta}(t)) \geq 0,\ \sum_{i=1}^{r} \mu_i(\boldsymbol{\theta}(t)) = 1. \tag{3}$$

According to the PDC schemes,[2] we use a controller structure incorporating a set of fuzzy rules expressed in the form that

$$\begin{aligned} &\text{if } \theta_1(t) \text{ is } F_{i1},\ \theta_2(t) \text{ is } F_{i2}, \ldots,\ \theta_g(t) \text{ is } F_{ig}, \\ &\text{then } \boldsymbol{u}(t) = \boldsymbol{K}_i\boldsymbol{x},\ i = 1, 2, \ldots, r, \end{aligned}$$

where $\boldsymbol{K}_i$ are the controller gains to be determined, and the overall state feedback fuzzy control law is represented by

$$\boldsymbol{u}(t) = \sum_{i=1}^{r} \mu_i(\boldsymbol{\theta}(t))\boldsymbol{K}_i\boldsymbol{x}. \tag{4}$$

Then, the resulting closed-loop system can be rewritten in the following form:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\boldsymbol{\theta}(t))\mu_j(\boldsymbol{\theta}(t)) \\ &\times [(\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_j)\boldsymbol{x}(t) + \boldsymbol{A}_{\mathrm{d}i}\boldsymbol{x}(t-h)]. \end{aligned} \tag{5}$$

In order to obtain the main results, the following lemma will be employed.

Lemma 1[25] Let $\boldsymbol{x}(t) \in \mathrm{R}^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any matrices $\boldsymbol{X}, \boldsymbol{M}_1, \boldsymbol{M}_2 \in \mathrm{R}^{n\times n}$ and $\boldsymbol{Z} \in \mathrm{R}^{2n\times 2n}$, and a scalar function $h := h(t) \geq 0$:

$$-\int_{t-h}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{X}\dot{\boldsymbol{x}}(s)\,\mathrm{d}s \leq \boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{\Upsilon}\boldsymbol{\xi}(t) + h\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{Z}\boldsymbol{\xi}(t),$$

where

$$\boldsymbol{\Upsilon} := \begin{bmatrix} \boldsymbol{M}_1^{\mathrm{T}} + \boldsymbol{M}_1 & -\boldsymbol{M}_1^{\mathrm{T}} + \boldsymbol{M}_2 \\ * & -\boldsymbol{M}_2^{\mathrm{T}} - \boldsymbol{M}_2 \end{bmatrix},$$

$$\boldsymbol{\xi}(t) := \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}(t-h) \end{bmatrix},\quad \boldsymbol{Z} = \begin{bmatrix} \boldsymbol{M}_1^{\mathrm{T}} \\ \boldsymbol{M}_2^{\mathrm{T}} \end{bmatrix} \boldsymbol{X}^{-1} \begin{bmatrix} \boldsymbol{M}_1 & \boldsymbol{M}_2 \end{bmatrix}.$$

Remark 1 Lemma 1 was proposed by He *et al.* in 2005, and it has been applied widely in the literature. However, it is rarely used in T–S fuzzy systems with time delays. On the other hand, it is deduced from the Newton–Leibniz formula, so it can use fewer free-weighting matrices to acquire better results than some exiting references, such as Refs. [26]–[30], in which a number of free-weighting matrices were introduced by the Newton–Leibniz formula. Therefore, we use this Lemma to reduce the number of free-weighting matrices, and the numerical examples in Section 5 can prove the effectiveness of this method.

3. Nominal system without control

For the smooth development of the theory, we will consider a T–S fuzzy system without control first,

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} \mu_i(\boldsymbol{\theta}(t))[\boldsymbol{A}_i \boldsymbol{x}(t) + \boldsymbol{A}_{\mathrm{d}i} \boldsymbol{x}(t-h)]. \quad (6)$$

For this system, we will have the following stability criteria.

Theorem 1 For a given scalar $h > 0$, the fuzzy system in expression (1) without any uncertainty is asymptotically stable if there exist $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} > 0$, $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} > 0$, $\boldsymbol{W} = \boldsymbol{W}^{\mathrm{T}} > 0$, and any appropriately dimensioned matrices $\boldsymbol{M} = [\boldsymbol{M}_1^{\mathrm{T}}, \boldsymbol{M}_2^{\mathrm{T}}]^{\mathrm{T}}$ such that the linear matrix inequalities (LMIs) are feasible for $i = 1, 2, \ldots, r$, i.e.,

