Characteristics of anomalous Hall effect in spin-polarized two-dimensional electron gases in the presence of both intrinsic, extrinsic, and external electric-field induced spin–orbit couplings*

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The various competing contributions to the anomalous Hall effect in spin-polarized two-dimensional electron gases in the presence of both intrinsic, extrinsic and external electric-field induced spin–orbit coupling were investigated theoretically. Based on a unified semiclassical theoretical approach, it is shown that the total anomalous Hall conductivity can be expressed as the sum of three distinct contributions in the presence of these competing spin–orbit interactions, namely an intrinsic contribution determined by the Berry curvature in the momentum space, an extrinsic contribution determined by the modified Bloch band group velocity and an extrinsic contribution determined by spin–orbit-dependent impurity scattering. The characteristics of these competing contributions are discussed in detail in the paper.

Keywords: anomalous Hall effect, intrinsic spin–orbit coupling, extrinsic spin–orbit coupling, external electric-field induced spin–orbit coupling

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1. Introduction

An external electric field can induce a Hall current in a ferromagnetic conductor even in the absence of external magnetic fields. This phenomenon is called the anomalous Hall effect (AHE).[1] In recent years, due to the potential application of this effect in the emerging field of spintronics and especially due to the creation of new types of diluted magnetic semiconductors, there has been a lot of theoretical interest in the various possible physical mechanisms of AHE in spin-polarized two-dimensional electronic systems with intrinsic spin–orbit splitting of band structures.[1−18] For AHE in conventional ferromagnetic metals, it was well known that there exist two different types of possible physical mechanisms that can lead to the effect, namely the *intrinsic* and the *extrinsic* mechanisms.[19−24] The intrinsic mechanisms attribute AHE to the intrinsic spin–orbit splitting of band structures of ferromagnetic conductors (namely intrinsic spin–orbit coupling).[19,20] In contrast, the extrinsic (side-jump or skew scattering) mechanisms ascribes the effect to spin–orbit coupling induced by impurity scatterings (namely extrinsic spin–orbit coupling).[21−24] Though both the intrinsic and the extrinsic mechanisms were proposed several decades ago, whether intrinsic or extrinsic spin–orbit couplings play the dominant roles for the formation of AHE in conventional ferromagnetic conductors is still not quite well understood and remains a keenly debated issue.[1,25−34] For the possible physical mechanisms of AHE in spin-polarized two-dimensional electronic systems, most of the recent studies focused on the intrinsic mechanisms.[1−18] This is mainly due to the fact that, compared with the cases in conventional ferromagnetic metals, the possible forms of intrinsic spin–orbit coupling in two-dimensional electronic systems are much well understood and their strengths may be significantly enhanced by the host crystal field.[1−18,35] However, from the strict theoretical point of view, in order to obtain a more clear understanding of the possible theoretical aspects for AHE in spin-polarized two-dimensional electronic systems, in addition to the intrinsic mechanisms, other possible physical mechanisms should also be checked. This is due to the fact that in two-dimensional electronic systems not only the strength of the intrinsic spin–orbit coupling but

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also the strength of the extrinsic spin–orbit coupling may also be significantly enhanced by the host crystal field.[35] Considering this fact, in this paper we shall develop a unified semiclassical theoretical description of AHE in spin-polarized two-dimensional electron gas in the presence of both intrinsic and extrinsic spin–orbit couplings. In addition to the usual intrinsic and the usual impurity-induced extrinsic spin–orbit couplings, the external electric field that drives the Hall current may also induce a spin–orbit coupling between the spin and orbit degrees of freedom of charge carriers. In the present paper we refer to this kind of spin–orbit interaction as external electric-field induced spin–orbit coupling. From the theoretical point of view, this kind of spin–orbit interaction is also extrinsic but it is not present in the absence of external electric fields. Like impurity-induced extrinsic spin–orbit coupling, the strength of this kind of extrinsic spin–orbit interaction may also be significantly enhanced by the host crystal field in two-dimensional electronic systems.[35] The possible AHE contribution due to this kind of extrinsic spin–orbit interaction will also be investigated in the present paper. We will show that the inclusion of this kind of extrinsic spin–orbit interaction may lead to some interesting new characteristics, which seem to be overlooked in many recent studies.

The rest of the paper is organized as follows. In Section 2 we will first introduce some details of the theoretical model considered in the paper and then establish a semiclassical theoretical formalism for the calculation of the Hall current in the presence of intrinsic, extrinsic and external electric-field induced spin–orbit couplings. In Section 3 we will discuss in detail the characteristics of the main results obtained by this formalism.

2. Model and theoretical formulation

We consider a spin-polarized two-dimensional electron gas that lies in the xy plane, with the intrinsic spin–orbit coupling taking the form of the usual Rashba term. Taking also the spin–orbit coupling induced by impurity scatterings and that induced by the external electric field into account, the total Hamiltonian of the system will read

$$\begin{aligned}\hat{H} = & \left[\frac{\hbar^2 k^2}{2m} - \Delta\hat{\sigma}_z\right] + \alpha_{\rm R}(\hat{\sigma}_x k_y - \hat{\sigma}_y k_x) \\ & + [e\boldsymbol{E}\cdot\boldsymbol{r} + e\alpha_{\rm E}\hat{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{k})] \\ & + [V_{\rm im}(\boldsymbol{r}) + \alpha_{\rm im}\hat{\boldsymbol{\sigma}}\cdot(\boldsymbol{\nabla}_{\boldsymbol{r}} V_{0{\rm im}}(\boldsymbol{r})\times\boldsymbol{k})]\,. \quad (1)\end{aligned}$$

