A method of measuring micro-impulse with torsion pendulum based on multi-beam laser heterodyne*

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In this paper, we propose a novel method of multi-beam laser heterodyne measurement for micro-impulse. The measurement of the micro-impulse, which is converted into the measurement of the small tuning angle of the torsion pendulum, is realized by considering the interaction between pulse laser and working medium. Based on Doppler effect and heterodyne technology, the information regarding the small tuning angle is loaded to the frequency difference of the multi-beam laser heterodyne signal by the frequency modulation of the oscillating mirror, thereby obtaining many values of the small tuning angle after the multi-beam laser heterodyne signal demodulation simultaneously. Processing these values by weighted-average, the small tuning angle can be obtained accurately and the value of the micro-impulse can eventually be calculated. Using Polyvinylchlorid+2%C as a working medium, this novel method is used to simulate the value of the micro-impulse by MATLAB which is generated by considering the interaction between the pulse laser and the working medium, the obtained result shows that the relative error of this method is just 0.5%.

Keywords: laser heterodyne, Doppler effect, Fourier analysis

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1. Introduction

Laser micro-thrusters will have wide application in the control field of satellite attitude and the orbit of satellites, because it has advantages of high dynamic range of impulse, low power consumption, high energy coupling efficiency, easy realization, lightweight and digital control.[1−3] The impulse is an important parameter for the performance of laser micro-thruster whose feature is a small order of magnitude (10^{-7} N·s–10^{-5} N·s). In 1999, Phipps *et al.*[4−6] proposed using a torsion pendulum system to measure the micro-impulse which was generated by a laser micro-thruster, meanwhile they tested the performance parameter of micro-thruster. Later, Li and Tang[7] carried out the relevant research work. To analyse these results, we will find that the noise of the measuring system will affect system accuracy, if the order of impulse is small, the system error may even reach 50%; in the meantime, if the target plane deviates from the focal plane or the coupling efficiency is quite low within the limited time of interaction between pulse laser and target, the measurement accuracy of micro impulse can be influenced. Therefore the routine micro-impulse measuring system is difficult to satisfy the requirements of ultra-high accuracy measurement.

Laser interferometry can effectively solve these problems which will improve the measurement accuracy of measuring system.[8−11] Differential measurement was adopted through using two corner prisms, instead of a light-pointer, to measure the tuning angle of torsion pendulum. The new method will improve the system accuracy efficiently. The mass of the torsion pendulum was increased from 0.2 g to 58 g which coped with the defocus problems in 2010. Research shows that the laser interferometry has greatly improved the performance of the torsion pendulum system, which can satisfy the requirements of micro-impulse measurement in laser micro-thrusters. However, numerous indirect parameters of measurement and accidental error are still significant, which leads to a low measurement accuracy.

Among the optical measurements, the laser heterodyne technique, which inherits many advantages of laser heterodynes technology and the laser Doppler technique, which is one of ultra-accuracy measurement methods, have received more attention.[12−27] The former method has the advantage of high spatial and temporal resolutions, higher accuracy, better linearity, fast dynamic response, better reproducibility, and

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wide measurement range and has become one of the representative techniques in modern ultra-accuracy detection and measurement. It is also widely used in ultra-accuracy measurement, detection, process unit, lidar system, etc.

The principle of traditional heterodyne detection is all dual-beam interference, the spectrum of the heterodyne signal contains only single frequency information. So, one can obtain a single value of the parameter under test after demodulation. To further improve the measurement accuracy and to solve the problem of single measured values in one measurement, according to Doppler effect and heterodyne technology, we propose an experimental scheme of the multi-beam laser heterodyne measurement for the micro impulse in this article. On the basis of this scheme, we propose a novel method of the multi-beam laser heterodyne measurement for micro impulse, which can improve the accuracy of traditional laser heterodyne. The measurement of the micro-impulse which is converted into the measurement of the small tuning angle of the torsion pendulum is realized by considering the interaction between pulse laser and working medium. We modulate the frequency of light incident at different time moments by the oscillating mirror in the optical path and load the information about the small tuning angle to the frequency difference of the multi-beam laser heterodyne signal, whose spectrum of signal contains multiple frequencies simultaneously and each frequency contains the information about the parameter under test. We can then obtain multiple values of the parameter under test after the heterodyne signal demodulation simultaneously. Processing multiple values of the parameter by the weighted-average, the accuracy of parameter under test is increased. A detailed analysis of this method is made theoretically in this article and finally a simulation test is carried out.

