Demonstration of the approximation of eliminating atomic excited populations in an atom–cavity system*

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Using the master equation approach to a V-type three-level atom inside a high-finesse single-mode cavity in the strong coupling condition, we demonstrate the approximation of eliminating populations of atomic excited states, which is widely used in the field of the atom–cavity systems [Hechenblaikner G, Gangl M, Horak P and Ritsch H 1998 *Phys. Rev.* A **58** 3030]; Liu L W, Tan T and Xu Y 2008 *J. Mod. Opt.* **56** 968; Cho J, Angelakis D G and Bose S 2008 *Phys. Rev.* A **78** 062338. This is reflected in the deviation of the population δ, of which the value is $10^{-3} \sim 10^{-2}$. We further find the deviation of the dipole force and demonstrate that the deviation of atomic population will not notably affect the dipole force of the atom in the strong coupling condition. A relevant experimental case is also presented.

Keywords: V-type three-level atom, high finesse optical cavity, dipole force, strong coupling condition

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1. Introduction

Atom–cavity systems in the strong coupling condition are an important means for the investigation of the nature of quantum phenomena which are not apparent in daily life. As the coupling of the system with the environment disturbs coherent dynamics of the system, the realization of strong coupling of the atom with a cavity mode[1–4] is of great importance. An atom–cavity system in the strong coupling condition can remain coherent dynamics of the system and can be applied in cavity cooling,[3] the generation of single photons on demand,[5] and the controlling of the motion of an atom inside the cavity.[6–8] Then atom–cavity systems under the strong coupling condition are becoming important objects in the research of the foundations of quantum mechanics and their applications such as Bose–Einstein condensates,[9–11] quantum information processing,[12] etc.

Experiments are actually carried out on multilevel atoms,[13,14] therefore in many conditions atoms are modeled as three-level systems, which include three types (V-type, Λ-type and Ξ-type).[15,16] In this paper we consider a three-level atom in the V configuration interacting with a single mode of a cavity.

For simplicity, population of atomic excited states is usually eliminated.[17–19] This approximation is usually called adiabatic elimination. This approximation reduces the degree of freedom of the system and greatly simplifies the problems. Adiabatic elimination means that the effects of the populations of atomic excited states are ignored. However, to what degree does the approximation still hold true? In this paper we will answer this question.

To find more accurate populations of atomic excited levels, we consider the relation of atomic populations, atomic coherences and field operators simultaneously. The deviation of the population between the master equation approach and the adiabatic elimination can reflect the degree of the validity of the adiabatic elimination approximation.

As the investigation of the dipole force on the atom in a high-finesse optical cavity is of great significance (e.g. in the field of atomic cooling[17]), we will calculate the dipole force acting on the atom and consider the deviation of the dipole force acting on the atom according to more accurate populations of atomic excited levels.

The remainder of the paper is organized as follows. In Section 2 we give the model of the atom–

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cavity system and the steady solutions of atomic excited populations of the atom by using the master equation. Then we study the deviations of the atomic populations and the dipole force in Section 3 and Section 4, respectively. Some concluding remarks are presented in Section 5.

2. Approach to calculating atomic populations

A schematic sketch of the atom–cavity system is given in Fig. 1. The upper levels are $|1\rangle$ (energy $\hbar\omega_1$) and $|3\rangle$ (energy $\hbar\omega_3$) and the lower level is $|2\rangle$ (energy 0). The transition between $|2\rangle$ and level $|1\rangle$ ($|2\rangle$ and level $|3\rangle$) is mediated by cavity mode of frequency ω_{12} (ω_{32}). Coupling strengths are denoted by g_1 and g_2. The transition between state $|3\rangle$ and state $|1\rangle$ is forbidden. The state $|3\rangle$ decays into state $|2\rangle$ (state $|1\rangle$ decays into state $|2\rangle$) at rate γ_3 (γ_1). In general, the system is driven by a monochromatic field with frequency $\omega_{\rm p}$ and effective amplitude η. We assume that the cavity has only one mode with frequency $\omega_{\rm c}$ and cavity photons decay with rate κ. The master equation describing the system in Fig. 1(a) is given in the rotating-wave approximation and in a frame rotating with the pump frequency $\omega_{\rm p}$ as[17]

$$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H_{\rm AC}, \rho] - \frac{\mathrm{i}}{\hbar}[H_{\rm P}, \rho] + L_\kappa\rho + L_\gamma\rho, \qquad (1)$$

