Detrended cross-correlation analysis of electroencephalogram*

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In the paper we use detrended cross-correlation analysis (DCCA) to study the electroencephalograms of healthy young subjects and healthy old subjects. It is found that the cross-correlation between different leads of a healthy young subject is larger than that of a healthy old subject. It was shown that the cross-correlation relationship decreases with the aging process and the phenomenon can help to diagnose whether the subject's brain function is healthy or not.

Keywords: detrended cross-correlation analysis, electroencephalogram, brain function, aging process

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1. Introduction

In many cases, there exists cross correlation[1] between different signal cross correlations. In seismology, the cross correlations can be regarded as a warning signal between different antenna signals on the detection array detecting earthquakes and volcanic eruptions; in finance, the risk estimates are also based on different portfolios of assets and the correlation matrix; and they are also discussed in many other fields.[2–12]

There is no good way to measure the cross-correlation index between two non-stationary time series. In 2008 Podobnik and Stanley[1] proposed a new cross correlation method to analyse nonstationary time series, which is mainly based on covariance. The method is mainly applied to physical, geographical, financial data analyses.

In this paper we use the detrended cross-correlation analysis to study the electroencephalograms (EEGs) of healthy young subjects and healthy old subjects. Some interesting phenomena are found.

2. Detrend cross-correlation analysis theory

For discrete functions x_j and y_j, the cross-correlation is defined as

$$R_{xy}(\tau) = \sum_{j=-\infty}^{\infty} x(j)y(j-n). \tag{1}$$

Generally, the cross correlation power-law of two time series is defined as following. Supposing that there are two stationary time series $\{y_i\}$ and $\{y'_i\}$, $i = 1, 2, \ldots, N$, their sequence can be expressed respectively as the formula of length k

$$\begin{aligned} R_k &\equiv y_1 + y_2 + \cdots + y_k, \\ R'_k &\equiv y'_1 + y'_2 + \cdots + y'_k, \quad k \geq N. \end{aligned} \tag{2}$$

The average of sequence $\{y_i\}$ is μ and the variance is σ^2. The average of sequence $\{y'_i\}$ is μ' and the variance is σ'^2. Assuming that the autocorrelation functions $A(n)$ and $A'(n)$ are expressed respectively as the following power-law forms:

$$\begin{aligned} A(n) &\sim n^{-\gamma}, \\ A'(n) &\sim n^{-\gamma'}, \quad 0 < \gamma, \gamma' < 1. \end{aligned} \tag{3}$$

We further assume that the cross correlation function between the time series $\{y_i\}$ and $\{y'_i\}$ can be expressed as

$$X(n) \sim n^{-\gamma x}, \quad 0 < \gamma x < 1. \tag{4}$$

The covariance of two stationary time series was calculated as

$$\begin{aligned} &\langle (R_n - \langle R_n \rangle)(R'_n - \langle R'_n \rangle) \rangle \\ &= nX(0) + \sum_{k=1}^{n-1} [n-k] \times [X(k) + X(-k)]. \end{aligned} \tag{5}$$

Taking two long-term cross-correlation time series with the same length of N, they are expressed as

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synthesized signals as follows:

$$R_k \equiv \sum_{i=1}^{k} y_i,$$
$$R'_k \equiv \sum_{i=1}^{k} y'_i, \quad k = 1, \ldots, N. \tag{6}$$

Then the whole time series is covered by $N - n$ overlapping boxes and each box contains $n+1$ values. For the two time series, each box begins from i and ends at $i + n$. The local trend is fitted by using the linear minimum mean-square method. Next, the covariance is calculated for two detrended time series in each box as

$$f^2_{\mathrm{DCCA}}(n, i) \equiv \frac{1}{(n-1)} \sum_{k=i}^{i+n} (R_k - \tilde{R}_{k,i}) \times (R'_k - \tilde{R}'_{k,i}). \tag{7}$$

Finally, we calculate the sum and the average of covariance for all the boxes. Let

$$F^2_{\mathrm{DCCA}}(n) \equiv (N-n)^{-1} \sum_{i=1}^{N-n} f^2_{\mathrm{DCCA}}(n, i), \tag{8}$$

if there exists the power-law relationship, then $F_{\mathrm{DCCA}} \propto n^{\lambda}$, where λ is the scaling exponent. When $R_k = R'_k$, the detrended covariance $F^2_{\mathrm{DCCA}}(n)$ turns into detrended variance $F^2_{\mathrm{DFA}}(n)$.

