Propagation dynamics and experimental observation of quadratic vortex beam*

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The concept of a quadratic vortex beam is proposed, in which phase term of the beam is given by $\exp(\mathrm{i}\, m\theta^2)$. The phase of the quadratic vortex beam increases with azimuthal angle nonlinearly. This change in phase produces several unexpected effects. Unlike the circularly symmetric beam spot of normal vortex beams, the intensity distribution of the quadratic vortex beam is shown to be asymmetric. The phase singularities will shift in the transverse beam plane on propagation.

Keywords: vortex beam, phase singularity, propagation dynamics

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1. Introduction

A beam with helical phase structure of $\exp(\mathrm{i}\, l\theta)$, where l is the topological charge and θ is the azimuthal angle, is regarded as the vortex beam. Laguerre–Gaussian (LG) beam is an example of vortex beam. It was predicted by Allen that an LG beam possesses a well-defined orbital angular momentum (OAM).[1] Since then, considerable attention has been paid to the generation of vortex beams and their applications.[2−5] The OAM of a vortex beam can be transferred to the particles and can set them into rotation. Therefore, they can be useful in designing optical tweezers and spanners.[6−8] The OAM can also be used to encode data onto a laser beam for transmitting information in free-space optical systems.[9−12] Such an application provides a possibility to increase the information density together with an inherent security enhancement.[9]

Vortex beams with asymmetric intensity distribution are referred to as noncanonical optical vortices, which can be constructed by continuously deforming the canonical vortices.[13] It is shown in the study of noncanonical optical vortices that the beam experiences rotation and the phase singularity shift in linear and nonlinear media.[14,15] Such an anormalous motion of optical vortices may be affected by amplitude gradient, phase gradient and nonlinear factors that depends on the intensity gradient.[14,15] The shift of the optical vortex in the transverse beam plane is an interesting property and has many potential applications. Very recently, a shift of helical waves through off-axis circular apertures was observed, which can be used to perform subdiffraction imaging.[16]

To the best of our knowledge, most of the research on vortex beams is limited to the case of beams with phase term of $\exp(\mathrm{i}\, l\theta)$, i.e. the phase depends linearly on azimuthal angle θ. This kind of vortex beam can be defined as the normal vortex beam. In this paper, we propose the concept of a quadratic vortex beam. The phase of quadratic vortex beams can be described by a phase term $\exp(\mathrm{i}\, m\theta^2)$. The non-linear dependence on the azimuthal angle of the phase will introduce many interesting phenomena including the emergence and the evolution of phase singularity and the rotation of beam intensity.

2. Theoretical analysis

It is clear for the normal vortex beam with topological charge $l = 1$ that the phase will gradually increases from 0 to 2π as the azimuthal angle increases from 0 to 2π. To compare with such a phase change of the normal vortex beam, a special case of the quadratic vortex beam is considered. The parameters of phase term $\exp(\mathrm{i}\, m\theta^2)$ are chosen to be $m = 1/(2\pi)$. For a normal vortex beam with phase term of $\exp(\mathrm{i}\, \theta)$,

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the phase increases linearly with the azimuthal angle as shown in Fig. 1(a). While for a quadratic vortex beam with a phase term of $\exp[\mathrm{i}\,\theta^2/(2\pi)]$, the phase increases with the azimuthal angle nonlinearly. The nonlinear increase of the phase of the quadratic vortex beam may have interesting characteristics.

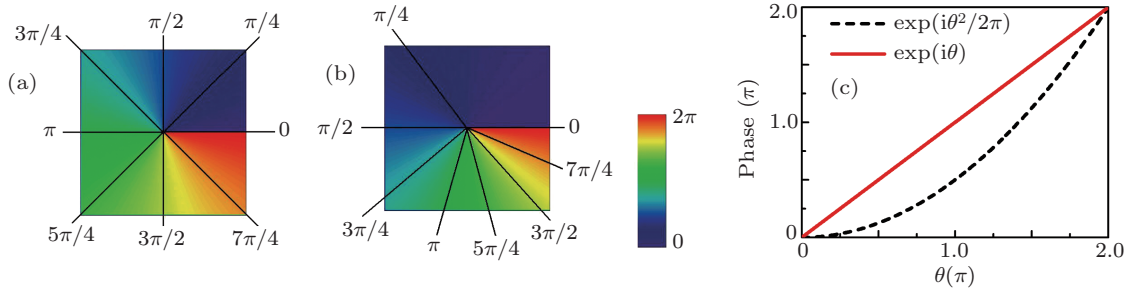


Fig. 1. (colour online) Phase of vortex beams: (a) the normal vortex beam with a phase term of $\exp(\mathrm{i}\,\theta)$; (b) the quadratic vortex beam with a phase term of $\exp[\mathrm{i}\,\theta^2/(2\pi)]$; (c) phases of vortex beams each as a function of azimuthal angle.