where

$$\begin{aligned} H_{\rm AC} &= -\hbar\Delta_1 A_{11} - \hbar\Delta_3 A_{33} - \hbar\Delta_{\rm c} a^\dagger a \\ &\quad + \hbar g_1(a^\dagger A_{21} + A_{12} a) \\ &\quad + \hbar g_2(a^\dagger A_{23} + A_{32} a), \qquad (2a) \\ H_{\rm P} &= \mathrm{i}\hbar\eta(a^\dagger - a), \qquad (2b) \\ L_\kappa\rho &= \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a), \qquad (2c) \\ L_\gamma\rho &= \gamma_1(2A_{21}\rho A_{12} - \rho A_{11} - A_{11}\rho) \\ &\quad + \gamma_2(2A_{23}\rho A_{32} - \rho A_{22} - A_{22}\rho), \qquad (2d) \end{aligned}$$

with atomic detunings $\Delta_1 = \omega_{\rm p} - \omega_{12}$, $\Delta_3 = \omega_{\rm p} - \omega_{32}$, and a detuning of the cavity $\Delta_{\rm c} = \omega_{\rm p} - \omega_{\rm c}$. $H_{\rm AC}$ represents the interaction Hamiltonian of the atom with the cavity field, and $H_{\rm P}$ is the pump Hamiltonian. $L_\kappa\rho$ and $L_\gamma\rho$ describe the decay of the cavity mode and atomic damping to background modes, respectively. We assume that the cavity is weakly driven, so weak atomic saturation and weak excitation are fulfilled, i.e., either there is one photon in the cavity while the atom is in the ground state or there is no photon in the cavity while the atom is in an excited state. It follows that we can eliminate the dynamics of the cavity field from the atomic system[19] and factorize all expectation values, e.g., $\langle A_{ij}a\rangle = \langle A_{ij}\rangle\langle a\rangle$.[17,20,21] Using Eq. (1) and Eqs. 2(a)–2(d), the evolutions of the average values of field operator a, population A_{ii}, and coherence A_{jk} can be obtained as follows:[22]

$$\begin{aligned} \langle\dot{a}\rangle &= -\mathrm{i}\,g_1\langle A_{21}\rangle - \mathrm{i}\,g_2\langle A_{23}\rangle \\ &\quad - \mathrm{i}\,\Delta_{\rm c}\langle a\rangle - \kappa\langle a\rangle + \eta, \qquad (3a) \\ \langle\dot{A}_{11}\rangle &= -2\gamma_1\langle A_{11}\rangle - \mathrm{i}\,g_1\langle A_{12}\rangle\langle a\rangle \\ &\quad + \mathrm{i}\,g_1\langle a^\dagger\rangle\langle A_{21}\rangle, \qquad (3b) \\ \langle\dot{A}_{33}\rangle &= \mathrm{i}\,g_2\langle A_{23}\rangle\langle a^\dagger\rangle - \mathrm{i}\,g_2\langle A_{32}\rangle\langle a\rangle - 2\gamma_3\langle A_{33}\rangle, \qquad (3c) \\ \langle\dot{A}_{21}\rangle &= \mathrm{i}\,g_1\langle A_{11}\rangle\langle a\rangle - (\gamma_1 - \mathrm{i}\,\Delta_1)\langle A_{21}\rangle \\ &\quad - \mathrm{i}\,g_1\langle A_{22}\rangle\langle a\rangle + \mathrm{i}\,g_2\langle A_{31}\rangle\langle a\rangle, \qquad (3d) \\ \langle\dot{A}_{23}\rangle &= \mathrm{i}\,g_1\langle A_{13}\rangle\langle a\rangle - \mathrm{i}\,g_2\langle A_{22}\rangle\langle a\rangle \\ &\quad - (\gamma_3 - \mathrm{i}\,\Delta_3)\langle A_{23}\rangle + \mathrm{i}\,g_2\langle A_{33}\rangle\langle a\rangle, \qquad (3e) \\ \langle\dot{A}_{13}\rangle &= -\mathrm{i}\,g_2\langle A_{12}\rangle\langle a\rangle + i(\Delta_3 - \Delta_1)\langle A_{13}\rangle \\ &\quad - (\gamma_1 + \gamma_3)\langle A_{13}\rangle + \mathrm{i}\,g_1\langle a^\dagger\rangle\langle A_{23}\rangle. \qquad (3f) \end{aligned}$$