Here, $\hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ denotes the usual Pauli matrix, Δ denotes the exchange field, $V_{\rm im}(\boldsymbol{r})$ represents the impurity scattering potential and $\boldsymbol{E}$ refers to the external electric field that drives the Hall current. For clarity, we denote the coupling constant of the intrinsic Rashba spin–orbit coupling by $\alpha_{\rm R}$ and that of the spin–orbit couplings induced by impurity scatterings and by the external electric field by $\alpha_{\rm im}$ and $\alpha_{\rm E}$, respectively. If the host crystal field and the chemical properties of impurities are neglected, both $\alpha_{\rm E}$ and $\alpha_{\rm im}$ will be given by the vacuum value $\alpha_{\rm vac} \equiv \hbar^2/4m_0^2c^2$ (m_0 is the vacuum electron mass and c the velocity of light). However, for charge carriers in two-dimensional semiconducting electronic systems, the values of both $\alpha_{\rm E}$ and $\alpha_{\rm im}$ may be tremendously enhanced by the host crystal field compared with the vacuum value or the corresponding values in conventional ferromagnetic metals.[35] If the short-range contributions from the chemical properties of impurities are disregarded, the values of $\alpha_{\rm im}$ and $\alpha_{\rm E}$ should be equal in principle.[35] But for clarity, in this paper we will denote them by different symbols explicitly.

To develop a perturbation approach to the calculation of the Hall current, the Hamiltonian (1) can be regrouped as

$$\hat{H} = \hat{H}_0 + \hat{H}', \quad (2)$$

where $\hat{H}_0$ and $\hat{H}'$ are defined as

$$\begin{aligned}\hat{H}_0 = & \frac{\hbar^2 k^2}{2m} - \Delta\sigma_z + \alpha_{\rm R}(\hat{\sigma}_x k_y - \hat{\sigma}_y k_x) \\ & + e\alpha_{\rm E}\hat{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{k}), \quad (3)\end{aligned}$$

$$\hat{H}' = e\boldsymbol{E}\cdot\boldsymbol{r} + V_{\rm im}(\boldsymbol{r}) + \alpha_{\rm im}\hat{\sigma}\cdot(\boldsymbol{\nabla}_{\boldsymbol{r}} V_{\rm im}(\boldsymbol{r})\times\boldsymbol{k}). \quad (4)$$

The Hamiltonian $\hat{H}_0$ is independent of the spatial coordinates of charge carriers and can be diagonalized exactly. $\hat{H}'$ depends on the spatial coordinates of charge carriers and can be treated as a perturbation in the perturbation approach. The eigenfunctions of $\hat{H}_0$ can expressed as

$$|\psi^{(0)}_{\lambda\boldsymbol{k}}\rangle = \frac{1}{\sqrt{A}}\,{\rm e}^{{\rm i}\,\boldsymbol{k}\cdot r}|u_{\lambda\boldsymbol{k}}\rangle,$$

where A is the area of the two-dimensional system and $|u_{\lambda\boldsymbol{k}}\rangle$ ($\lambda = \pm$) the spinor eigenfunction. Assuming that the external electric field is applied in the x direction, $\boldsymbol{E} = E_x\hat{\boldsymbol{x}}$ (we use $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{z}}$ to denote the unit vectors along the x, y, z directions), then the spinor eigenfunction $|u_{\lambda\boldsymbol{k}}\rangle$ and the corresponding eigenvalue $\varepsilon_{\lambda\boldsymbol{k}}$ can be expressed explicitly as follows:

$$|u_{\lambda\boldsymbol{k}}\rangle = \sqrt{\frac{G(\boldsymbol{k}) + \lambda(\Delta - e\alpha_{\rm E}E_x k_y)}{2G(\boldsymbol{k})}}$$

$$\times \begin{pmatrix} 1 \\ \dfrac{\mathrm{i}\,\alpha_{\mathrm{R}}(k_x + \mathrm{i}\,k_y)}{\Delta - e\alpha_{\mathrm{E}}E_x k_y + \lambda G(\boldsymbol{k})} \end{pmatrix}, \quad (5)$$

$$\varepsilon_{\lambda\boldsymbol{k}} = \frac{\hbar^2 k^2}{2m} - \lambda G(\boldsymbol{k}), \quad (6)$$

where $G(\boldsymbol{k})$ is defined as

$$G(\boldsymbol{k}) = \sqrt{\alpha_{\mathrm{R}}^2 k^2 + (\Delta - e\alpha_{\mathrm{E}}E_x k_y)^2}. \quad (7)$$

In the semiclassical description, for an external electric field applied in the x direction, the density of the induced Hall current in the y direction can be calculated by the following formula:[1]

$$j_y(\boldsymbol{r}) = -e\sum_{\lambda\boldsymbol{k}} v_{\lambda,y}(\boldsymbol{k}) f_\lambda(\boldsymbol{k}, r), \quad (8)$$

where $f_\lambda(\boldsymbol{k}, r)$ is the nonequilibrium distribution function of electrons and $v_{\lambda,y}(\boldsymbol{k})$ the y component of the expectation value of the velocity operator $\hat{\boldsymbol{v}}$ with respect to the scattering wave function $|\Psi_{\lambda\boldsymbol{k}}\rangle_{\mathrm{sc}}$ of an incident electron with wave vector $\boldsymbol{k}$ and spin index λ, which are related to each other by

$$\boldsymbol{v}_\lambda(\boldsymbol{k}) = \langle\Psi_{\lambda\boldsymbol{k}}|\hat{\boldsymbol{v}}|\Psi_{\lambda\boldsymbol{k}}\rangle_{\mathrm{sc}}. \quad (9)$$