It is well known that the intensity distribution of a normal vortex beam with integer topological charge is a bright annulus with a dark core inside and the intensity distribution of the beam is circularly symmetric. A phase singularity exists in the centre of the beam and its position keeps invariant on propagation. The intensity distribution and the phase of the normal vortex beam with topological charge $l = 1$ are illustrated in Figs. 2(a) and 2(b), respectively. Now we turn to study the propagation of the quadratic vortex beam in free space. At the $z = 0$ plane, the quadratic vortex beam, with the phase being quadratically dependent on the azimuthal angle, can be expressed as

$$E^{(0)}(r,\theta) = A(r,\theta)\exp\left(-\frac{r^2}{w^2}\right)\exp(\mathrm{i}\,m\theta^2), \qquad (1)$$

where $A(r,\theta)$ is the amplitude factor, which is dependent on r and θ, and w is a constant characterizing the beam size. It is assumed that the quadratic vortex beam is generated by imposing the nonlinear phase dependence on the beam, which can be done by passing a Gaussian beam through a spiral phase plane (SPP) or computer-generated hologram (CGH), therefore just behind the SPP, we can choose $A(r,\theta) = A_0$. Here A_0 is a constant representing the amplitude of the beam.

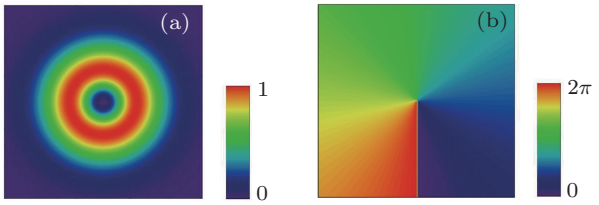


Fig. 2. (colour online) Intensity (a) and phase (b) of normal vortex beams with a phase term of $\exp(\mathrm{i}\,\theta)$.

Based on the Fresnel diffraction integral, the electric field for a quadratic vortex beam propagating in free space can be given by[17]

$$E(\rho,\phi,z) = \int E^{(0)}(r,\theta)\exp\left\{-\frac{\mathrm{i}\,k}{2z}[r^2+\rho^2 - 2r\rho\cos(\theta-\phi)]\right\} r\,\mathrm{d}r\,\mathrm{d}\theta, \qquad (2)$$

where ρ and ϕ denote the radial coordinate and the azimuthal angle in propagation field respectively, and $k = \omega/c$ is the wave number of the incident beam.

Substituting Eq. (1) into Eq. (2), and after tedious integration, we obtain

$$\begin{aligned} E(\rho,\varphi,z) = {} & \left(\frac{-\mathrm{i}}{\lambda z}\right)\exp(-\mathrm{i}\,kz)A_0\exp\left(\frac{\mathrm{i}\,k}{2z}\rho^2\right)\left\{\sum_{l=0}^{\infty}\left\{\frac{\sqrt{\mathrm{i}\,\pi}}{2\sqrt{m}}\exp\left(-\frac{\mathrm{i}\,l^2}{4m}\right)\left[\mathrm{erf}\left(\frac{l+4m\pi}{2\sqrt{\mathrm{i}\,m}}\right)-\mathrm{erf}\left(\frac{l}{2\sqrt{\mathrm{i}\,m}}\right)\right]\right.\right. \\ & \left.\times(-\mathrm{i})^l\exp(-\mathrm{i}\,l\phi)\frac{\sqrt{\pi}\beta}{8\alpha^{3/2}}\exp\left(-\frac{\beta^2}{8\alpha}\right)\exp\left[\mathrm{I}_{l/2-1/2}\left(\frac{\beta^2}{8\alpha}\right)-\mathrm{I}_{l/2+1/2}\left(\frac{\beta^2}{8\alpha}\right)\right]\right\} \\ & +\left\{\sum_{l=0}^{\infty}\left\{-\frac{\sqrt{\mathrm{i}\,\pi}}{2\sqrt{m}}\exp\left(-\frac{\mathrm{i}\,l^2}{4m}\right)\left[\mathrm{erf}\left(\frac{l-4m\pi}{2\sqrt{\mathrm{i}\,m}}\right)-\mathrm{erf}\left(\frac{l}{2\sqrt{\mathrm{i}\,m}}\right)\right]\right.\right. \\ & \left.\left.\times(-\mathrm{i})^l\exp(\mathrm{i}\,l\phi)\frac{\sqrt{\pi}\beta}{8\alpha^{3/2}}\exp\left(-\frac{\beta^2}{8\alpha}\right)\exp\left[\mathrm{I}_{l/2-1/2}\left(\frac{\beta^2}{8\alpha}\right)-\mathrm{I}_{l/2+1/2}\left(\frac{\beta^2}{8\alpha}\right)\right]\right\}\right\}, \end{aligned} \qquad (3)$$