2. Measuring principle

2.1. Measuring principle of micro-impulse using torsion pendulum system and the experimental scheme

The scheme of multi-beam laser heterodynes measurement for micro-impulse is shown in Fig. 1. The scheme mainly consists of; a solid state laser, a pulse laser, mirror, a quarter wave plate, an oscillating mirror, a polarizing beamsplitter (PBS), a lens, the standard bar, a target, a standard mirror, a photodetector and signal processing system which is composed of a preamplifier, a band-pass filter, an A/D converter and a digital signal processor (DSP). The oscillating mirror moves in the manner of uniform acceleration motion under the action of the signal source. The advantage of the oscillating mirror is that the laser incident on the oscillating mirror in different time moments can be modulated in frequency.

First, the pulse laser acts on the working medium producing a plasma jet, regurgitation of plasma makes the standard bar rotate. Meanwhile, we make the linearly polarized beams go through the PBS and quarter wave plate after turning on the solid-state laser, then reach the front surface of the oscillating mirror, therefore, the reflected light at different time moments, modulated by the oscillating mirror, propagates through quarter-wave plate and PBS and obliquely impinges on the mirror which is agglutinated on the surface of the standard bar. The light which is reflected by the last surface and light which is reflected by the former surface are converged to the sensitive surface of photodetector by a convex lens. Then converged

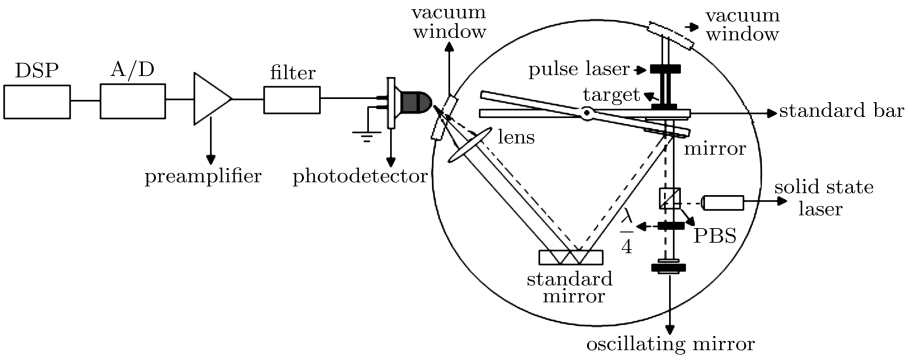


Fig. 1. Schematic diagram of experimental setup.

light is converted into an electrical signal by the photodetector, and the signal propagates through the preamplifier, A/D converter, and DSP, and then we obtain small tuning angle under test. Eventually, we can obtain the micro impulse.

The relationship of impulse and rotation angle can be shown by:[3]

$$I = \frac{2J\omega}{L}\theta' = \frac{4\pi J}{LT}\theta', \tag{1}$$

where J is the rotational inertia of torsion pendulum, ω is the damped frequency, T is the damping period, L is the length of the standard bar, and θ' is the rotation angle of the standard bar. Assuming that $k = 4\pi J/LT$, then

$$I = k\theta'. \tag{2}$$

Equation (2) shows that under the condition of small angle approximation, micro-impulse can be obtained by the rotation angle of the standard bar. The range of rotating angle is 0°–90° for the standard bar in Fig. 1. Therefore, the measuring range of impulse is 0–k in the experimental scheme. Meanwhile, systematic errors which affect measuring accuracy result from rotating friction of the standard bar, homogeneity of working medium target, and power stability of pulse laser.

2.2. Principle of multi-beam laser heterodyne for measuring small angle

It is shown in Fig. 2 that the beam is reflected and refracted continuously between the front surface and the rear surface of the plane standard mirror, thereby inducing the interference between the reflected light and the transmitted light at the infinite distance or the focal plane of the lens. When discussing interference we must consider the multiple reflection and refraction effects. Hence the multi-beam laser interference should be discussed.

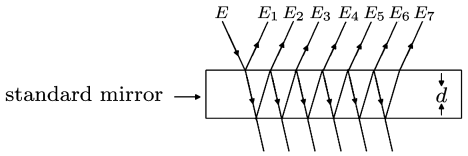


Fig. 2. Multi-beam laser interference principle.

