Seeing time-reversal transmission characteristics through kinetic anti-ferromagnetic Ising chain*

Chen Ying-Ming(陈英明)[a)b)] and Wang Bing-Zhong(王秉中)[a)†]

[a)] *Institute of Applied Physics, University of Electronic Science and Technology of China, Chengdu* 610054, *China*
[b)] *School of Physics and Electronic Information, Huaibei Normal University, Huaibei* 235000, *China*

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As an example of our new approach to complex near-field (NF) scattering of electromagnetic waves, the time-reversal (TR) transmission process on an NF current-element array is mapped to the statistical process on a kinetic Ising transmission chain. Equilibrium statistical mechanics and non-equilibrium Monte Carlo (MC) dynamics help us to find signal jamming, aging, annihilating, creating, and TR symmetry breaking on the chain with inevitable background noises; and these results are general in NF systems where complex electromagnetic scattering arises.

Keywords: time-reversal, near-field scattering, symmetry breaking, temporal focusing

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1. Introduction

The problem of complex near-field (NF) scattering of the electromagnetic wave is widely present in NF scanning microscopy,[1−5] surface plasmon (SP),[6−9] meta-materials,[10−12] and time-reversal (TR) microwave NF scattering system (SS).[13−16] In these hot concepts there are many interesting physics phenomena, such as super resolution beyond diffraction limit, SP resonance, extraordinary optical transmission, negative refraction, temporal focusing, TR symmetry, etc. Existing static analysis and dynamic analysis both are based on Maxwell's equations.[17−19] However, it is always difficult to directly solve Maxwell's equations for these NF scattering problems with complex boundary conditions. As a new attempt, we will map a TR NF SS to a kinetic Ising model,[20−23] and then use statistical-mechanical method to analyse the signal transmission properties and TR symmetry of the system.

A TR NF SS includes two parts: one is far-field TR mirror (TRM) which receives signals, reverses the signals, and transmits the TR signals; the other is NF SS which transmits original signals and receives the TR signals through complex NF scattering. We focus only on the NF behaviour; the far-field behaviour is esteemed to be ideal. For the region very near a current element the most significant term in the magnetic field is the one that varies as r^{-2} and the dominant terms in the electric field are those that vary as r^{-3}, where r is the distance between the element and the field point. Near the element, magnetic field may be identified as the usual induction field obtained from Ampere's law. Electric field in this region may be identified as the one that is calculated from an electrostatic dipole. If there are no high magnetic permeability materials in the NF SS, the electrostatic field will dominate NF interaction. We suppose that the NF SS may be described by an array of NF current elements (or electric dipoles).

2. Mapping NF scattering to Ising model

We will map the NF current element array to an Ising chain by the following four steps.

First, we decompose the multipath according to scattering times: Paths = {path_0, path_1, ..., path_N...}; where Paths represents all possible paths from the source to the TRM; the path_0 is the direct path without scattering; and any path along which scatterings take place N times from the source to the TRM belongs to the path_N. It is evident that our decomposition is linear as long as the system is linear. We concentrate our attention on the path_N. In

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†Corresponding author. E-mail: bzwang@uestc.edu.cn

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Fig. 1(a), S_k denotes the dipole of the k-th scattering on the path$_N$. If S_k and S_j happen to be the same dipole, we will continue to decompose the same dipole into two different dipoles. In other words, the same current element can be linearly decomposed into several dipoles between which there are no interactions. The result of the decomposition is that the complex NF scattering problem is converted into the signal transmission problem along the paths.