where $\alpha = 1/w^2 - \mathrm{i}\,k/2z$, $\beta = k\rho/z$, and erf denotes the error function, and J_l the Bessel function of the first kind.

The propagation dynamics of quadratic vortex beam with a phase term of $\exp[\mathrm{i}\,\theta^2/(2\pi)]$ is shown in Fig. 3. Like a normal vortex beam, there is also a dark core in the centre of the beam and a bright annulus surrounds the core. While unlike the symmetric distribution of normal vortex beam, the intensity distribution in the annulus varies with position, i.e., it is stronger in some part but it is weaker in other part. The stronger part of the annulus rotates on propagation. The rotation-like behaviour of quadratic vortex beam is similar to that of an off-axis optical vortex beam.[18] In particular, we are interested in the evolution of the phase singularity. For a normal vortex beam, the phase singularity is located in the centre of the beam and keeps its position in propagation. However, for the quadratic vortex beam with a phase term of $\exp[\mathrm{i}\,\theta^2/(2\pi)]$, the location of phase singularity moves on propagation. The phase singularity deviates from axis as propagation distance increases.

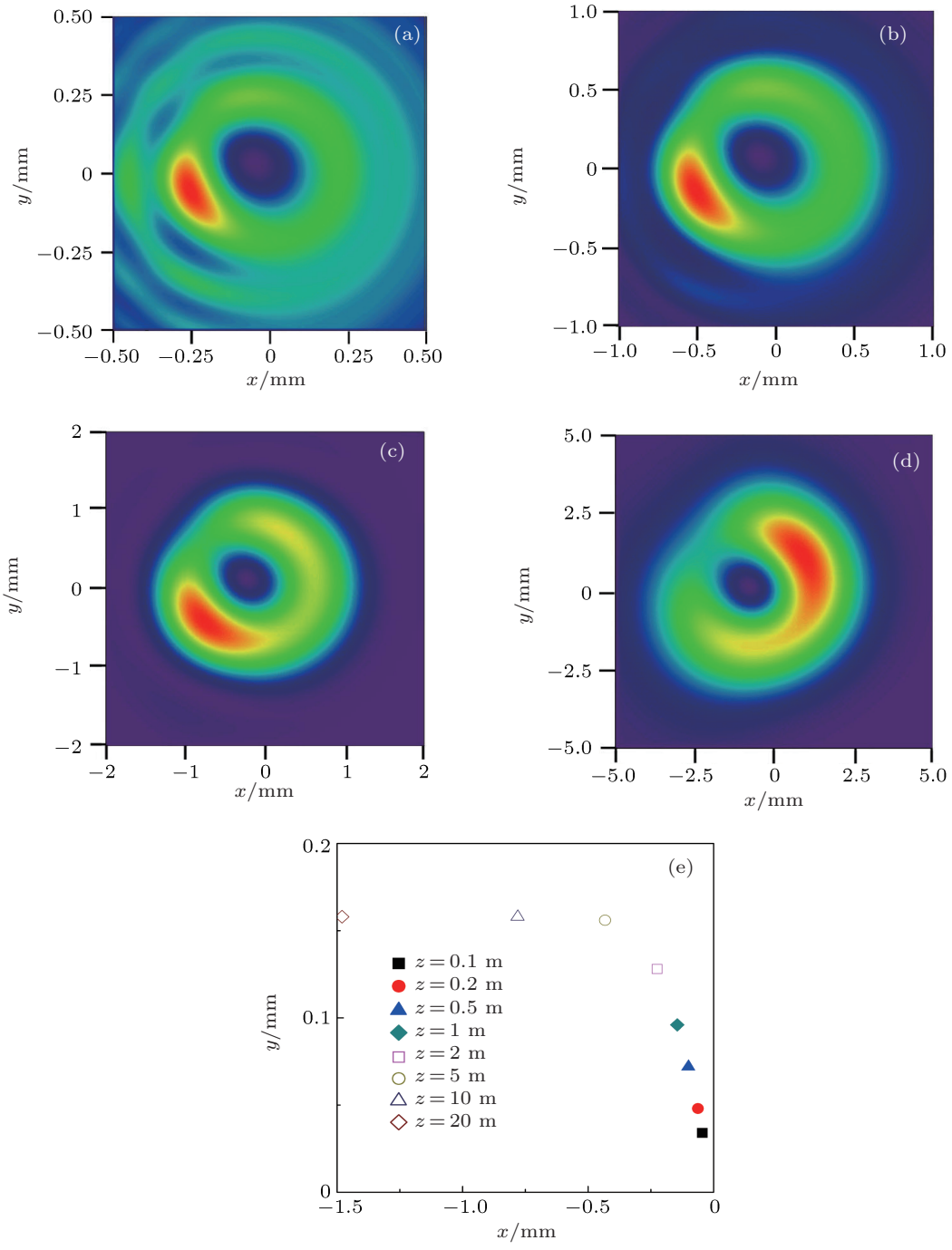


Fig. 3. (colour online) Propagation dynamics of a quadratic vortex beam with a phase term of $\exp[\mathrm{i}\,\theta^2/(2\pi)]$, with $z = 0.1$ m (a), 0.5 m (b), 2 m (c), and 10 m (d), and the evolution of phase singularity (e). The chosen parameters are $w = 1$ mm and $\lambda = 633$ nm.

More interesting properties are found in the propagation of beam with a phase term of $\exp[\mathrm{i}\,2\theta^2/(2\pi)]$. The intensity distribution and the phase singularity are plotted in Fig. 4. Different from one dark core of beam with a phase term of $\exp[\mathrm{i}\,\theta^2/(2\pi)]$, two dark cores emerge for a sufficiently long propagation distance and these two dark cores overlap into one in a short propagation distance. These characterizations can be understood with the help of distribution of phase singularities as shown in Fig. 4(e). It is clear that there are two phase singularities corresponding to the two dark cores. The phase singularities are first close to the axis and then away from the axis on propagation. The longer the propagation distance, the larger the displacement is. Furthermore, the separation between the two phase singularities increases as well with the increase in propagation distance. Therefore, one dark core gradually evolves into two. The sizes of the two dark cores are different, one is larger and the other is smaller.