Suppose that the incidence angle of laser is θ_0, and the incident field is $E(t) = E_0 \exp(\mathrm{i}\omega_0 t)$. The uniform acceleration motion equation and rate equation of the oscillating mirror are $x(t) = a(t^2/2)$ and $v(t) = at$, respectively. Because of the movement of the oscillating mirror, based on Doppler effect, the frequency of reflected light changes into $\omega = \omega_0(1 + at/c)$. In this expression, ω_0 is the angular frequency of laser, a is the vibration acceleration, and c is the light speed. So the optical field arrived at front surface of the plane standard mirror at $t - l/c$ time is

$$E_1(t) = \alpha_1 E_0 \exp\left\{\mathrm{i}\left[\omega_0\left(1 + \frac{a\,(t - l/c)}{c}\right)t + \omega_0\frac{a\,(t - l/c)^2/2}{c}\right]\right\}, \tag{3}$$

where l is the distance between the mirror and the front surface of the standard mirror.

The beam is reflected at different time moments by the rear surface of the standard mirror and the reflected light can be described as follows:

$$\begin{cases} E_2(t) = \alpha_2 E_0 \exp\left\{\mathrm{i}\left[\omega_0\left(1 + a\dfrac{t - l/c - 2nd\cos\theta/c}{c}\right)t \right.\right. \\ \qquad \left.\left. + \omega_0\dfrac{\left(a(t - l/c - 2nd\cos\theta/c)^2/2 + 2nd\cos\theta\right)}{c}\right]\right\}, \\ \quad \vdots \\ E_m(t) = \alpha_m E_0 \exp\left\{\mathrm{i}\left[\omega_0\left(1 + a\dfrac{t - l/c - 2(m-1)nd\cos\theta/c}{c}\right)t \right.\right. \\ \qquad \left.\left. + \omega_0\dfrac{\left(a(t - l/c - 2(m-1)nd\cos\theta/c)^2/2 + 2(m-1)nd\cos\theta\right)}{c}\right]\right\}, \end{cases} \tag{4}$$

where $\alpha_1 = r$, $\alpha_2 = \beta\beta' r', \ldots, \alpha_m = \beta\beta' r'(2m-3)$, r represents the reflectivity of light from the surrounding media into standard mirror, β the transitivity, r' the reflectivity of the rear surface of the plane standard mirror, β' the transitivity when the light is reflected by the front and the rear surfaces and then transmits through the standard mirror, d the thickness of standard mirror, n the refractive index of standard mirror, θ the refraction angle and m is a positive integer.

So the total optical field received by the photodetector is

$$E(t) = E_1(t) + E_2(t) + \cdots + E_m(t). \tag{5}$$

Then the output photocurrent of the photodetector is

$$\begin{aligned} I &= \frac{\eta e}{h\nu}\frac{1}{Z}\iint_S \frac{1}{2}[E_1(t) + E_2(t) + \cdots + E_m(t)] \\ &\quad \times [E_1(t) + E_2(t) + \cdots + E_m(t)]^* \mathrm{d}s \\ &= \frac{\eta e}{2h\nu}\frac{1}{Z}\iint_S \left[\sum_{j=1}^{m} E_j^2(t) \right. \\ &\quad \left. + \sum_{p=1}^{m-1}\sum_{j=1}^{m-p}\left(E_j(t)E_{j+p}^*(t) + E_j^*(t)E_{j+p}(t)\right)\right]\mathrm{d}s, \end{aligned} \tag{6}$$

where e denotes the electronic charge, Z the intrinsic impedance of the medium in the surface of the photodetector, η the quantum efficiency, S the sensitive surface area of the photodetector, h the Planck constant and v the laser frequency. As the DC component can be filtered after passing through the low-pass filter, only the AC component is considered and called the intermediate frequency current. We can coordinate and obtain the intermediate frequency current as

$$\begin{aligned} I_{\mathrm{if}} &= \frac{\eta e}{2h\nu}\frac{1}{Z}\iint_s \sum_{p=1}^{m-1}\sum_{j=1}^{m-p}(E_j(t)E_{j+p}^*(t) \\ &\quad + E_j^*(t)E_{j+p}(t))\mathrm{d}s. \end{aligned} \tag{7}$$