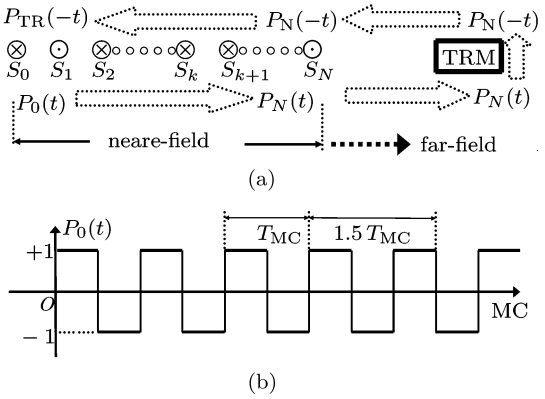


Fig. 1. (a) TR transmission process including the NF path$_N$ and far-field TRM, TR transmission including a round trip; (b) extrinsic excitation signal $P_0\,(t)$, square wave with period $T_{\rm MC}$.

Secondly, considering the short range of NF interactions between different current elements, we write the effective Hamiltonian including only the nearest-neighbour interaction energy as

$$H_{\rm eff} = \sum_{k=0}^{N-1} J_k^{k+1} P_k P_{k+1}, \tag{1}$$

where P_k represents the S_k dipole moment and J_k^{k+1} is the interaction coefficient between S_k and S_{k+1}. The energy of the source dipole is associated with the external input intensity; the other dipoles refer to the source. The source energy is absent in Eq. (1), because it can be treated as a constant. The term $J_k^{k+1} P_k P_{k+1}$ means the electrostatic energy of S_{k+1} if a signal is transmitted from S_0 to S_N; the same term means the energy of S_k if a signal is transmitted from S_N to S_0.

Thirdly, those interaction coefficients can be obtained in an actual array by calculation. As a simple example, we suppose that N dipoles are regularly aligned abreast to form a chain and P_k can take only values of either $+1$ or -1. Because two dipoles put abreast will counter each other, the $H_{\rm eff}$ can be accepted as an anti-ferromagnetic Ising model; that is to say, $J_k^{k+1} \geq 0$. For open space, the signal energy will trail off step by step. In Fig. 1(a), the dipoles are spaced evenly apart. When the signal energy is transmitted from S_0 to S_N, we set $J_k^{k+1} = J_0^1 \alpha^k$, $0 < \alpha < 1$, where parameter α is closely related to the distance between dipoles and J_0^1 is mainly related to the volume of the dipoles and the intensity of the extrinsic excitation signal. Symmetrically, we may set $J_k^{k+1} = J_0^N \alpha^{N-k-1}$, when the signal is transmitted from S_N to S_0.

Fourthly, an actual NF SS is inevitably disturbed by random background noises such as extraneous noise, thermal noise, crosstalk noise from parallel multipath, etc. A reduced temperature β is introduced to characterize them. That $\beta = 0^+$ means infinite noise and that $\beta = +\infty$ means no noise. For convenience, we merge J_0^1 into β, namely $J_0^1 = 1$. Thus β is a token of input signal-to-noise ratio (ISNR).

It is well known that temperature is a symbolic quantity of thermodynamics and statistical physics. Does an NF SS mapped into statistical physics still keep TR symmetry and why? Interaction strength is usually related to the system structure design. How does the interaction regulate the TR transmission behaviours? There are generally more noises in a more complex system. How do noises affect the TR transmission? How do the TR transmission properties change with the input signal period?

In Fig. 1(a), one complete TR transmission process is a round trip. Square waves $P_0\,(t)$ are inputted on S_0, then S_N receives the signals $P_N\,(t)$ through NF interactions and at the same time S_N transmits $P_N\,(t)$ to far-field TRM; TRM records $P_N\,(t)$ and reverses $P_N\,(t)$ into $P_N\,(-t)$, then TRM transmits $P_N\,(-t)$ back to S_N; S_0 receives the TR signals $P_{\rm TR}\,(-t)$ through NF interactions finally. In Fig. 1(b) the time unit is one MC step which is one tentative flip by MC single-moment-flip dynamics. It is well known that the conductivity of non-magnetic metal extremely approaches the direct current conductivity in a wide frequency range which includes even the infrared spectrum. The relaxation time of the direct current in common metal is about 10^{-5} picosecond (ε_0/σ), therefore the rising and the falling edges of the square wave are negligible in the wide frequency range including the infrared spectrum. The range in which the conductivity is practically equal to the direct current conductivity implies the application limit of the mapping kinetic Ising model. So long as the $P_0\,(t)$ period is not too short, the foregoing supposition that $P_k = \pm 1$ is reasonable.