3. Data analysis

We take 10 healthy young and 10 healthy elderly EEGs from Neuroelectric and Myoelectric Databases database for example. The young subjects' data are taken from the 'yng' documents (including MT1415, MT1416, MT1419 $\sim$ MT1424, MT1427, MT1428) and the old subjects' data are taken from the 'eh' documents (including the MT1502 $\sim$ MT1504, MT1506, MT1509, MT1510, MT1519, MT1520, MT1521, MT1523).

We analyse the same length data (the distribution of DFA index and DCCA index for young subjects, which are was shown in Table 1 and Fig. 1. The distributions of DFA index and DCCA index for old subjects are shown in Table 2 and Fig. 2).

Table 1. Distributions of DFA index and DCCA index of young people.

Recording name	DFA(a-1)	DFA(a-2)	DCCA
Young1	0.788595	0.688537	0.738715
Young2	0.746323	0.771024	0.837702
Young3	0.734296	0.733433	0.750841
Young4	0.757056	0.786946	0.806229
Young5	0.783593	0.767872	0.837373
Young6	0.745056	0.786863	0.726352
Young7	0.812433	0.776624	0.848955
Young8	0.809831	0.798018	0.936038
Young9	0.752140	0.809935	0.925724
Young10	0.774173	0.810382	0.871788
Mean±std.	0.770345±0.023380	0.772962±0.026198	0.827971±0.057950

Table 2. Distributions of DFA index and DCCA index of old people.

Recording name	DFA(a-1)	DFA(a-2)	DCCA
Older1	0.690286	0.779388	0.746922
Older2	0.783162	0.757834	0.709742
Older3	0.759315	0.766096	0.784346
Older4	0.790916	0.766672	0.833312
Older5	0.684636	0.789987	0.746862
Older6	0.765386	0.723822	0.788726
Older7	0.704134	0.667865	0.667782
Older8	0.785088	0.739026	0.725943
Older9	0.692367	0.710528	0.707923
Older10	0.717113	0.683682	0.702158
Mean±std	0.737240±0.038533	0.73849±0.029083	0.741369±0.046039

The DFA method is used to determine the power-law autocorrelated behaviour of a single time series, while the DCCA method is used to determine the power-law cross-correlated behaviours of two time series. Tables 1 and 2 and figures 1 and 2 show that each time series of the magnitudes exhibits a power-law autocorrelated behaviour, indicating that a large increment in one variable is more likely to be followed by a large increment in the same variable. And they also show that besides autocorrelations, the time series of magnitudes exhibits power-law cross-correlation indicating that a large increment in one variable is more likely to be followed by large increment in the other variable.