The velocity operator $\hat{\boldsymbol{v}}$ is determined by the Heisenberg equation of motion,

$$\hat{\boldsymbol{v}} = \dot{\boldsymbol{r}} = \frac{1}{\mathrm{i}\,\hbar}[\boldsymbol{r}, \hat{H}],$$

from which we have

$$\hat{v}_y = \frac{\hbar k_y}{m} + \frac{\alpha_{\mathrm{R}}}{\hbar}\hat{\sigma}_x + \frac{e\alpha_{\mathrm{E}}E_x}{\hbar}\hat{\sigma}_z + \frac{\alpha_{\mathrm{im}}}{\hbar}\frac{\partial V_{\mathrm{im}}(\boldsymbol{r})}{\partial x}\hat{\sigma}_z. \quad (10)$$

The scattering wave function $|\Psi_{\lambda\boldsymbol{k}}\rangle_{\mathrm{sc}}$ can be obtained through the Schlieman–Schwinger equation,[36] which reads

$$|\Psi_{\lambda\boldsymbol{k}}\rangle_{\mathrm{sc}} = |\psi^{(0)}_{\lambda\boldsymbol{k}}\rangle + \hat{G}_0^{(+)}\hat{H}'|\Psi_{\lambda\boldsymbol{k}}\rangle_{\mathrm{sc}}, \quad (11)$$

where $|\psi^{(0)}_{\lambda\boldsymbol{k}}\rangle$ is the initial wave function of an incident electron with wave vector $\boldsymbol{k}$ and spin index λ and $\hat{G}_0^{(+)} = (\varepsilon_{\lambda\boldsymbol{k}} - \hat{H}_0 + \mathrm{i}\,0^+)^{-1}$ is the unperturbed retarded Green's function. In the first Born approximation, from Eq. (11) we can obtain

$$\begin{aligned} &|\Psi_{\lambda\boldsymbol{k}}\rangle_{\mathrm{sc}} \\ &= |\psi^{(0)}_{\lambda\boldsymbol{k}}\rangle + \sum_{(\lambda'\boldsymbol{k}')\neq(\lambda\boldsymbol{k})} \frac{\langle\hat{V}_{\mathrm{im}}\rangle_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}}}{\varepsilon_{\lambda\boldsymbol{k}} - \varepsilon_{\lambda'\boldsymbol{k}'} + \mathrm{i}\,0^+}|\psi^{(0)}_{\lambda'\boldsymbol{k}'}\rangle \\ &\quad + \sum_{(\lambda'\boldsymbol{k}')\neq(\lambda\boldsymbol{k})} \frac{\mathrm{i}\,\alpha_{\mathrm{im}}\langle\hat{V}_{\mathrm{im}}\hat{\sigma}_z\rangle_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}}(\boldsymbol{k}'\times\boldsymbol{k})}{\varepsilon_{\lambda\boldsymbol{k}} - \varepsilon_{\lambda'\boldsymbol{k}'} + \mathrm{i}\,0^+}|\psi^{(0)}_{\lambda'\boldsymbol{k}'}\rangle \\ &\quad + \sum_{(\lambda'\boldsymbol{k}')\neq(\lambda\boldsymbol{k})} \frac{\langle\psi^{(0)}_{\lambda'\boldsymbol{k}'}|\,\mathrm{i}\,eE_x\frac{\partial}{\partial k_x}|\psi^{(0)}_{\lambda\boldsymbol{k}}\rangle}{\varepsilon_{\lambda\boldsymbol{k}} - \varepsilon_{\lambda'\boldsymbol{k}'} + \mathrm{i}\,0^+}|\psi^{(0)}_{\lambda'\boldsymbol{k}'}\rangle, \end{aligned} \quad (12)$$

where

$$\langle\hat{V}_{\mathrm{im}}\rangle_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}} \equiv \langle\psi^{(0)}_{\lambda'\boldsymbol{k}'}|V_{\mathrm{im}}(\boldsymbol{r})|\psi^{(0)}_{\lambda\boldsymbol{k}}\rangle$$
$$\langle\hat{V}\hat{\sigma}_z\rangle_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}} \equiv \langle\psi^{(0)}_{\lambda'\boldsymbol{k}'}|V_{\mathrm{im}}(\boldsymbol{r})\hat{\sigma}_z|\psi^{(0)}_{\lambda\boldsymbol{k}}\rangle.$$

Substituting Eqs. (10) and (12) into Eq. (9) and after some tedious algebras, one can show that the y component of the semiclassical velocity $\boldsymbol{v}_\lambda(\boldsymbol{k})$ can be expressed as follows:

$$v_{\lambda,y}(\boldsymbol{k}) = \hbar^{-1}\frac{\partial\varepsilon_{\lambda\boldsymbol{k}}}{\partial k_y} - eE_x\Omega_{\lambda,z}(\boldsymbol{k}) - \frac{\alpha_{\mathrm{im}}k_x\langle\hat{\sigma}_z\rangle_{\lambda\boldsymbol{k}}}{\tau_\lambda(\boldsymbol{k})}, \quad (13)$$

where $\langle\hat{\sigma}_z\rangle_{\lambda\boldsymbol{k}} \equiv \langle u_{\lambda\boldsymbol{k}}|\hat{\sigma}_z|u_{\lambda\boldsymbol{k}}\rangle$, $\tau_\lambda(\boldsymbol{k})$ a transport relaxation time defined by