Substituting Eqs. (3) and (4) into Eq. (7) and calculating the integral by software, then we obtain the results as

$$I_{\mathrm{if}} = \frac{\eta e}{h\nu}\frac{\pi}{Z}\sum_{p=1}^{m-1}\sum_{j=1}^{m-p}\alpha_j\alpha_{j+p}E_0^2\cos\left[\left(\frac{4\omega_0 anpd\cos\theta}{c^2} - \frac{2\omega_0 anpd\cos\theta}{c^3}\right) + \omega_0\left(\frac{an^2p^2d^2\cos^2\theta}{c^3} - \frac{2npd\cos\theta}{c}\right)\right]. \tag{8}$$

After ignoring the minor term of $1/c^3$, equation (8) can be simplified into

$$I_{\mathrm{if}} = \frac{\eta e}{h\nu}\frac{\pi}{Z}\sum_{p=1}^{m-1}\sum_{j=1}^{m-p}\alpha_j\alpha_{j+p}E_0^2\cos\left(\frac{4\omega_0 anpd\cos\theta}{c^2}t - \omega_0\frac{2npd\cos\theta}{c}\right). \tag{9}$$

Equation (9) can be written as

$$I_{\mathrm{if}} = \frac{\eta e}{h\nu}\frac{\pi}{Z}\sum_{p=1}^{m-1}\sum_{j=1}^{m-p}\alpha_j\alpha_{j+p}E_0^2\cos[\Omega(p)t - \Phi(p)] = \frac{\eta e}{h\nu}\frac{\pi}{Z}E_0^2\sum_{p=1}^{m-1}\cos[\Omega(p)t - \Phi(p)]\left(\sum_{j=1}^{m-p}\alpha_j\alpha_{j+p}\right), \tag{10}$$

where

$$\Omega(p) = 4\omega_0 anpd\cos\theta/c^2, \tag{11}$$

$$\Phi(p) = 2\omega_0 npd\cos\theta/c, \tag{12}$$

with p and j being defined as a positive integer each.

Equation (10) contains the information about the refraction angle θ of plane standard mirror for frequency difference of intermediate frequency (IF) component and the phase retardation obtained by the multi-beam heterodyne measurement. The frequency difference of the IF component is analysed primarily since it is easy to implement frequency measurements by using the Fast-Fourier Transform (FFT). According to Eq. (11), the frequency of heterodyne signal can be recorded as

$$f_p = \Omega(p) = K_p\cos\theta, \tag{13}$$

where, by the refraction law,

$$\cos\theta = \cos[\arcsin(\sin\theta_0/n)]. \tag{14}$$

Equations (13) and (14) show that the frequency of heterodyne signal is in inverse proportion to the incidence angle of standard mirror, the scale factor can be expressed as

$$K_p = 4\omega_0 anpd/c^2, \tag{15}$$

where ω_0 is the angular frequency of light source, d the thickness of standard mirror, and a the vibration acceleration of oscillating mirror. It can be found that $K_{\rm p}$ is related to ω_0, n, d, and a by Eq. (15).

By Eqs. (14) and (2), it is easy to obtain the relationship between impulse I and refraction angle θ as

$$I = k \cdot \arcsin\{n \sin[\arccos(\cos\theta)]\}/2. \qquad (16)$$

Therefore, by Eq. (16), the micro-impulse could be obtained when refraction angle is given.

3. Numerical simulation and result analysis

By using the method mentioned in Ref. [3], we obtained the period variations of the system in the cases with and without adding the standard bar to the system. Then the rotational inertia of the system was calibrated and the results are shown in Table 1. The value of k is obtained by calibrated results.

To verify the feasibility of this novel method, based on the system of multi-beam laser heterodyne measurement for the small angle in Fig. 1, the value of the micro-impulse was simulated in 10.0 Pa pressure condition by MATLAB when working medium was Polyvinylchlorid (PVC)+2%C with a thickness of 180 μm and the width of the pulse laser was 50 ms. Meanwhile, taking the holmium (H_o) solid laser for example, which operated at 2050 nm, the length of standard bar was 15 cm, and the refractive index of standard mirror was 1.493983 with a thickness of 2 cm, the diameter of the detector photosurface was 1 mm with a sensitivity of 1 A/W, acceleration a of the oscillating mirror was 2×10^3 m/s^2.