The atomic operators $A_{ij} = |i\rangle\langle j|$ $(i, j = 1, 2, 3)$ satisfy the commutator relations $[A_{ij}, A_{kl}] = A_{il}\delta_{jk} - A_{kj}\delta_{il}$, and the condition $\langle A_{11}\rangle + \langle A_{22}\rangle + \langle A_{33}\rangle = 1$. In contrast to the method involving evolution equations just for the coherences of the atom and the annihilation operators of the cavity mode,[17] we consider the relations between field operators and all the elements of the reduced density operator of the atom. The evolution equations for the averages of these variables are given. Then we obtain a set of equations for the average values of atomic operators (including both the populations and the coherences) and equations for the field operators. Using these equations, we obtain the populations of each atomic level in different conditions. As finding an exact solution of Eqs. (3a)–(3f) is formidable, we turn to numerical method to obtain the detailed atomic populations.

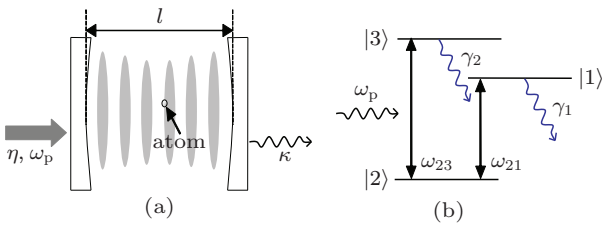


Fig. 1. (a) Sketch of the atom–cavity system under consideration: a V-type three-level atom is coupled to a single-mode Fabry–Perot cavity and is driven by a laser field. The length of the cavity is l. The gray shadings represent the standing wave of the light field inside the cavity. (b) Configuration of the V-type three-level atom. γ_1 and γ_3 are the spontaneous emission rates of levels $|1\rangle$ and $|3\rangle$ respectively.

3. Population of V-type three-level atom

In this section, we calculate the populations and plot them each as a function of detuning Δ_3. The parameters are selected to satisfy the strong coupling condition. We can obtain the steady solution in the strong coupling condition for the average values of the atomic populations $\langle A_{11}\rangle$ and $\langle A_{33}\rangle$ from Eqs. 3(a)–3(f). For clarity, the average values of populations of levels $|1\rangle$ and $|3\rangle$ are plotted each as a function of Δ_3 when $\omega_{21} = \omega_{\rm p}$ ($\Delta_1 = 0$) in Fig. 2 and Fig. 3, respectively. It is shown that both populations are on the order of 10^{-3}. When the pump field is resonant to the $|2\rangle \leftrightarrow |3\rangle$ transition, the population of $|1\rangle$ decreases greatly. We denote the deviation of the population of level $|i\rangle$ by δ_i ($i = 1, 2, 3$). When $\Delta_1 = 0$, the largest deviations of the populations are $(\delta_1)_{\max} \simeq 0.007$ for the level $|1\rangle$ and $(\delta_3)_{\max} \simeq 0.002$ for the level $|3\rangle$. We find that both of them are less than 0.01.

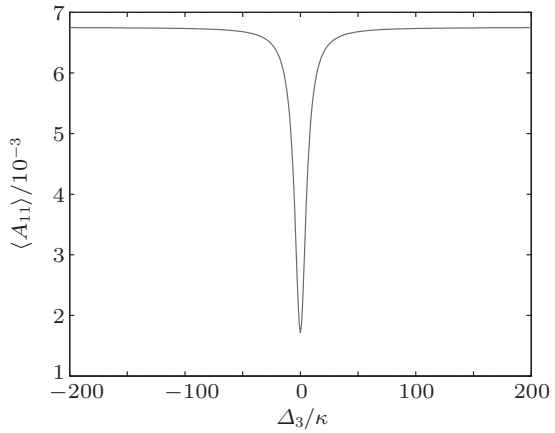


Fig. 2. Average population of the excited state $|1\rangle$ of the atom when $\Delta_1 = 0$. The parameters are $(\Delta_{\rm c}, g_1 = g_2, \gamma_1 = \gamma_3, \kappa, \eta) = 2\pi \times (0, 12, 3, 1, 1)$ MHz respectively.