3. Model calculations and analysis

By introducing the reduced temperature, the transmission behaviours are no longer deterministic in the kinetic statistical model. Probabilities will dominate all details of the transmission. When the $T_{\rm MC}$ approaches infinity, we may use standard equilibrium statistics method to describe the behaviours of the chain as follows:

$$p_+^+ = \frac{\sum\limits_{\sigma} \exp\left(-\beta H_{+\sigma}^+\right)}{\sum\limits_{\sigma} \exp\left(-\beta H_{\sigma}^+\right)}, \quad p_-^- = \frac{\sum\limits_{\sigma} \exp\left(-\beta H_{-\sigma}^-\right)}{\sum\limits_{\sigma} \exp\left(-\beta H_{\sigma}^-\right)}. \tag{2}$$

In Eqs. (2) the superscripts $\pm$ means $P_0 = \pm 1$, the subscript $\pm$ means $P_N = \pm 1$. For example, the interaction energy H_σ^+ of a specific configuration labeled by σ is

$$H_\sigma^+ = P_1 + \sum_{k=1}^{N-1} \alpha^k P_k P_{k+1}. \tag{3}$$

In Eq. (3) the σ configuration satisfies the constraint $P_0 = \pm 1$. In Eq. (2), the summation runs over all qualified configurations; the p_+^+ is the probability that S_0 sends $+1$ and S_N receives $+1$; the p_-^- is the probability that S_0 sends -1 and S_N receives -1. For the space-reversal symmetry of the $H_{\rm eff}$, we can present $p_+^+ = p_-^-$, then

$$p_{\rm TR}^+ = 1 - 2p_+^+ + 2p_+^+ p_+^+. \tag{4}$$

In Eq. (4), $p_{\rm TR}^+$ is the probability that S_0 sends $+1$ and S_0 receives $+1$ after TR transmission in Fig. 1(a). The received signal is completely definite if $p_{\rm TR}^+ = 1$ and the received signal is completely random if $p_{\rm TR}^+ = 0.5$.

Figure 2(a) is the distribution of the $p_{\rm TR}^+$, given $\beta = 10$; on the white definite land, $p_{\rm TR}^+ = 1$; in the black random sea, $p_{\rm TR}^+ = 0.5$; in the grey fuzzy shoreline, the $p_{\rm TR}^+$ transits from 0.5 to 1. The step of N is 1, so the shoreline is not very smooth. The shoreline trend shows the competition between α and N; the $p_{\rm TR}^+$ will fall rapidly into the random sea with the increase of N, but enhancing α will make the $p_{\rm TR}^+$ climb up to the definite land nimbly. Because the signal energy will fall significantly with the increase of the scattering times, bigger N means smaller contribution. That is to say, it is improvident to place too many NF scatterers when we design a TR NF SS. Figure 2(b) is also the distribution of the $p_{\rm TR}^+$, given $N = 7$. The trend of the fuzzy shoreline shows the cooperation between α and β; the resultant force including big interaction and big ISNR pulls the $p_{\rm TR}^+$ away from the random sea. But, there is a black noise channel to stop the correct transmission, so the interaction cannot suppress all noises. The channel tells designers that a TR NF SS will collapse under over strong background noises. In a dense integrated TR NF SS, the channel will become a significant problem by reason of great crosstalk noises. We will discuss