$$\frac{1}{\tau_\lambda(\boldsymbol{k})} = \frac{2\pi}{\hbar}\sum_{\lambda'\boldsymbol{k}'}|\langle\hat{V}_{\mathrm{im}}\rangle_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}}|^2\left(1 - \frac{k'_x}{k_x}\right)\delta(\varepsilon_{\lambda\boldsymbol{k}} - \varepsilon_{\lambda'\boldsymbol{k}'}), \quad (14)$$

and $\Omega_{\lambda,z}(\boldsymbol{k})$ is the so-called Berry curvature in the momentum space,[37] which is defined by

$$\Omega_{\lambda,z}(\boldsymbol{k}) = [\boldsymbol{\nabla}_{\boldsymbol{k}} \times \boldsymbol{A}_\lambda(\boldsymbol{k})]_z, \quad (15)$$

with $\boldsymbol{A}_\lambda(\boldsymbol{k})$ denoting the gauge potential in the momentum space,

$$\boldsymbol{A}_\lambda(\boldsymbol{k}) = -\,\mathrm{i}\langle u_{\lambda\boldsymbol{k}}|\frac{\partial}{\partial\boldsymbol{k}}|u_{\lambda\boldsymbol{k}}\rangle. \quad (16)$$

Inserting Eq. (5) into Eq. (16), we obtain

$$A_{\lambda,x}(\boldsymbol{k}) = -\frac{\alpha_{\mathrm{R}}^2 k_y}{2G(\boldsymbol{k})[\Delta - \alpha_{\mathrm{E}}E_x k_y + \lambda G(\boldsymbol{k})]}, \quad (17)$$

$$A_{\lambda,y}(\boldsymbol{k}) = \frac{\alpha_{\mathrm{R}}^2 k_x}{2G(\boldsymbol{k})[\Delta - \alpha_{\mathrm{E}}E_x k_y + \lambda G(\boldsymbol{k})]}. \quad (18)$$

And then from Eq. (15) we have

$$\Omega_{\lambda,z}(\boldsymbol{k}) = \lambda\frac{\alpha_{\mathrm{R}}^2\Delta}{2[\alpha_{\mathrm{R}}^2 k^2 + (\Delta - e\alpha_{\mathrm{E}}E_x k_y)^2]^{3/2}}. \quad (19)$$

From Eq. (13) one can see that after taking the intrinsic, the extrinsic and the external electric-field induced spin–orbit couplings into account, the semiclassical velocity of charge carriers in the direction perpendicular to the applied external electric field can be expressed as the sum of three distinct contributions. The first term on the right-hand side in Eq. (13) is the modified Bloch band group velocity. The second term is an anomalous velocity determined by the Berry curvature in the momentum space, which will vanish if the intrinsic Rashba spin–orbit coupling is neglected. The last term is an anomalous velocity originating from spin–orbit-dependent impurity scattering. This term will vanish if the extrinsic spin–orbit coupling induced by impurity scattering is neglected.

In order to calculate the Hall current by use of Eq. (8), the nonequilibrium distribution function $f_\lambda(\boldsymbol{k},\boldsymbol{r})$ of charge carriers in the presence of the external electric field also needs to be determined. This can be obtained by solving the Boltzmann transport equation, which reads

$$\begin{aligned}&\frac{\partial f_\lambda(\boldsymbol{k},r)}{\partial t}-e\boldsymbol{E}\cdot\frac{\partial f_\lambda(\boldsymbol{k},r)}{\hbar\partial\boldsymbol{k}}+\frac{\partial f_\lambda(\boldsymbol{k},r)}{\partial\boldsymbol{r}}\cdot\boldsymbol{v}_\lambda(\boldsymbol{k})\\&=\sum_{\lambda'\boldsymbol{k}'}w_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}}f_{\lambda'}(\boldsymbol{k}',\boldsymbol{r})[1-f_\lambda(\boldsymbol{k},\boldsymbol{r})]\delta(\varepsilon_{\lambda'\boldsymbol{k}'}-\varepsilon_{\lambda\boldsymbol{k}})\\&\quad-\sum_{\lambda'\boldsymbol{k}'}w_{\lambda\boldsymbol{k},\lambda'\boldsymbol{k}'}f_\lambda(\boldsymbol{k},r)[1-f_{\lambda'}(\boldsymbol{k}',\boldsymbol{r})]\delta(\varepsilon_{\lambda'\boldsymbol{k}'}-\varepsilon_{\lambda\boldsymbol{k}}),\end{aligned}\tag{20}$$

where $w_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}}\equiv(2\pi/\hbar)|\langle\hat{V}_{\rm im}\rangle_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}}|^2$ denotes the impurity scattering rate. In this paper we will consider only the cases of homogeneous steady-states. In such cases, the nonequilibrium distribution function $f_\lambda(\boldsymbol{k},r)$ is independent of the spatial coordinates of charge carriers and can be expressed as $f_\lambda(\boldsymbol{k})=f^{(0)}(\varepsilon_{\lambda\boldsymbol{k}})+\delta f_\lambda(\boldsymbol{k})$, where $f^{(0)}(\varepsilon_{\lambda\boldsymbol{k}})$ is the equilibrium Fermi distribution function and $\delta f_\lambda(\boldsymbol{k})$ denotes the deviation from the equilibrium state. In the linear-response approximation, the deviation term can be expressed in the following general form:

$$\begin{aligned}\delta f_\lambda(\boldsymbol{k})=eE_x\frac{\partial f^{(0)}(\varepsilon_{\lambda\boldsymbol{k}})}{\partial\varepsilon_{\lambda\boldsymbol{k}}}[&A_{\lambda\boldsymbol{k}}(\hat{\boldsymbol{x}}\cdot\boldsymbol{v}_\lambda(\boldsymbol{k}))\\&+B_{\lambda\boldsymbol{k}}(\hat{\boldsymbol{y}}\cdot\boldsymbol{v}_\lambda(\boldsymbol{k}))],\end{aligned}\tag{21}$$

where $A_{\lambda\boldsymbol{k}}$ and $B_{\lambda\boldsymbol{k}}$ are two coefficients that need to be determined self-consistently from Eq. (20). Substituting Eq. (21) into Eq. (20) and after some tedious algebras, one can show that the solutions of $A_{\lambda\boldsymbol{k}}$ and $B_{\lambda\boldsymbol{k}}$ can be expressed in the following form:

$$A_{\lambda\boldsymbol{k}}=\frac{\tau_\lambda(\boldsymbol{k})}{1+[\tau_\lambda(\boldsymbol{k})/\tau'_\lambda(\boldsymbol{k})]^2},\tag{22}$$

$$B_{\lambda\boldsymbol{k}}=-\frac{\tau'_\lambda(\boldsymbol{k})}{1+[\tau'_\lambda(\boldsymbol{k})/\tau_\lambda(\boldsymbol{k})]^2},\tag{23}$$

where $\tau_\lambda(\boldsymbol{k})$ is defined in Eq. (14) and $\tau'_\lambda(\boldsymbol{k})$ is defined by

$$\begin{aligned}\frac{1}{\tau'_\lambda(\boldsymbol{k})}=&\sum_{\lambda'\boldsymbol{k}'}w_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}}\delta(\varepsilon_{\lambda'\boldsymbol{k}'}-\varepsilon_{\lambda\boldsymbol{k}})\\&\times\frac{|\boldsymbol{v}_{\lambda'\boldsymbol{k}'}|}{|\boldsymbol{v}_{\lambda\boldsymbol{k}}|}\sin(\theta_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}}),\end{aligned}\tag{24}$$

with $\theta_{\lambda'\boldsymbol{k}',\lambda\boldsymbol{k}}$ denoting the angle between $\boldsymbol{v}_{\lambda'\boldsymbol{k}'}$ and $\boldsymbol{v}_{\lambda\boldsymbol{k}}$.

After $v_{\lambda,y}(\boldsymbol{k})$ and $f_\lambda(\boldsymbol{k})$ are determined, the anomalous Hall current induced by an external electric field in the x direction can be calculated by use of Eq. (8). Corresponding to Eq. (13), the anomalous Hall current can also be expressed as the sum of three distinct contributions, i.e., $j_y=j_y^{(\rm I)}+j_y^{(\rm II)}+j_y^{(\rm III)}$, where $j_y^{(\rm I,II,III)}$ are defined respectively by

$$j_y^{(\rm I)}=e^2E_x\sum_{\lambda\boldsymbol{k}}\Omega_{\lambda,z}(\boldsymbol{k})f_\lambda(\boldsymbol{k}),\tag{25}$$

$$j_y^{(\rm II)}=-\frac{e}{\hbar}\sum_{\lambda\boldsymbol{k}}(\partial\varepsilon_{\lambda\boldsymbol{k}}/\partial k_y)f_\lambda(\boldsymbol{k}),\tag{26}$$

$$j_y^{(\rm III)}=e\alpha_{\rm im}\sum_{\lambda\boldsymbol{k}}\frac{\langle\hat{\sigma}_z\rangle_{\lambda\boldsymbol{k}}k_x}{\tau_\lambda(\boldsymbol{k})}f_\lambda(\boldsymbol{k}).\tag{27}$$

Here, the first contribution is determined by the Berry curvature in the momentum space, the second contribution is determined by the modified Bloch band group velocity and the third contribution originates from the spin–orbit-dependent impurity scattering.

3. Results and discussion

In the formalism established above, the various anomalous Hall current contributions in the presence of intrinsic, extrinsic and external electric-field induced spin–orbit couplings have been described in a unified way based on equal footing. Below we will discuss the characteristics of the anomalous Hall conductivity in the presence of these competing spin–orbit interactions based on this formalism. Corresponding to Eqs. (25)–(27), the anomalous Hall conductivity can also be expressed as the sum of three distinct contributions, i.e., $\sigma_{xy}=\sigma_{xy}^{(\rm I)}+\sigma_{xy}^{(\rm II)}+\sigma_{xy}^{(\rm III)}$, where $\sigma_{xy}^{(\rm I,II,III)}$ are defined respectively by $\sigma_{xy}^{(\rm I,II,III)}=j_y^{(\rm I,II,III)}/E_x$. To the first order of the electric field, we obtain

$$\sigma_{xy}^{(\rm I)}=\frac{1}{2}e^2\alpha_{\rm R}^2\Delta\sum_{\boldsymbol{k}}\frac{f^{(0)}(\epsilon_{+\boldsymbol{k}})-f^{(0)}(\epsilon_{-\boldsymbol{k}})}{(\alpha_{\rm R}^2k^2+\Delta^2)^{3/2}},\tag{28}$$

$$\sigma_{xy}^{(\rm II)}=-\frac{e^2\alpha_{\rm E}\Delta}{\hbar}\sum_{\boldsymbol{k}}\frac{f^{(0)}(\epsilon_{+\boldsymbol{k}})-f^{(0)}(\epsilon_{-\boldsymbol{k}})}{(\alpha_{\rm R}^2k^2+\Delta^2)^{1/2}},\tag{29}$$