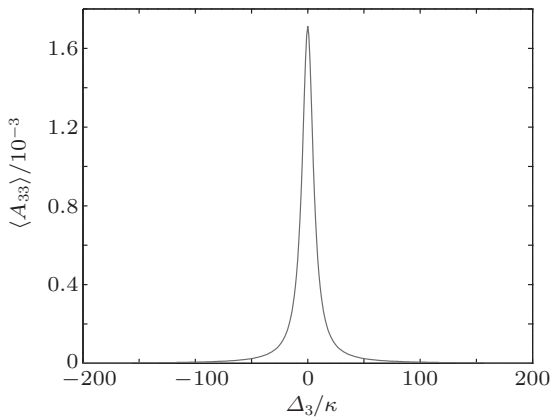


Fig. 3. Average population of the excited state $|3\rangle$ of the atom when $\Delta_1 = 0$. The maximum of the deviation $(\delta_3)_{\max} \simeq 0.002$. The parameters are $(\Delta_{\rm c}, g_1 = g_2, \gamma_1 = \gamma_3, \kappa, \eta) = 2\pi \times (0, 12, 3, 1, 1)$ MHz respectively.

When $\Delta_1 = 0$, $\Delta_3 = 0$, levels $|1\rangle$ and $|3\rangle$ are degenerate and they are expected to have the same value of atomic population. When Δ_3 is large enough, we also expect that the transition frequency of $|3\rangle \leftrightarrow |2\rangle$ will detune to the frequency of the pump field, so $\langle A_{33}\rangle$ will tend to be 0. Indeed, these phenomena are included in our results. From Figs. 2 and 3, we find $\langle A_{11}\rangle = \langle A_{33}\rangle \simeq 0.002$ when $\Delta_3 = 0$. In Fig. 3, when $|\Delta_3| \geq 2\pi \times 50$ MHz, $\langle A_{33}\rangle \simeq 1 \times 10^{-4} \ll 10^{-3}$. The two results show that our approach to calculating atomic populations is self-consistent.

To test the approximation of adiabatic elimination in a broader range, we calculate the populations of the atomic excited levels when the atomic detuning ranges from 0 to $2\pi \times 100$ MHz. In Fig. 4, we plot the average populations of the excited state $|1\rangle$ of the atom, each as a function of detunings Δ_3 for $\Delta_1 = 2\pi \times 5$ MHz and $\Delta_1 = 2\pi \times 50$ MHz. In Fig. 4(a), the largest deviation of the population, $(\delta_1)_{\max} \simeq 0.01$, and in Fig. 4(b), $(\delta_1)_{\max} \simeq 0.03$. In Fig. 5, we also plot the average populations of the excited state $|3\rangle$ of the atom for $\Delta_1 = 2\pi \times 5$ MHz and $\Delta_1 = 2\pi \times 50$ MHz. In Fig. 5(a), the largest deviation of the population, $(\delta_3)_{\max} \simeq 0.006$, and in Fig. 5(b), $(\delta_3)_{\max} \simeq 0.04$. As in the most of cavity cooling experiments, the atomic detunings each have a value on the order of 10^2 MHz,[1] we find that the largest deviation of the population under the condition of the excitation eliminated approximation from the population is less than 0.05. This demonstrates that the approximation is exactly appropriate when one studies the effects of the atomic populations in the strong coupling condition.

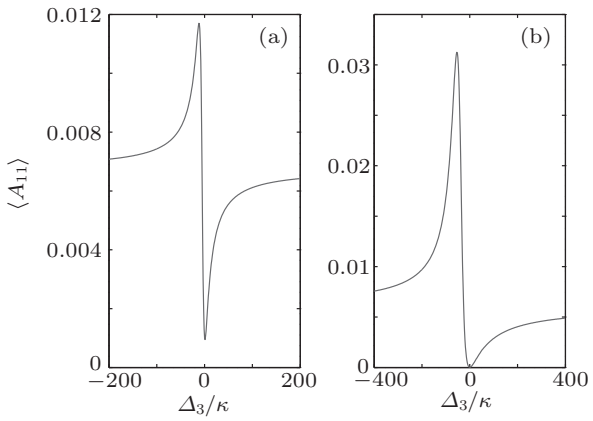


Fig. 4. Average populations of the excited state $|1\rangle$ of the atom when (a) $\Delta_1 = 2\pi \times 5$ MHz and (b) $\Delta_1 = 2\pi \times 50$ MHz. (a) The maximum of the deviation $(\delta_1)_{\max} \simeq 0.002$; (b) the maximum of the deviation $(\delta_1)_{\max} \simeq 0.03$. The parameters are $(\Delta_{\rm c}, g_1 = g_2, \gamma_1 = \gamma_3, \kappa, \eta) = 2\pi \times (0, 12, 3, 1, 1)$ MHz respectively.