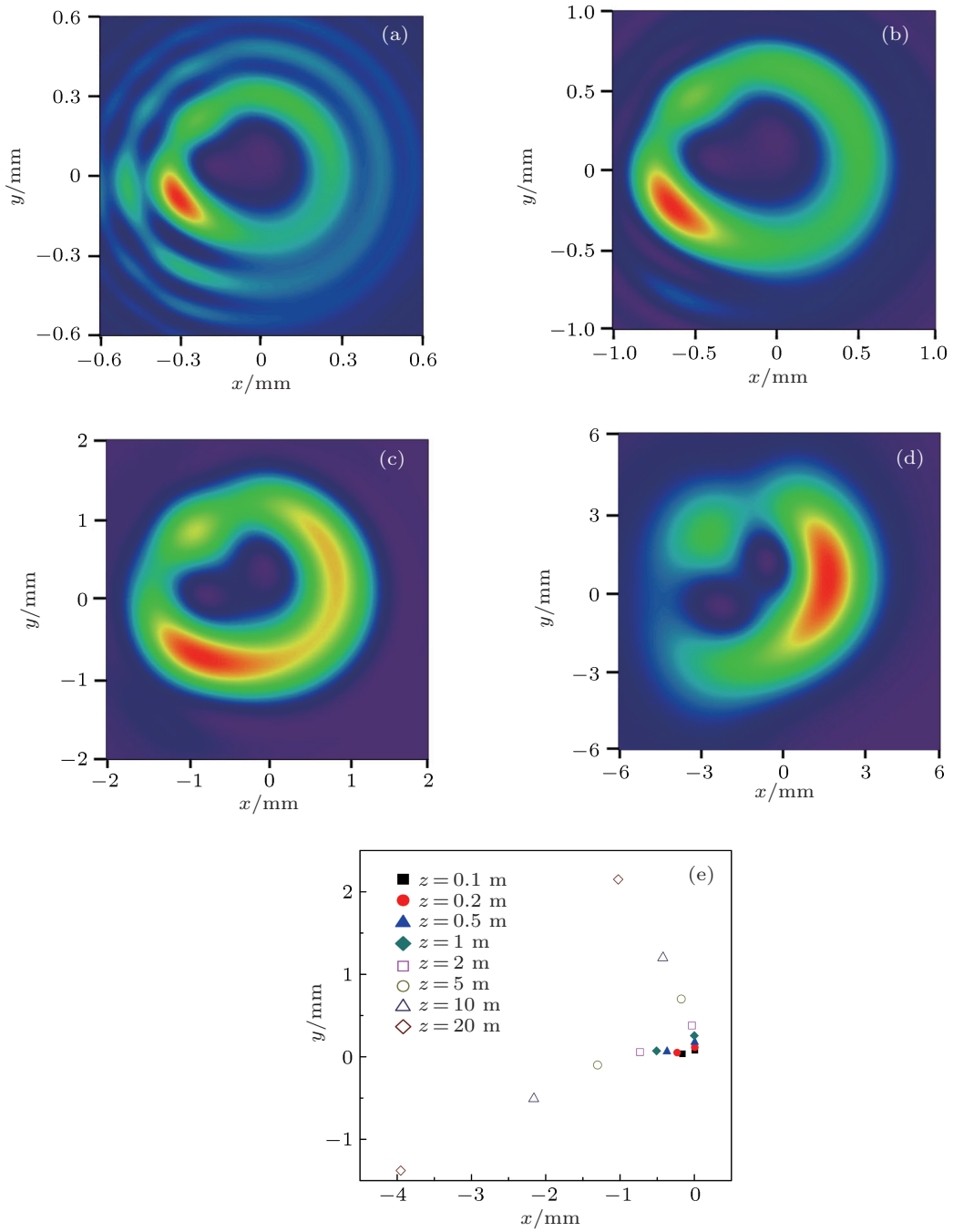


Fig. 4. (colour online) Propagation dynamics of a quadratic vortex beam with a phase term of $\exp[\mathrm{i}\,2\theta^2/(2\pi)]$, with $z = 0.1$ m (a), 0.5 m (b), 2 m (c), and 10 m (d), and the evolution of phase singularities (e). The other parameters are the same as those in Fig. 3.

3. Experimental observation

An experimental setup based on computer-generated hologram (CGH) is introduced to generate such a quadratic vortex beam. The CGH formed by interference of the required quadratic vortex beam with a plane wave in computer is displayed on a programmable spatial light modulator (SLM) (LCR2500, Holoeye). Several reconstructed beams emerge as the laser beam impinges on the SLM. The quadratic vortex beam desired is selected by a spatial filter and recorded by a CCD camera. The quadratic vortex beam with a phase term of $\exp[\mathrm{i}\,\theta^2/(2\pi)]$ obtained in our experiment is shown in Fig. 5(b). According to the experimental condition, we also simulate the corresponding theoretical result. The simulation presented in Fig. 5(a) coincides well with the experimental observation.

The properties of higher-order quadratic vortex beams propagating in free space are experimentally observed as well, which is shown in Fig. 6. The intensity distributions show different characteristics. For the vortex beam with a phase term of $\exp[\mathrm{i}\,\theta^2/(2\pi)]$, a dark core is formed and the intensity in the outer-ring varies around the dark core. For the vortex beam with a phase term of $\exp[\mathrm{i}\,2\theta^2/(2\pi)]$, two dark cores emerge and the intensity varies around every dark core. More generally, m dark cores emerge for vortex beam with a phase term of $\exp[\mathrm{i}\,m\theta^2/(2\pi)]$. The dark cores vary in size, which accords well with the theoretical prediction.