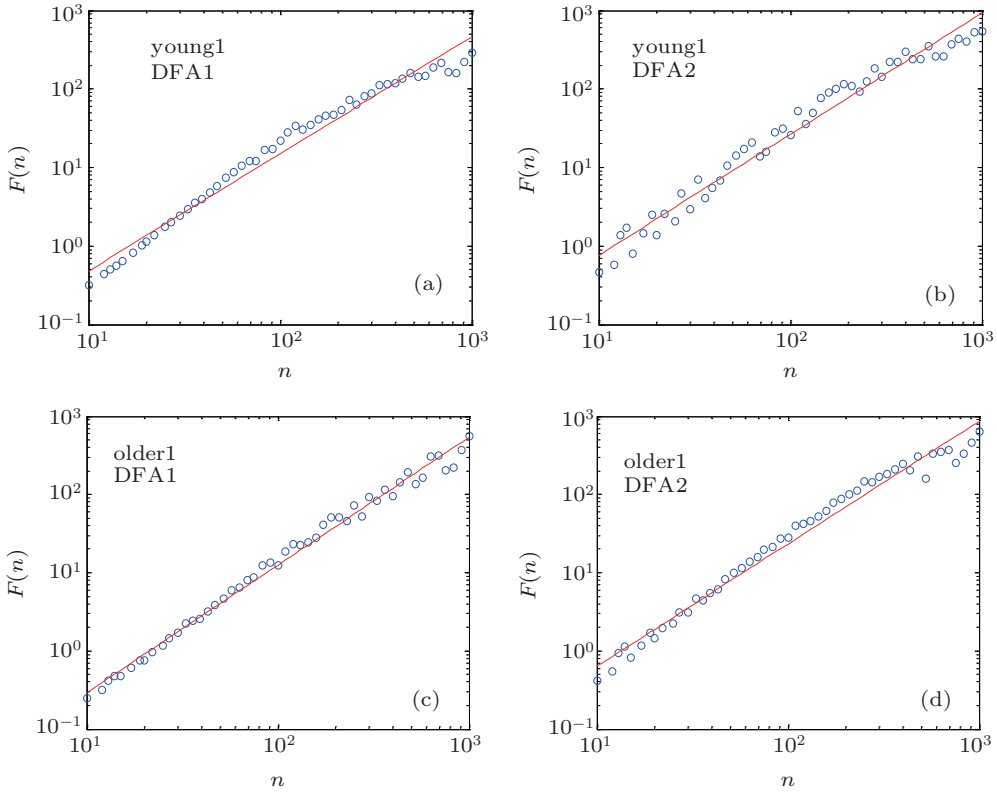


Fig. 1. (colour online) DFA indexes of four leads for two subjects, where panel (a) shows the DFA index of lead 1 for young people, panel (b) the DFA index of lead 2 for young people, panel (c) the DFA index of lead 1 for old people and (d) the DFA index of lead 2 for old people.

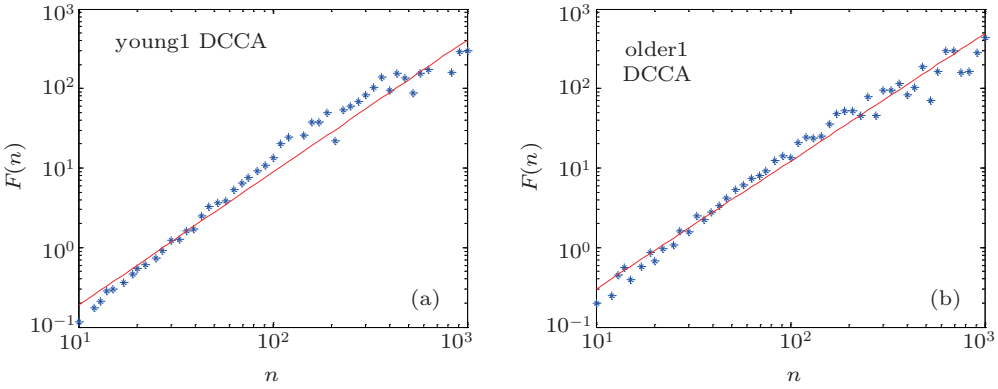


Fig. 2. (colour online) DCCA indexes for young and old subjects, where panel (a) shows the DCCA of two leads for young subjects and panel (b) the DCCA of two leads for old subjects.

It follows from Tables 1 and 2 that the average DCCA indexes of young people are slightly larger than the average those of old people. Student's test shows $t = 2.483 > t_{0.05(18)} = 2.101$. So $P < 0.05$, there is a significant difference.

So, now, we can reach a conclusion that with age growing, the cross-correlation power law between the different leads becomes weaker. A comparison of

power-law index between different leads of EEG with the common power-law index range show that doctors can pre-detect whether the target individuals have the brain aging phenomena and provide further clinical interventions.

4. Conclusion

In this paper we use the DCCA method to analyse the EEG signals of young and old subjects. It is shown that each EEG signal exhibits a power-law autocorrelated behaviour, indicating that a large increment in one variable is more likely to be followed by a large increment in the same variable. Besides autocorrelations, the EEG signals exhibit a power-law cross-correlation, indicating that a large increment in one variable is more likely to be followed by large increment in the other variable. It is found that for EEG signals of different leads exists a cross correlation power law and the correlation is of long-term and stable. By comparing the cross correlation index with the average values of the same age range, it can be detected in advance whether the target individual EEG is abnormal, which has important clinical significance.

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