Figure 3 shows the simulation results of the FFT spectrum of the signal of the multi-beam laser heterodyne corresponding to the measurement of the small angle by the multi-beam laser heterodyne measurement at different values of incidence angle θ_0. It shows that with the increase of the incidence angle, the relative position of the spectrum shifts toward lower frequency. It is said that the frequency decreases as a result of the decrease of incidence angle. The reason is that when the scale factor $K_{\rm p}$ is invariant, the relationship between frequency $f_{\rm p}$ and incidence angle θ_0 can be expressed as $f_{\rm p} = K_{\rm p}\cos\theta = K_{\rm p}\cos[\arcsin(\sin\theta_0/n)]$. Therefore, as the incidence angle increases, $f_{\rm p}$ value reduces, i.e., the relative position of the spectrum moves towards the lower frequency.

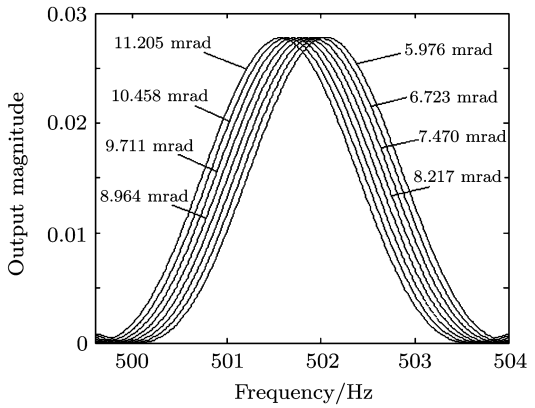


Fig. 3. Measured frequency spectra of the micro impulse at different incidence angles.

It must be explained that in Fig. 3 the frequency variation with the incidence angle gives only a single peak spectrum of the multi-beam heterodyne after FFT, while deconvoluted spectrum of Fig. 3 can present a multi-peak FFT spectrum. At the same time, since heterodyne is a detection method with near-diffraction limit, its sensitivity of detection is extremely high, so the signal-to-noise ratio (SNR) of the heterodyne signal is also extremely high in Fig. 3.

Eight sets of data were obtained when multi-beam laser heterodyne measurement was used and the simulation results of micro-impulse with different incidence angles are shown in Table 2.

Table 1. Calibrated results of torsion pendulum parameters.

Moment of inertia of standard bar J_1/kg·m^2	System period without standard bar T_1/s	System period with standard bar T/s	Moment of inertia of system J/kg·m^2
1.125×10^{-5}	0.7940	1.151	1.02×10^{-4}

Table 2. Measured values of I and simulation values of I_i for micro-impulse at different values of incidence angle θ_0.

Time	1	2	3	4	5	6	7	8
θ_0/mrad	5.976	6.723	7.470	8.217	8.964	9.711	10.458	11.205
$I/10^{-6}$ N·s	22.183	24.956	27.729	30.502	33.275	36.048	38.820	41.593
$I_i/10^{-6}$ N·s	22.299	24.832	27.867	30.392	33.415	35.930	38.943	41.449

Using the simulation data in Table 2, the maximum relative error can be calculated to be less than 0.5%, showing that accuracy of the novel method is very high. At the same time, the environmental and reading errors could be ignored under the condition of the small angle approximation, the error of simulation comes mainly from the precision error after FFT and the rounding error in the processing of calculation.

4. Conclusion

To overcome the shortcomings and the deficiencies of the traditional micro-impulse measurement system, in this paper we proposed a novel method of multi-beam laser heterodyne measurement for the micro impulse, in which the PVC+2%C is used as a working medium. The Value of the micro-impulse is simulated by MATLAB which is generated by considering the interaction between the pulse laser and the working medium and the simulation error is less than 0.5%. The result shows that the linear range of this method is wide and its accuracy is very high. The advantage of this method is that it is sensitive to the torsion pendulum rotation, but not to translational motion. This means that the system also has a strong ability to resist the mechanical vibration, especially the low frequency vibration. Therefore, it can restore the original working state of the measurement system in a few seconds which reduces not only the measurement error, but also the requirements for the measurement equipment and the experimental environment. Meanwhile, when the tuning angle is small (less than 5°), the relationship between micro impulse and incident angle can be considered to be linear, this novel method can satisfy the requirement of measuring micro impulse in the laser micro-thruster and provide an effective measuring means, which can be used to evaluate the performance of the laser micro-thruster.

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