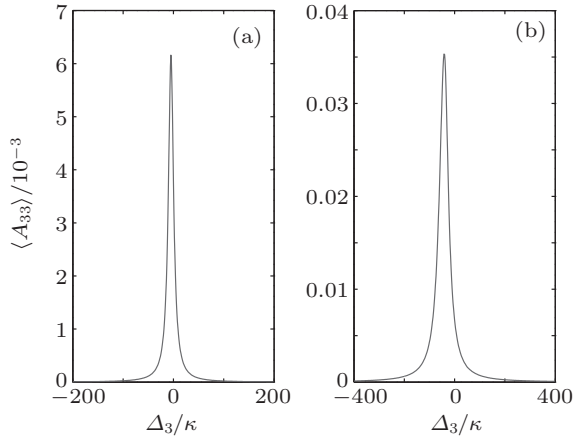


Fig. 5. Average populations of the excited state $|3\rangle$ of the atom. (a) $\Delta_1 = 2\pi \times 5$ MHz, the maximum of the deviation $(\delta_3)_{\max} \simeq 0.006$; (b) $\Delta_1 = 2\pi \times 50$ MHz, the maximum of the deviation $(\delta_3)_{\max} \simeq 0.04$. The parameters are $(\Delta_c, g_1 = g_2, \gamma_1 = \gamma_3, \kappa, \eta) = 2\pi \times (0, 12, 3, 1, 1)$ MHz respectively.

The approximation of adiabatic elimination is valid when $\eta \sim \kappa$. For example, if $\eta > 3\kappa$, the populations of the excited states are larger than 0.1. In this condition, the approximation is not accurate and the approximation results in more significant errors. As an example, we plot the average population of level $|3\rangle$ when $\eta = 2\pi \times 5$ MHz, $\kappa = 2\pi \times 1$ MHz in Fig. 5. We find that the population of level $|3\rangle$ is more than 0.2. However, in usual experiments, we have $\eta = (1 \sim 2)\kappa$, so the approximation of adiabatic elimination is reasonable.

We find that $\langle A_{11}\rangle$ reaches the minimum in both Fig. 4(a) and Fig. 4(b) when $\Delta_3 = 0$. Because the frequency of the pump field equals the frequency of the transition $|2\rangle \leftrightarrow |3\rangle$, an atom in the ground state has a bigger probability to jump to level $|3\rangle$ than level $|1\rangle$. This also demonstrates the reliability of our approach in calculating the atomic populations as well.

4. Dipole force acting on the atom

We can now find how much the dipole force will be effected by the deviation of atomic populations. In the Fabry–Perot cavity, the photon creation and annihilation operators $a^\dagger$ and a refer to the cavity mode with the spatial mode function

$$\phi(\boldsymbol{r}) = \cos(kz)\exp[-(x^2+y^2)/w_0^2]. \tag{4}$$

Here $\boldsymbol{k}$ is the wave vector and w_0 is the waist of the cavity mode. The dipole force can be calculated from this Hamiltonian using the Heisenberg equation of the atomic momentum

$$\boldsymbol{F}(\boldsymbol{r}) = \dot{\boldsymbol{P}} = \frac{\mathrm{i}}{\hbar}[H, \boldsymbol{P}]. \tag{5}$$

Then the dipole force is

$$\begin{aligned}\boldsymbol{F}(\boldsymbol{r}) \;=\; & -\hbar\nabla g_1(\boldsymbol{r})(a^\dagger A_{21} + A_{12}a) \\ & -\hbar\nabla g_2(\boldsymbol{r})(a^\dagger A_{23} + A_{32}a).\end{aligned} \tag{6}$$

Next we calculate the dipole force and compare it with the one in the approximation of adiabatic elimination. For simplicity, we consider the average value of Eq. (6)

$$\begin{aligned}\langle\boldsymbol{F}(\boldsymbol{r})\rangle \;=\; & -\hbar\langle\nabla g_1(\boldsymbol{r})(a^\dagger A_{21} + A_{12}a)\rangle \\ & -\hbar\langle\nabla g_2(\boldsymbol{r})(a^\dagger A_{23} + A_{23}a)\rangle.\end{aligned} \tag{7}$$