$$\boldsymbol{\Xi}_i^{(1)} = \begin{bmatrix} \boldsymbol{H} & h\boldsymbol{\Gamma}_1^{\mathrm{T}}\boldsymbol{W} & h\boldsymbol{\Gamma}_2^{\mathrm{T}} \\ * & -h\boldsymbol{W} & 0 \\ * & * & -h\boldsymbol{W} \end{bmatrix} < 0, \quad (7)$$

where

$$\boldsymbol{\Gamma}_1 = [\boldsymbol{A}_i, \boldsymbol{A}_{\mathrm{d}i}], \quad \boldsymbol{\Gamma}_2 = [\boldsymbol{M}_1, \boldsymbol{M}_2], \quad \boldsymbol{H} = \begin{bmatrix} \boldsymbol{P}\boldsymbol{A}_i + \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{Q} + \boldsymbol{M}_1^{\mathrm{T}} + \boldsymbol{M}_1 & \boldsymbol{P}\boldsymbol{A}_{\mathrm{d}i} - \boldsymbol{M}_1^{\mathrm{T}} + \boldsymbol{M}_2 \\ * & -\boldsymbol{Q} - \boldsymbol{M}_2^{\mathrm{T}} - \boldsymbol{M}_2 \end{bmatrix}.$$

Proof Choose a fuzzy Lyapunov–Krasovskii functional candidate as

$$V(t) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{x}(t) + \int_{t-h}^{t} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}\boldsymbol{x}(s)\mathrm{d}s + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta, \quad (8)$$

then the time derivative of $V(t)$ along the trajectory will satisfy

$$\dot{V}(t) = 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{x}}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{x}(t) - \boldsymbol{x}^{\mathrm{T}}(t-h)\boldsymbol{P}\boldsymbol{x}(t-h) + h\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{W}\dot{\boldsymbol{x}}(t) - \int_{t-h}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s. \quad (9)$$

From Lemma 1, for any $\boldsymbol{M}_1$, $\boldsymbol{M}_2$ with an appropriate dimension, we have the following integral inequality:

$$\begin{aligned} &- \int_{t-h}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}\dot{\boldsymbol{x}}(s)\mathrm{d}s \\ &\leq \boldsymbol{\eta}^{\mathrm{T}}(t) \begin{bmatrix} \boldsymbol{M}_1^{\mathrm{T}} + \boldsymbol{M}_1 & -\boldsymbol{M}_1^{\mathrm{T}} + \boldsymbol{M}_2 \\ * & -\boldsymbol{M}_2^{\mathrm{T}} - \boldsymbol{M}_2 \end{bmatrix} \boldsymbol{\eta}(t) \\ &\quad + h\boldsymbol{\eta}^{\mathrm{T}}(t) \begin{bmatrix} \boldsymbol{M}_1^{\mathrm{T}} \\ \boldsymbol{M}_2^{\mathrm{T}} \end{bmatrix} \boldsymbol{W}^{-1} \begin{bmatrix} \boldsymbol{M}_1^{\mathrm{T}} & \boldsymbol{M}_2 \end{bmatrix} \boldsymbol{\eta}(t), \end{aligned} \quad (10)$$

where

$$\boldsymbol{\eta}^{\mathrm{T}}(t) = [\boldsymbol{x}^{\mathrm{T}}(t), \boldsymbol{x}^{\mathrm{T}}(t-h)].$$

Substituting the proposed equalities, we can obtain

$$\dot{V}(t) \leq \sum_{i=1}^{r} [\mu_i(\boldsymbol{\theta}(t))\boldsymbol{\eta}^{\mathrm{T}}(t)(\boldsymbol{H} + h\boldsymbol{\Gamma}_1^{\mathrm{T}}\boldsymbol{W}\boldsymbol{\Gamma}_1 + h\boldsymbol{\Gamma}_2^{\mathrm{T}}\boldsymbol{W}^{-1}\boldsymbol{\Gamma}_2)\boldsymbol{\eta}(t)]. \quad (11)$$

According to the Schur complement, $\boldsymbol{H} + h\boldsymbol{\Gamma}_1^{\mathrm{T}}\boldsymbol{W}\boldsymbol{\Gamma}_1 + h\boldsymbol{\Gamma}_2^{\mathrm{T}}\boldsymbol{W}^{-1}\boldsymbol{\Gamma}_2$ is equivalent to expression (7). We can easily obtain that if $\boldsymbol{\Xi}_i^{(1)} < 0$, then $\dot{V}(t) < -\rho||\boldsymbol{x}(t)||^2 < 0$, which means that system (6) is asymptotically stable. Then the proof is completed.