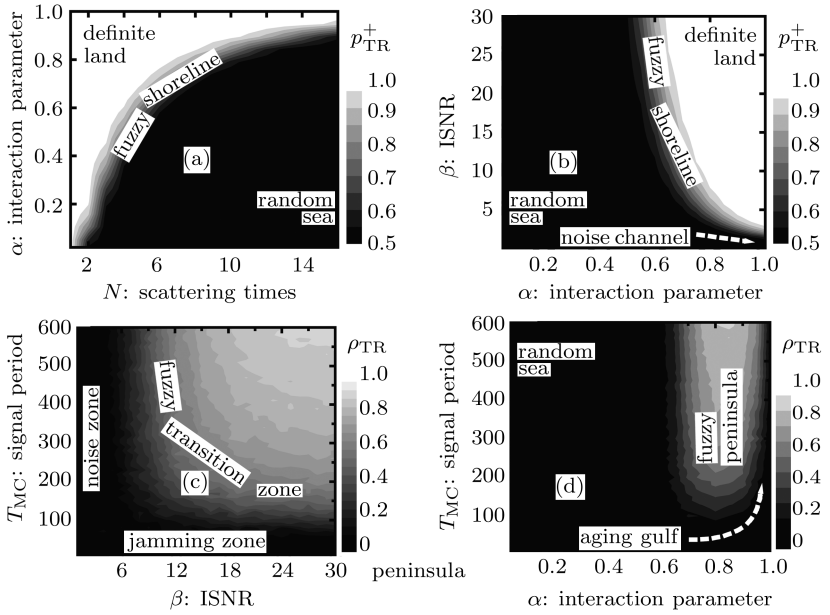


Fig. 2. TR transmission properties of the kinetic Ising chain; (a), (b) distribution of $p_{\rm TR}^+$ as $T_{\rm MC} = +\infty$; (a) given $\beta = 10$, α step is 0.05, N step is 1; (b) given $N = 7$, β step is 0.5, α step is 0.05; (c), (d) distribution of $\rho_{\rm TR}$ as $T_{\rm MC}$ is finite; (c) given $N = 7$, $\alpha = 0.7$, β step is 1, $T_{\rm MC}$ step is 10 MCss; (d) given $N = 7$, $\beta = 10$, $T_{\rm MC}$ step is 10 MCss, α step is 0.05.

the physical mechanism of the noise channel in Fig. 3 below. In Figs. 2(a) and 2(b), the definite lands where information is recovered perfectly imply that TR operation can play a perfect temporal focusing role; but that the $T_{\rm MC}$ approaches infinity is the premise.

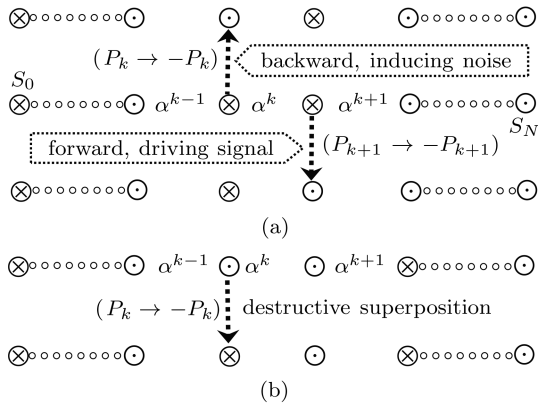


Fig. 3. (a) Moment-flip probabilities that can drive signal in two directions, signal steps backward induces noise; (b) The annihilation of two contiguous signals that can stop signal transmission.

When the $T_{\rm MC}$ is finite, we measure the time-dependent sequence of $P_{\rm TR}(-t)$ by the Glauber single-moment-flip MC dynamic with updates at randomly chosen sites. Each attempted moment-flip from P_k to $-P_k$ is accepted with the probability

$$W(P_k \to -P_k) = [1 + \exp(\beta \Delta E_k)]^{-1}, \qquad (5)$$

where ΔE_k is the energy change of the system resulting from an accepted moment-flip. The correlation coefficient $\rho_{\rm TR}$ is defined as

$$\rho_{\rm TR} = \frac{\langle P_{\rm TR}(-t), P_0(-t)\rangle}{\|P_{\rm TR}(-t)\|_2 \|P_0(-t)\|_2}. \qquad (6)$$