$$\begin{aligned}\sigma_{xy}^{(\rm III)}=&\frac{e^2\hbar\alpha_{\rm im}\Delta}{m}\sum_{\boldsymbol{k}}\frac{k_x^2}{(\alpha_{\rm R}^2k^2+\Delta^2)^{1/2}}\\&\times\left[\frac{\partial f^{(0)}(\epsilon_{+\boldsymbol{k}})/\partial\epsilon_{+\boldsymbol{k}}}{1+(\tau_+(\boldsymbol{k})/\tau'_+(\boldsymbol{k}))^2}\right.\\&\left.-\frac{\partial f^{(0)}(\epsilon_{-\boldsymbol{k}})/\partial\epsilon_{-\boldsymbol{k}}}{1+(\tau_-(\boldsymbol{k})/\tau'_-(\boldsymbol{k}))^2}\right],\end{aligned}\tag{30}$$

where

$$\epsilon_{\lambda\boldsymbol{k}}=\frac{\hbar^2k^2}{2m}-\lambda\sqrt{\alpha_{\rm R}^2k^2+\Delta^2}$$

is the eigenenergy of electrons in the absence of extrinsic and external electric-field induced spin–orbit couplings. The anomalous Hall conductivity contribution

given by Eq. (28) has an intrinsic origin, which will disappear in the absence of the intrinsic Rashba spin–orbit coupling (i.e., $\alpha_{\rm R} = 0$.) and is determined by the Berry curvature in momentum space. The contribution given by Eq. (30) is the usual extrinsic (side-jump and skew scattering) contribution due to spin–orbit-dependent impurity scattering. The contribution given by Eq. (29) is an extrinsic contribution determined by the modified Bloch band group velocity and it will disappear if the external electric-field induced spin–orbit coupling is neglected (i.e., $\alpha_{\rm E} = 0$).

Several interesting characteristics can be seen from Eqs. (28)–(30). First, from Eq. (28) one can see that the anomalous Hall conductivity contribution determined by the Berry curvature in momentum space is independent of disorder scattering effect and all electron states below the Fermi level will contribute to it (i.e., it is a Fermi sea property). This characteristic is in agreement with that obtained by several different theoretical approaches to anomalous Hall effect in spin-polarized two-dimensional electronic systems with Rashba spin–orbit coupling.[1] Since the Berry-curvature-related anomalous Hall conductivity is independent of disorder scattering effect and all electron states below the Fermi level contribute to it, in some recent literature such intrinsic anomalous Hall conductivity contributions are said to be dissipationless and the Berry-curvature dependence of the anomalous Hall conductivity is argued to be a reflection of the topological nature of the Hall currents. For detailed review, please refer to Ref. [1]. From Eq. (28) one can also see that the Berry-curvature-related anomalous Hall conductivity contribution depends only on the intrinsic Rashba spin–orbit coupling but is independent of the impurity-induced extrinsic spin–orbit coupling or the external electric-field induced spin–orbit coupling when these competing spin–orbit interactions are present simultaneously.

Secondly, from Eq. (29) one can see that the anomalous Hall conductivity contribution due to impurity-induced extrinsic spin–orbit coupling is proportional to the derivative of the equilibrium Fermi distribution function with respect to the Fermi level, which is nonvanishing only for electron states in the vicinity of the Fermi level. So, like usual dissipative electron transport coefficients, the anomalous Hall conductivity contribution due to impurity-induced extrinsic spin–orbit coupling is a Fermi surface property.

Thirdly, from Eq. (30) one can see that unlike the AHE contribution due to impurity-induced extrinsic spin–orbit coupling but similar to the intrinsic Berry-curvature-related AHE contribution, the anomalous Hall conductivity contribution due to external electric-field induced spin–orbit coupling is also a Fermi sea property (i.e., all electron states below the Fermi level contribute to it) and also independent of disorder scattering effect and in this sense one can say that it is also dissipationless. From the theoretical point of view, the dissipationless character of this anomalous Hall conductivity contribution originates from the fact that the spin-dependent anomalous velocity component of charge carriers due to external electric-field induced spin–orbit coupling is independent of the wave vectors of electrons when they are confined to move in a two-dimensional plane. To see this fact explicitly, we neglect the intrinsic and the impurity-induced extrinsic spin–orbit couplings (i.e., setting $\alpha_{\rm R} = 0$ and $\alpha_{\rm im} = 0$). Then from Eqs. (9) and (10) one can see that in the direction perpendicular to the applied external electric field, the velocity component of a charge carrier with a given wave vector $\boldsymbol{k}$ and spin index λ is given by

$$v_{\lambda,y}(\boldsymbol{k}) = \frac{\hbar k_y}{m} + \lambda \frac{e\alpha_{\rm E} E_x}{\hbar}. \tag{31}$$

Then the anomalous Hall current induced by the external electric field, $j_y = -e\sum_{\lambda,\boldsymbol{k}} v_{\lambda,y}(\boldsymbol{k}) f_\lambda(\boldsymbol{k})$, is given by (to the first order of the electric field)

$$j_y = -\frac{e^2\alpha_{\rm E} E_x}{\hbar} \sum_{\boldsymbol{k}} [f_+(\boldsymbol{k}) - f_-(\boldsymbol{k})] \approx \sigma_{xy}^{(II)} E_x, \tag{32}$$

where

$$\sigma_{xy}^{(\rm II)} = -\frac{e^2\alpha_{\rm E}}{\hbar}(N_+ - N_-),$$

with $N_\pm = \sum_{\boldsymbol{k}} f^{(0)}(\varepsilon_{\lambda\boldsymbol{k}})$ denoting the equilibrium spin-resolved charger carrier densities. From Eqs. (31) and (32) one can see clearly that though the distribution function $f_\lambda(\boldsymbol{k})$ depends on disorder scattering effect (see Eqs. (21)–(23)), the anomalous Hall conductivity contribution due to external electric-field induced spin–orbit coupling is independent of disorder scattering effect and all electron states below the Fermi level will contribute to it, similar to the dissipationless anomalous Hall conductivity contribution due to the intrinsic Rashba spin–orbit interaction.