4. Controller design

This section deals with the stabilization problem of closed-loop T–S fuzzy systems. A delay-dependent stabilization method will be developed. The following theorem summarizes our results for the design of a memoryless PDC controller to stabilize T–S fuzzy system (2).

Theorem 2 For a given scalar $h > 0$, the fuzzy system in expression (2) is asymptotically stable if there exist $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} > 0$, $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} > 0$, and any appropriately dimensioned matrices, $\boldsymbol{M} = [\boldsymbol{M}_1^{\mathrm{T}}, \boldsymbol{M}_2^{\mathrm{T}}]^{\mathrm{T}}$ such that the LMIs are feasible for $i = 1, 2, \ldots, r$. Let $\boldsymbol{W} = \boldsymbol{P}$, then we can obtain

$$\boldsymbol{\Xi}_i^{(2)} = \begin{bmatrix} \boldsymbol{\Xi}_{11} & \boldsymbol{A}_{\mathrm{d}i}\boldsymbol{X} - \boldsymbol{X}\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{X} + \boldsymbol{X}\boldsymbol{M}_2\boldsymbol{X} & h\boldsymbol{X}\boldsymbol{A}_i^{\mathrm{T}} + h\boldsymbol{Y}_j^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}} & h\boldsymbol{X}\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{X} \\ * & -\boldsymbol{X}\boldsymbol{Q}\boldsymbol{X} - \boldsymbol{X}\boldsymbol{M}_2^{\mathrm{T}}\boldsymbol{X} - \boldsymbol{X}\boldsymbol{M}_2\boldsymbol{X} & h\boldsymbol{X}\boldsymbol{A}_{\mathrm{d}i}^{\mathrm{T}} & h\boldsymbol{X}\boldsymbol{M}_2^{\mathrm{T}}\boldsymbol{X} \\ * & * & -h\boldsymbol{X} & 0 \\ * & * & * & -h\boldsymbol{X} \end{bmatrix} < 0, \quad (12)$$

where

$$\begin{aligned}\boldsymbol{\Xi}_{11} = {} & \boldsymbol{A}_i\boldsymbol{X} + \boldsymbol{B}_i\boldsymbol{Y}_j + \boldsymbol{X}\boldsymbol{A}_i^{\mathrm{T}} + \boldsymbol{Y}_j^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}} \\ & + \boldsymbol{X}\boldsymbol{Q}\boldsymbol{X} + \boldsymbol{X}\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{X} + \boldsymbol{X}\boldsymbol{M}_1\boldsymbol{X}.\end{aligned}$$

Proof Based on Theorem 1, the feedback gain in the fuzzy system without any uncertainty is given by $\boldsymbol{K}_j = \boldsymbol{Y}_j\boldsymbol{X}^{-1}$. Replace $\boldsymbol{A}_i$ with $\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_j$, let $\boldsymbol{W} = \boldsymbol{P}$, pre- and post-multiply expression (7) with $\mathrm{diag}(\boldsymbol{X}, \boldsymbol{X}, \boldsymbol{X}, \boldsymbol{X})$, where $\boldsymbol{X}^{-1} = \boldsymbol{P}$, then we can obtain the above criteria.

Remark 2 In this paper, we obtain some delay-dependent conditions. As is well known, delay-dependent criteria, in which the information about the length of delay is used, are usually less conservative than the delay-independent ones, especially when the size of the time delay is small.

Remark 3 In order to reduce the conservatism of the results, many researchers have devoted much effort to the complicated Lyapunov function. However, too many variables will increase the computational demand. Therefore, we use a less-redundant Lyapunov function and a useful inequality to analyze the stability problem for the general T–S fuzzy systems, which can improve the upper bound of time delay and alleviate the amount of calculation required. Therefore, our method can achieve a significant improvement on performance compared with that in many exiting papers, such as Refs. [18]–20] and [26]–[30].

Remark 4 According to the obtained stability results, we can derive a stabilization criterion, which is expressed within the framework of LMIs and can be easily checked in practice. The controller gain and the maximum allowable delay bound of the closed-loop control system can be obtained simultaneously, and their comparisons with the results in some existing references[22−24] are made in Example 2, which will show the effectiveness of our results.

5. Numerical examples

In this section, we demonstrate the effectiveness of the proposed approach through numerical examples. For comparisons with existing methods in some references, we choose the same fuzzy systems used in those references.