Furthermore, we assume that the atom is at rest in the centre of the cavity. In the strong coupling, the maximum of $g_i(\boldsymbol{r})$ (g_{i0}) is on the order of 10 MHz. As

$$g_i(\boldsymbol{r}) = g_{i0}\phi(\boldsymbol{r}) = g_{i0}\cos(kz)\exp[-(x^2+y^2)/w_0^2] \tag{8}$$

has the form of sinusoid for z, we can estimate the value of $\langle\nabla g_1(\boldsymbol{r})\rangle$ by

$$\langle\nabla g_1(\boldsymbol{r})\rangle \simeq \frac{g_{10}}{\lambda_\mathrm{p}/4}. \tag{9}$$

Here λ_p is the wavelength of the pump field and it is on the order of 10^{-6} m. We find $\langle\nabla g_1(\boldsymbol{r})\rangle \sim 10^7$ MHz/m. In the following calculations, we suppose $g_1 = g_2$, so we also obtain $\langle\nabla g_2(\boldsymbol{r})\rangle \sim 10^7$ MHz/m. Let $x = y = 0$ in Eq. (4) and insert it into Eq. (7), then we find that the average value of the dipole force is on the order of 10^{-21} N.

To compare the dipole forces in the two situations (with or without considering the elimination of the excited state populations), we will work out the dipole force under the approximation at the same time. Under this approximation, $\langle A_{11}\rangle = \langle A_{33}\rangle = 0$ and $\langle A_{22}\rangle = 1$.[17,20,21] Using Eq. (1), we obtain the evolution of the average value of the field operator and the atomic coherences as follows:[22,23]

$$\begin{aligned}\langle\dot{a}\rangle = & -\mathrm{i}\,g_1\langle A_{21}\rangle - \mathrm{i}\,g_2\langle A_{23}\rangle \\ & -\mathrm{i}\,\Delta_c\langle a\rangle - \kappa\langle a\rangle + \eta, \qquad (10a)\\ \langle\dot{A}_{21}\rangle = & -(\gamma_1 - \mathrm{i}\,\Delta_1)\langle A_{21}\rangle \\ & -\mathrm{i}\,g_1\langle a\rangle + \mathrm{i}\,g_2\langle A_{31}\rangle\langle a\rangle, \qquad (10b)\\ \langle\dot{A}_{23}\rangle = & \mathrm{i}\,g_1\langle A_{13}\rangle\langle a\rangle - \mathrm{i}\,g_2\langle a\rangle \\ & -(\gamma_3 - \mathrm{i}\,\Delta_3)\langle A_{23}\rangle, \qquad (10c)\\ \langle\dot{A}_{13}\rangle = & -\mathrm{i}\,g_2\langle A_{12}\rangle\langle a\rangle + i(\Delta_3 - \Delta_1)\langle A_{13}\rangle \\ & -(\gamma_1 + \gamma_3)\langle A_{13}\rangle + \mathrm{i}\,g_1\langle a^\dagger\rangle\langle A_{23}\rangle. \qquad (10d)\end{aligned}$$

From Eqs. (10a)–(10d), we can obtain the dipole force. In Fig. 6, we plot the dipole force as a function of atomic detuning Δ_3.

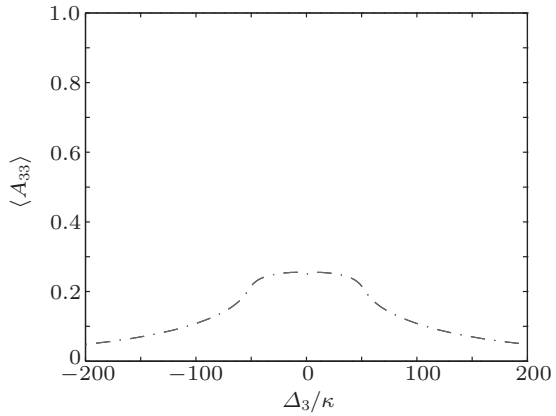


Fig. 6. Average population of the excited state $|3\rangle$ of the atom when $\eta = 2\pi \times 5$ MHz. The value of $\langle A_{33}\rangle$ is larger than 0.2 when $\eta = 2\pi \times 5$ MHz. The parameters are $(\Delta_c, \Delta_1, g_1 = g_2, \gamma_1 = \gamma_3, \kappa, \eta) = 2\pi \times (0, 0, 12, 3, 1, 1)$ MHz respectively.