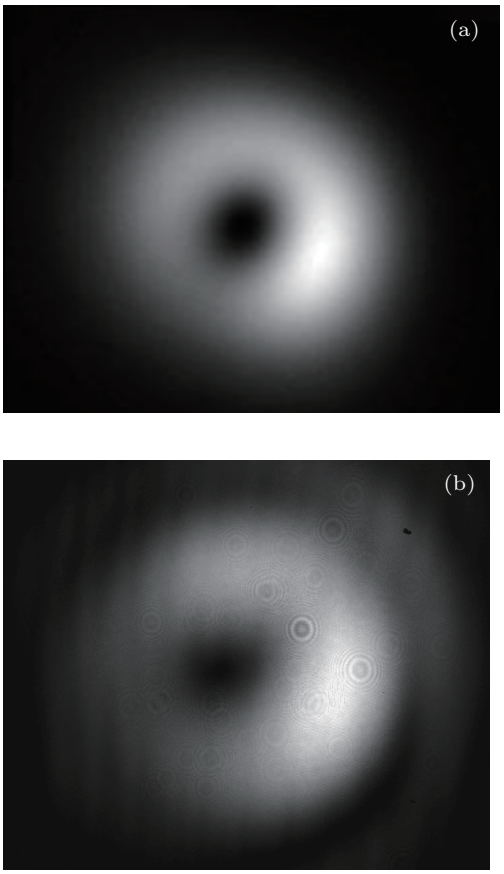


Fig. 5. Intensity distributions of the quadratic vortex beam with a phase term of $\exp[\mathrm{i}\,\theta^2/(2\pi)]$. Panel (a) shows the theoretical result; panel (b) displays the experimental observation.

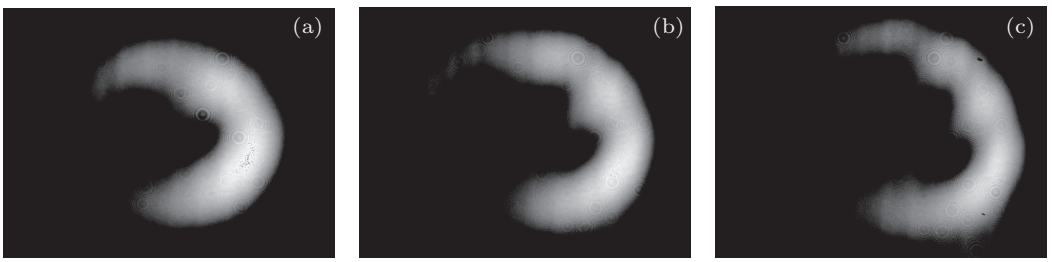


Fig. 6. Experimental observations of higher-order quadratic beam with a phase term of $\exp[\mathrm{i}\,m\theta^2/(2\pi)]$ propagating in free space, with $m = 2$ (a), 3 (b), and 4 (c).

It is known that the intensity distribution of a normal vortex beam is a dark core surrounded by a bright annulus. This property is useful in particle trapping. Particles can be trapped inside the high-intensity region because of optical gradient force. Furthermore, each photon of a normal vortex beam with topological charge l carries OAM of $l\hbar$ and the OAM of the beam can transfer to the particles. It can be found from the results of the quadratic vortex beam that the dark core still exists. Moreover, the number of dark cores which depend on the phase term may increase. The motion of optical vortices may be influenced by amplitude gradient and phase gradient.[14,15] The bright annulus of a quadratic vortex beam is no longer circularly symmetric and it will influence the optical gradient force within the bright annulus. The

phase of a quadratic vortex beam is dependent nonlinearly on azimuthal angle. Therefore, the peculiar property of a quadratic vortex beam may show its unique advantage in micro manipulation of particles.

4. Conclusions

We present a new kind of vortex beam, which is called the quadratic vortex beam. The phase of the quadratic vortex beam is dependent nonlinearly on the azimuthal angle. To investigate the property of quadratic vortex beams, a special example of a beam with a phase term of $\exp[\mathrm{i}\, m\theta^2/(2\pi)]$ is considered. It is found that beams with m phase singularities emerge, which is related to dark cores and the intensity distribution around the core is varying. Moreover, the position of phase singularity shifts on propagation. These unique properties may have important potential applications and deserve to be further investigated.

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