Example 1: Consider the nominal T–S fuzzy system without any uncertainty, then the parameters will be

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{2} \mu_i(\boldsymbol{\theta}(t))[\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{A}_{\mathrm{d}i}\boldsymbol{x}(t-h)],$$

where

$$\begin{aligned}&\boldsymbol{A}_1 = \begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix}, \quad \boldsymbol{A}_{\mathrm{d}1} = \begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix},\\ &\boldsymbol{A}_2 = \begin{bmatrix}-1 & 0.5\\ 0 & -1\end{bmatrix}, \quad \boldsymbol{A}_{\mathrm{d}2} = \begin{bmatrix}-1 & 0\\ 0.1 & -1\end{bmatrix},\\ &\mu_1 = \frac{1}{1+\exp(-2x_1(t))}, \quad \mu_2 = 1-\mu_1.\end{aligned}$$

The admissible delay upper bounds for h computed in the references and by us are shown in Table 1. It is obvious that our method gives a less conservative result in comparison with those in Refs. [18], [19] and [26]–[30]. The state trajectories of the system are shown in Fig. 1.

Table 1. Nominal system in Example 1.

Method	Upper h bound	Number of variables
Ref. [26]	1.00	5
Ref. [27]	1.5974	24
Ref. [28]	1.5974	8
Corollary 1 in Ref. [29]	1.5974	8
Ref. [30]	1.5974	8
Theorem 3 in Ref. [18]	1.5974	11
Lemma 2 in Ref. [19]	1.5974	7
Theorem 1 in this paper	1.5974	5

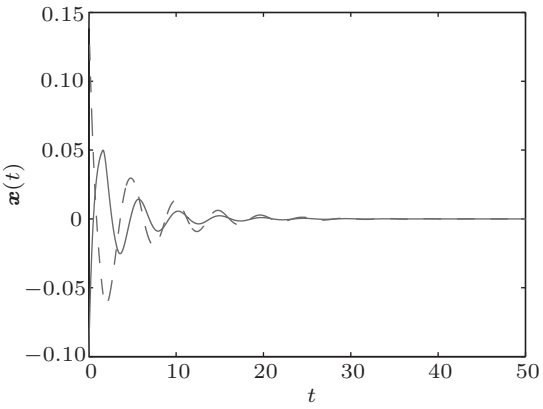


Fig. 1. State trajectories of $\boldsymbol{x}(t)$ in Example 1.

Example 2: Consider the stabilization of the fuzzy system without any uncertainty, then the parameters will be

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{2} \mu_i(\boldsymbol{\theta}(t))[\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{A}_{\mathrm{d}i}\boldsymbol{x}(t-h) + \boldsymbol{B}_i\boldsymbol{u}(t)],$$

where

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{A}_{\mathrm{d}1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix},$$

$$\boldsymbol{A}_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad \boldsymbol{A}_{\mathrm{d}2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix},$$

$$\boldsymbol{B}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{B}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Then the admissible delay upper bounds for h computed in the references and by us are shown in Table 2. It is obvious that our method gives a less conservative result in comparison with those in Refs. [22]–[24]. The state trajectories of the system are shown in Fig. 2.

Table 2. Stabilization for system in Example 2.

Method	Upper h bound	Number of variables
Ref. [22]	0.1524	>8
Ref. [23]	0.2302	11
Ref. [24]	0.2664	>9
Theorem 3 in this paper	0.2664	7

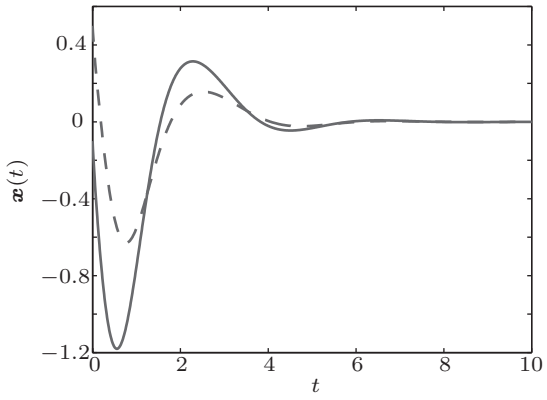


Fig. 2. State trajectories of $\boldsymbol{x}(t)$ in Example 2.

6. Conclusion

In this paper, the problems of stability and stabilization for T–S fuzzy time-delay systems are investigated. By utilizing a useful inequality, based on the Lyapunov–Krasovskii functional, some less conservative LMI-based criteria are obtained. Furthermore, we focus on the fuzzy state feedback controller design, which guarantees the closed-loop system to be stable. Finally, some numerical examples demonstrate that the proposed method is an improvement over the existing ones.

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