The dipole forces are plotted in Fig. 7(a) and Fig. 8(a), each as a function of Δ_3 for Δ_1=0 and $\Delta_1 = 2\pi \times 50$ MHz respectively. At the same time, their counterparts calculated under the adiabatic eliminating approximation are plotted in Fig. 7(b) and Fig. 8(b) respectively. From Fig. 7 and Fig. 8, we find that the two results of the dipole forces are consistent, or the deviations of the dipole forces are very small.

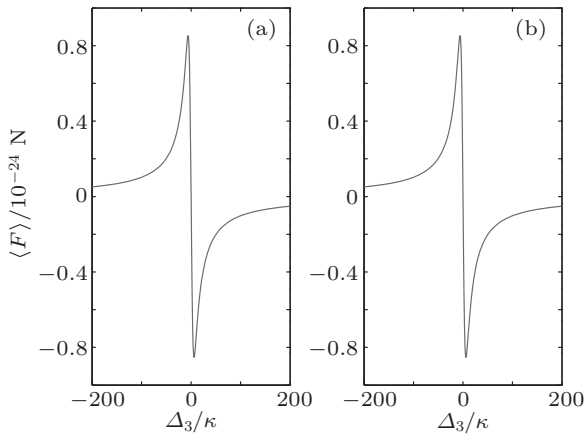


Fig. 7. Average dipole forces of the atom when $\Delta_1 = 0$: (a) the results obtained from the present model; (b) the results obtained from the model under the approximation of adiabatic elimination. The parameters are $(\Delta_c, g_1 = g_2, \gamma_1 = \gamma_3, \kappa, \eta) = 2\pi \times (0, 12, 3, 1, 1)$ MHz respectively.

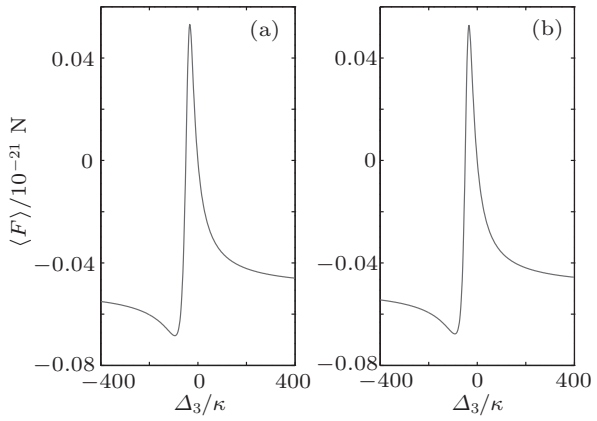


Fig. 8. Average dipole forces of the atom when $\Delta_1 = 2\pi \times 50$ MHz: (a) the result obtained from the present model; (b) the results obtained from the approximation of adiabatic elimination. The parameters are $(\Delta_c, g_1 = g_2, \gamma_1 = \gamma_3, \kappa, \eta) = 2\pi \times (0, 12, 3, 1, 1)$ MHz respectively.

To obtain the difference of the dipole force in our model from that under the approximation of adiabatic elimination, we calculated the deviation of the dipole force from that obtained under the approximation. The results are plotted in Fig. 9. The solid line corresponds to the results from the present model in Section 2 and the dashed one corresponds to the results from the approximation. Let the relative deviation of the dipole force be δ_f and we can find $\delta_f \simeq 1.2\%$. If Δ_1 increases, both the magnitude and the shape of the dipole force will change, but the deviation is not significant,which is illustrated in Fig. 8, where we have $\Delta_1 = 2\pi \times 50$ MHz. From Fig. 10, we can estimate the relative deviation δ_f, and we find $\delta_f \simeq 2.0\%$.