Finally, to make a quantitative estimation on the relative magnitudes of the three distinct anomalous

Hall conductivity contributions, below we will consider some actual material parameters suitable for some realistic physical systems. We take two-dimensional n-doped GaAs for example. For two-dimensional n-doped GaAs, the magnitude of the intrinsic Rashba spin–orbit coupling constant $\alpha_{\rm R}$ is on the order of 10^{-9} eV·cm$\sim 10^{-10}$ eV·cm and its value can be tuned by applying a gate voltage.[35] According to Eqs. (28)–(30), one can show that if the magnitudes of the extrinsic spin–orbit coupling constants $\alpha_{\rm E}$ and $\alpha_{\rm im}$ take the vacuum value ($\alpha_{\rm vac} \equiv \hbar^2/4m_0^2c^2$) and the intrinsic Rashba spin–orbit coupling constant $\alpha_{\rm R}$ takes a typical value of 10^{-9} eV·cm$\sim 10^{-10}$ eV·cm, then the extrinsic anomalous Hall conductivity contributions $\sigma_{xy}^{\rm (II)}$ and $\sigma_{xy}^{\rm (III)}$ due to the spin–orbit coupling induced by impurity scattering and by the applied external electric field will be several orders of magnitude smaller than the intrinsic anomalous conductivity contribution $\sigma_{xy}^{\rm (I)}$ and hence they can be neglected in principle. However, as is well known, in some realistic semiconducting materials such as n-doped GaAs, the extrinsic spin–orbit coupling strength may be enhanced significantly by the host crystal field and the extrinsic spin–orbit coupling constants $\alpha_{\rm E}$ and $\alpha_{\rm im}$ may hence be several orders of magnitude larger than the vacuum value.[35] For n-doped GaAs, the magnitudes of the extrinsic spin–orbit coupling constants $\alpha_{\rm E}$ and $\alpha_{\rm im}$ can be estimated by using the third-order perturbation theory based on the 8×8 Kane model.[35] Disregarding the short-range contributions from the chemical properties of impurities, the following result has been derived as

$$\alpha_{\rm E} = \alpha_{\rm im} = \frac{P^2}{3}\left[\frac{1}{E_0^2} - \frac{1}{(E_0+\Delta_{\rm SO})^2}\right], \qquad (33)$$

where P is the matrix element of the momentum operator between the conduction and valence bands, E_0 the energy gap and $\Delta_{\rm SO}$ the spin–orbit splitting. By use of this formula one estimates $\alpha_{\rm E} = \alpha_{\rm im} \approx 5.3$ Å^2 (1 Å=0.1 nm) for n-doped GaAs, which is about six orders of magnitude larger than the vacuum value.[35] In Fig. 1 we plot the total anomalous Hall conductivity $\sigma_{xy} = \sigma_{xy}^{\rm (I)} + \sigma_{xy}^{\rm (II)} + \sigma_{xy}^{\rm (III)}$ as a function of the Rashba spin–orbit coupling constant $\alpha_{\rm R}$ by setting $\alpha_{\rm E} = \alpha_{\rm im} = 5.3$ Å^2 and the exchange field $\Delta = 0.01$ eV and assuming a δ-function shaped impurity scattering potential. In Fig. 2(a) we show the three distinct anomalous Hall conductivity contributions each as a function of $\alpha_{\rm R}$ and in Fig. 2(b) we show the relative ratios of the three distinct contributions to the total anomalous Hall conductivity versus $\alpha_{\rm R}$. From Fig. 1 one can see that the magnitude of the total anomalous Hall conductivity is typically on the order of 10^{-2} (e^2/h) for the material parameters used. From Fig. 2 one can see that the relative ratio of the intrinsic Berry-curvature-related contribution to the total anomalous Hall conductivity increases substantially as the magnitude of $\alpha_{\rm R}$ increases, while the relative ratios of the two extrinsic contributions to the total anomalous Hall conductivity decreases substantially as the magnitude of $\alpha_{\rm R}$ increases. However, from Fig. 2 one can also note that except in the case where the intrinsic spin–orbit interaction is very strong, the anomalous Hall conductivity contribution from impurity-induced extrinsic spin–orbit coupling and that from external electric-field induced spin–orbit coupling are roughly on the same orders of magnitude as the Berry-curvature-related intrinsic contribution. Detailed analyses show that the extrinsic contributions $\sigma_{xy}^{\rm (II)}$ and $\sigma_{xy}^{\rm (III)}$ are negligible compared with the intrinsic contribution $\sigma_{xy}^{\rm (I)}$ only if in the case of $\alpha_{\rm im}/\alpha_{\rm R}$ $(\alpha_{\rm E}/\alpha_{\rm R}) \ll 10^{-7}$ cm·eV^{-1}. This suggests that except in the case where the intrinsic spin–orbit interaction is extremely strong, the possible AHE contributions from impurity-induced extrinsic spin–orbit coupling and that from external electric-field induced spin–orbit coupling may need to be considered in the study of the possible theoretical aspects of AHE in spin-polarized two-dimensional electronic systems or other diluted magnetic semiconductors.