In Eq. (6), the $P_0(-t)$ is the TR sequence of the input signal including $400T_{\rm MC}$ MC steps; $P_{\rm TR}(-t)$ is the received TR signal sequence also including $400T_{\rm MC}$ MC steps. From Cauchy inequality, the correlation coefficient $\rho_{\rm TR}$ satisfies $-1 \le \rho_{\rm TR} \le +1$. If the $\rho_{\rm TR}$ approaches $+1$ (or -1), the received signal $P_{\rm TR}(-t)$ is extremely positively (or negatively) correlative to the input signal $P_0(-t)$; if the $\rho_{\rm TR}$ approaches 0, the $P_{\rm TR}(-t)$ is extremely non-correlative to the $P_0(-t)$. That $|\rho_{\rm TR}| < 1$ means some information loss. In other words, the recovery information is incomplete; TR operation cannot play a perfect temporal focusing role. Figure 2(c) shows the distribution of the $\rho_{\rm TR}$, given $N = 7$ and $\alpha = 0.7$. Figure 2(d) also shows the distribution of the $\rho_{\rm TR}$, given $N = 7$ and $\beta = 10$.

In Fig. 3(a), the present signal energy is in the junction between S_k and S_{k+1}. If the moment-flip from P_{k+1} to $-P_{k+1}$ is accepted, the signal energy will step forward correctly; if the moment-flip from P_k to $-P_k$ is accepted, the signal energy will step backward incorrectly; and that the junction maintains unchanged is also incorrect. When a signal is transmitted from S_0 to S_N, the statistical-mechanical chain will be stimulated to some non-stationary non-equilibrium state; the dynamic restoring process back to the equilibrium state will produce additional noise inevitably. It is obvious that more noises will be induced if the $T_{\rm MC}$ becomes shorter. We say that the signals are jamming when the signal period is shortening. In Fig. 2(c), it is an evidence of signal jamming that shortening signal period has almost the same effects with the enhancement of noise.

The engine of the signal transmission is the difference between the forward probability and the backward probability, because the MC dynamics determines the signal junction forward or backward only by the moment-flip probability. Signals will be disoriented gradually with the difference in two directions vanishing. Especially, all signals will walk randomly just like noises on the chain as

$$\lim_{\alpha \to 1} W(P_k \to -P_k) = \frac{1}{2} = \lim_{\alpha \to 1} W(P_{k+1} \to -P_{k+1}). \qquad (7)$$

We say that signals are aging as $\alpha \to 1$. It is interesting that aging signals are digging the noise channel in Fig. 2(b) and a lot of random signals (noises) are massing near $\alpha = 1$ to form the aging Gulf in Fig. 2(d). It is easily understood that the noise zone of Fig. 2(c) is a logical result of the noise channel, because the noises cannot be suppressed completely by enhancing the interaction. The transition zone and the peninsula are fuzzy where information cannot be recovered completely and there are no completely correlative white zones in Figs. 2(c) and 2(d). TR symmetry is breaking because of inevitable noises. The TR transmission process is a process of entropy increase, because entropy increase and information loss are dual. In other words, the NF transmission is irreversible in an actual NF SS. But when the signal period approaches infinity, the process may touch the definite lands of Figs. 2(a) and 2(b).

In Figs. 2(c) and 2(d), when the $T_{\rm MC}$ is short enough, the $\rho_{\rm TR}$ will be almost zero. The result can be easily mapped back to the original scattering question. The NF interaction will be no longer the leading mechanism with the shortening of the signal period because the NF scattering will be diminishing and replaced by the far-field radiation.