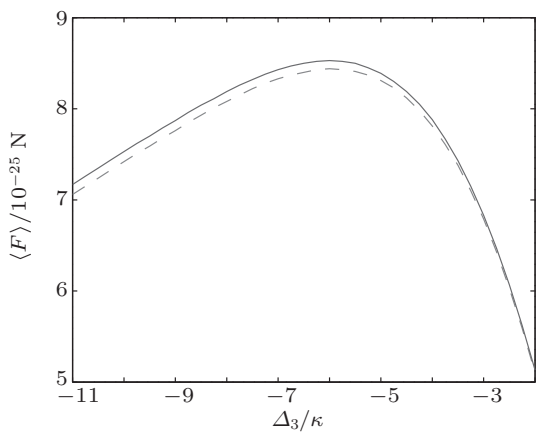


Fig. 9. Correction of the dipole force when $\Delta_1 = 0$. Solid and dashed lines correspond to the the present model and the model under the approximation of adiabatic elimination respectively. The parameters are $(\Delta_c, g_1 = g_2, \gamma_1 = \gamma_3, \kappa, \eta) = 2\pi \times (0, 12, 3, 1, 1)$ MHz respectively.

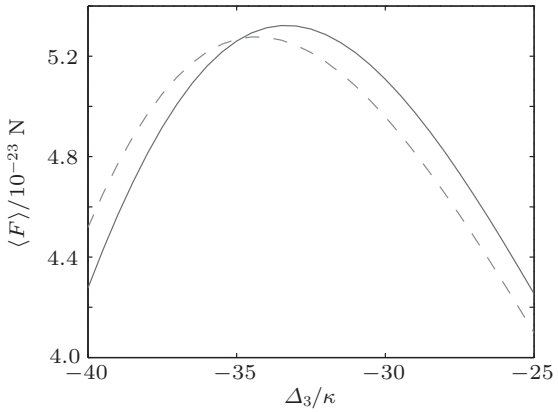


Fig. 10. Deviations of the dipole forces when $\Delta_1 = 2\pi \times 50$ MHz. Solid and dashed lines correspond to the the present model and the model under the approximation of adiabatic elimination respectively. The parameters are $(\Delta_c, g_1 = g_2, \gamma_1 = \gamma_3, \kappa, \eta) = 2\pi \times (0, 12, 3, 1, 1)$ MHz respectively.

By comparison with the dipole forces, we find the relative deviations of the dipole force are on the order of 10^{-2}. As the dipole force is expressed by atomic coherence, the result of the comparison of dipole forces can reflect the degree of deviation of atomic coherence. We infer that the deviation of the atomic populations will not notably affect the atomic coherence $\langle A_{ij} \rangle$ in the atomic detuning area $|\Delta_i| \simeq (0 \sim 10^2)$ MHz $(i = 1, 3)$ in the strong coupling condition. This range of the atomic detuning is the very detuning range in usual experiments of the cavity cooling. So we have demonstrated the validity of the approximation of adiabatic elimination in the atom–cavity system in the strong coupling condition.

For experimental demonstration of the approximation, we take ^{87}Rb inside a cavity as an example.[24] In Figs. 3 and 4(b), we find that the populations of atomic excited states $|1\rangle$ and $|3\rangle$ are extremely low, especially for lager values of Δ_1 and Δ_3. The ground state $|2\rangle$ can be achieved in $|F = 2, m = 2\rangle$ of $5S_{1/2}$ and the upper levels $|1\rangle$ and $|3\rangle$ correspond to $|F = 2, m = -1\rangle$ and $|F = 2, m = 1\rangle$ of $5P_{1/2}$, respectively. Levels $|1\rangle$ and $|3\rangle$ are assumed to be far off resonance to the cavity mode, then the Zeeman hyperfine levels $|F = 2, m = -1\rangle$ and $|F = 2, m = 1\rangle$ of $5P_{1/2}$ will not participate significantly in the population dynamics. Therefore, for a V-type three-level atom in a cavity, our results of atomic populations are very consistent with practical situations.

5. Conclusion

We have made a detailed calculation of the atomic populations of the excited states of a V-type three-level atom in the high-finesse optical cavity using the master equation approach. We have found the deviations of the populations from the approximation of adiabatic elimination (δ) are on the order of $10^{-3} \sim 10^{-2}$. In usual experimental conditions,[4] the deviation of the population is less than 0.05 and relative deviation of the dipole force is on the order 1%. The results demonstrate that the approximation of eliminating atomic excited population is exactly appropriate when $\eta \sim \kappa$. This also demonstrates that atomic coherence will not be affected significantly by the change of the population. Our calculation is based on V-type atom, other three-level atoms like Λ-type or Ξ-type can be studied using the same way.

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