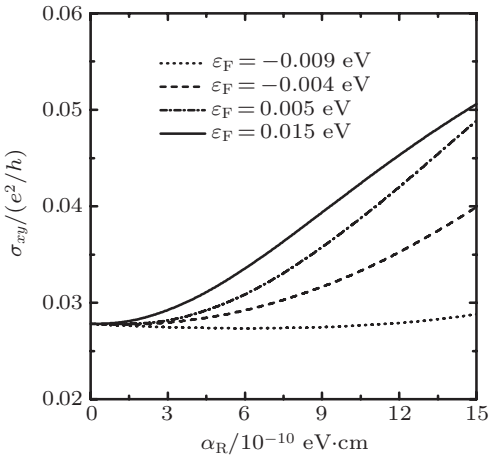


Fig. 1. Plots of the total anomalous Hall conductivity $\sigma_{xy} = \sigma_{xy}^{\rm (I)} + \sigma_{xy}^{\rm (II)} + \sigma_{xy}^{\rm (III)}$ as a function of the intrinsic Rashba spin–orbit coupling constant $\alpha_{\rm R}$ for different values of $\varepsilon_{\rm F}$ shown in the figure. The material parameters used are $\alpha_{\rm E} = \alpha_{\rm im} = 5.3$ Å^2 and $\Delta = 0.01$ eV.

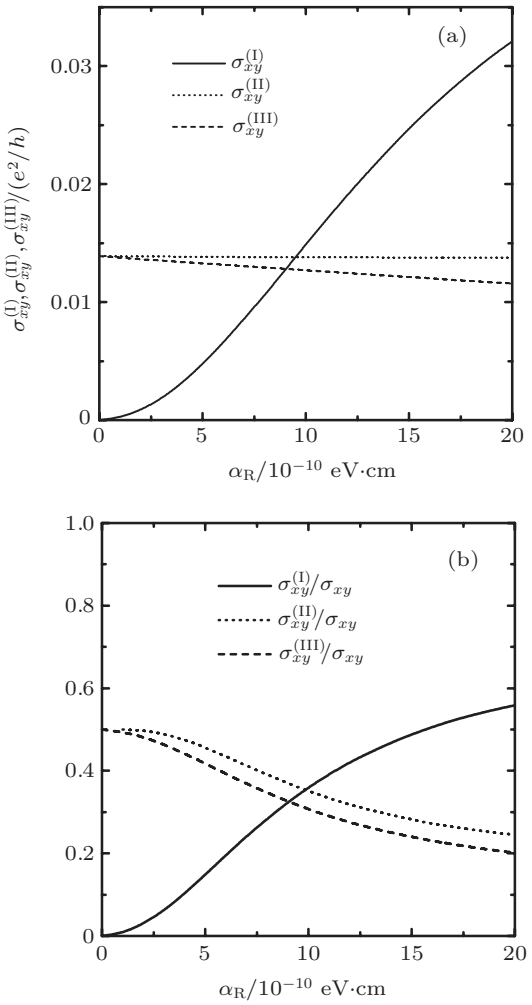


Fig. 2. (a) Plots of the three distinct anomalous Hall conductivity contributions $\sigma_{xy}^{(\mathrm{I})}$, $\sigma_{xy}^{(\mathrm{II})}$, and $\sigma_{xy}^{(\mathrm{III})}$ each as a function of the Rashba spin–orbit coupling constant α_{R}. (b) Plots of the relative ratios of the three distinct anomalous Hall conductivity contributions to the total anomalous Hall conductivity each as a function of the Rashba spin–orbit coupling constant α_{R}. The material parameters used are: $\alpha_{\mathrm{E}} = \alpha_{\mathrm{im}} = 5.3$ Å^2, $\Delta = 0.01$ eV, and $\varepsilon_{\mathrm{F}} = 0.015$ eV.

In summary, we have investigated theoretically the various competing contributions to anomalous Hall effect in spin-polarized two-dimensional electron gases in the presence of intrinsic, extrinsic and external electric-field induced spin–orbit couplings based on a unified semiclassical theoretical approach. It is shown that the total anomalous Hall conductivity can be expressed as the sum of three distinct contributions in the presence of these competing spin–orbit interactions, namely an intrinsic contribution determined by the Berry curvature in the momentum space and an extrinsic contribution determined by the modified Bloch band group velocity and an extrinsic (side-jump and skew scattering) contribution determined by spin–orbit-dependent impurity scatterings. Both the intrinsic contribution determined by the Berry curvature in the momentum space and the extrinsic contribution determined by the modified Bloch band group velocity are shown to be dissipationless, in the sense that they are independent of disorder scattering effect and all states below the Fermi level may contribute to them. The relative magnitudes of the three distinct anomalous Hall conductivity contributions are estimated numerically by considering some actual material parameters suitable for some realistic physical systems. The results suggest that in order to obtain a more clear understanding of the theoretical aspects for AHE in spin-polarized two-dimensional electronic systems (or other diluted magnetic semiconductors), in addition to the Berry-curvature-related intrinsic mechanisms, the possible AHE contributions from impurity-induced extrinsic spin–orbit coupling and that from external electric-field induced spin–orbit coupling may also need to be considered.

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