Why can signal jamming and signal aging destroy TR symmetry? In Fig. 3(b), from Eq. (5), the sequence of the probabilities of the four choices are

$$\begin{aligned} & W\left(P_k \to -P_k\right) \\ > & W\left(P_{k+1} \to -P_{k+1}\right) > \frac{1}{2} \\ > & W\left(P_k \to P_k\right) > W\left(P_{k-1} \to -P_{k-1}\right). \end{aligned} \tag{8}$$

Inequality (8) shows that the probability of the annihilation between two contiguous signals is larger than those of the other three choices (forward, stationary and backward). The signal must be lucky enough if the signal can reach correctly the end of the path. Because, every time there is a meeting between the signal junction and a random noise junction, a choice will be made from among the four options; the multiplicative probability that the signal can reach correctly the end will be declining exponentially with the increase of noises induced by signal jamming and signal aging. In other words, the noise channel and the aging Gulf are also the tombs of the signals. The annihilation between the signals and noises means that TRM cannot receive all information; then the difference between the signal received by TRM and the original signal is a noise sequence. And for the same reason, the TR signals will add the other random sequence on the voyage home. Certainly, the relation between the annihilation and the creation is dual. From Eq. (5), the probability of the creation is

$$W\left(-P_k \to P_k\right) = 1 - W\left(P_k \to -P_k\right), \tag{9}$$

where $W\left(P_k \to -P_k\right)$ is the probability of the annihilation at site S_k; the creation probability $W\left(-P_k \to P_k\right)$ denotes the inverse process probability of the annihilation. From inequality (8) and Eq. (9), we have at once

$$W\left(-P_k \to P_k\right) < \frac{1}{2} < W\left(P_k \to -P_k\right). \tag{10}$$

Inequality (10) means that the information loss is inevitable. So TR symmetry is breaking as long as there are random background noises and the T_{MC} is finite. The references [14]–[16] in which we do not find perfect temporal focus are the experimental supports for the TR symmetry breaking.

In Figs. 2(a) and 2(b), it is clear that the enhancement of interaction parameter can help the system to improve the signal transmission, but figure 2(d) shows that received signals are nearly completely noncorrelative to the original input signals in the aging Gulf. Why cannot we see the similar aging Gulfs in Figs. 2(a) and 2(b)? As the signal period approaches infinity, one ergodic process expresses only one bit ($P_0 = +1$ or $P_0 = -1$) by Eq. (2); all information is just the one bit. And then it is easily understood that figures 2(a) and 2(b) can present definite lands which show TR symmetry: although the probability that aging signals successfully sail through the chain is very low as $\alpha \to 1$, the only one bit will reach the end of the finite-length chain inevitably after numerous repetitions. So, as $T_{\mathrm{MC}} \to +\infty$ the aging Gulf will be shallower and shallower.

In an actual NF SS, the annihilation means destructive superposition of two fields and $\alpha = 1$ means no energy will be taken out of the system. Obviously, that numerous destructive superpositions and the energy conservation appear together is easily associated with the standing waves. In other words, the NF SS will approach electric resonance as $\alpha \to 1$. The aging Gulf reminds the designer of TR NF SS to search after an optimum interaction parameter for the TR transmission.

4. Discussions and conclusions

Further work will get closer to an actual NF SS, following two steps: one, expanding the incidence field by square waves approximately; two, calculating J_k^{k+1} (J_k^{k+1} may be negative) by the electric properties and the spatial distributions of the dipoles. Remarkably, higher-dimensional Ising model can also be employed to describe more complex structure and explain more profound physics, such as NF imaging detection system in which we can build a high-dimensional kinetic Ising network between the scanning probe and the imaging points including both ferromagnetic and antiferromagnetic interactions. Additionally, the temporal focusing role of TR technique in a standard statistical-mechanical model may be helpful to discuss some elementary problems in statistics such as irreversible process, ergodicity and time arrow.

In conclusion, in this work we present an example about mapping a complex NF SS to a kinetic statistical model. We obtain the transmission properties which are helpful to design TR NF SS and understand the physical mechanism of the TR temporal focusing. Signal jamming, aging, creating, annihilating and TR symmetry breaking are general in an NF SS. Because NF scattering is essentially a process in which fields

exchange energy with matters frequently, NF interactions will dominate the energy transmission. We expect that this work will stimulate research in other NF systems where complex scattering arises, such as NF microscopy, meta-materials and surface plasmon.

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