Multiple flux difference effect in the lattice hydrodynamic model*

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Considering the effect of multiple flux difference, an extended lattice model is proposed to improve the stability of traffic flow. The stability condition of the new model is obtained by using linear stability theory. The theoretical analysis result shows that considering the flux difference effect ahead can stabilize traffic flow. The nonlinear analysis is also conducted by using a reductive perturbation method. The modified KdV (mKdV) equation near the critical point is derived and the kink–antikink solution is obtained from the mKdV equation. Numerical simulation results show that the multiple flux difference effect can suppress the traffic jam considerably, which is in line with the analytical result.

Keywords: flux difference, lattice hydrodynamic model, traffic flow, mKdV equation

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1. Introduction

Traffic congestion is a serious issue in modern society, which has attracted much attention in the past decades. In order to explore the underlying mechanisms of jam formation, various traffic models have been proposed by scholars, such as cellular automata models, car following models, gas kinetic models (mesoscopic), and hydrodynamic models.[1−11] The flow breakdown, stop-and-go traffic phenomena, which occur in actual traffic, have been found in those models, but the nature of traffic flow cannot be described completely due to the complexity of the transportation system.

Recently, the nonlinear analysis has been performed by researchers to investigate the traffic jam. Kurtze and Hong[12] have derived the Korteweg-de Vries (KdV) equation from the macroscopic hydrodynamic model and showed that the traffic jam can be identified with a soliton solution of the KdV equation. Komatsu and Sasa[13] derived the modified Korteweg-de Vries (mKdV) equation and described traffic as a kink density wave by the use of car-following model. Incorporating the effect of acceleration delay, Nagatani[14] presented an extended model and obtained the KdV equation.

To improve the transportation level of service, it is necessary to raise the travelers' utility and prevent traffic jams efficiently. In the intelligent transport system (ITS), on-line traffic data provide important information, drivers can get the information about other vehicles on road, and then adjusts the vehicle's velocity to avoid accelerate or decelerate frequently. Some researchers have investigated the traffic jam based on ITS and proposed some extended models. Some models are extended based on position information of other cars to improve the stability of traffic. Nagatani[15] extended the car-following model by taking into account the next-nearest-neighbor interaction. Lenz[16] presented a model in which a driver looked at multiple vehicles ahead. Ge[17] extended car following model with the consideration of an arbitrary number of vehicles ahead and investigated the dynamic behaviour near the critical point. Hasebe[18,19] established a dynamical model which can be applicable to cooperative control system, and they obtained the most stable parameter set, which gives the highest stability. In Hasebe's model, a vehicle is controlled by the system using the information of arbitrary number of vehicles that precede or follow.

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The information of position is also used in the lattice hydrodynamic model. In 2004, Xue[20] improved Nagatani's[21,22] lattice model by taking the next-nearest-neighbour lattice site into account. Ge[23] proposed an extended lattice model by considering an arbitrary number of front lattice sites based on a cooperative driving system. In 2008, Ge[24] proposed another extended lattice model by incorporating the "backward looking effect", and the KdV equation near the neutral stability line was derived.

Besides considering the information of position, some car-following models were given to improve the stability of traffic by taking into account the velocity information. In order to solve the problem of too high acceleration and unrealistic deceleration, Helbing and Tilch[25] added a complementary term (negative velocity difference, $-\Delta v$) into OV model, and proposed a generalized force model (GFM). Jiang *et al.*[26] found that the positive Δv is also important to driver in real traffic, and the delay time of car motion and kinematic wave speed at jam density in GFM is in poor accordance with the real traffic. Then, they presented a full velocity difference model in which not only the negative Δv but also the positive Δv was considered. Zhao and Gao[27] found that the car-following model would produce collision, then they presented a new car-following model by adding a acceleration term. In order to further improve the stability of traffic flow, Wang[28] developed a multi-velocity difference model.

The above models used merely either the position information or the velocity information singly. In fact, two of the above information can be included and described by one variable, flux. The information of flux is more comprehensive because it contains not only the information of position (density) but also the information of the velocity. Based on Nagatani's model,[21,22] Tian[29] presented a new lattice model by taking into account the flux information, and analysed the model by using the linear and nonlinear analysis. Sun[30] extended the lattice model by considering the relative current information about next-nearest-neighbour sites ahead.

Due to the flux including two important information (density and velocity), we devote ourselves to making a clear understanding whether the more "forward looking" effect can make the traffic flow "most stable" in lattice hydrodynamical model. Then, we presented an extended lattice model to enhance the stability of traffic flow by considering the flux difference effect of arbitrary number lattices ahead on a single-lane highway. We analyse the new model by linear stability theory, and the neutral stability condition is obtained. Analytical result shows that the flux difference effect can stabilize the traffic flow efficiently. The nonlinear analysis is also conducted using reductive perturbation method. The mKdV equation is derived and the kink–antikink solution is obtained near the critical point. In addition, we carry out numerical simulation to validate the theoretical result.

The reminder of the present paper is organized as follows. In the following section, we put forward the extended lattice model. In Section 3, we derive the stability condition by making use of linear stability theory. In Section 4, the mKdV equation is derived and the kink–antikink solution is obtained. In Section 5, we carry out the simulation to prove our analytical result. Finally, the conclusions are given in Section 6.

2. Model

2.1. Origin model

In 1998, Nagatani[21,22] proposed a lattice hydrodynamic model, which is a lattice version of the macroscopic hydrodynamic model. The model is described by the following equation:

$$\partial_t \rho + \partial_x(\rho v) = 0, \tag{1}$$

$$\partial_t(\rho v) = a\rho_0 V(\rho(x+\delta)) - a\rho v, \tag{2}$$

where time is a continuous variable and space is a discrete variable. ρ_0 is the average density, a is the sensitivity of the driver, and δ represents the average headway, $\delta = 1/\rho_0$. The $\rho(x+\delta)$ is the local density at position $x+\delta$ at time t, which is related to the inverse of headway $h(x,t)$, i.e., $\rho(x+\delta) = 1/h(x,t)$. $V(\rho(x+\delta))$ is the optimal velocity. The idea is that a driver adjusts the vehicle's velocity according to the observed headway $h(x,t)$ or density ahead $\rho(x+\delta)$. This is similar to the idea in the car following model with the optimal velocity $V(h(x,t))$.

Then, for dimensionless space x, equation (1) is modified as follows:

$$\partial_t \rho + \rho_0 \partial_x(\rho v) = 0, \tag{3}$$

where $\partial_x(\rho v) = \partial_x(\rho v)/\delta\partial_x(x/\delta) = \rho_0\partial_{x^*}(\rho v)$, x^* is indicated as x hereafter. Then, the lattice model is expressed as

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1}v_{j-1}) = 0, \tag{4}$$

$$\partial_t(\rho_j v_j) = a\rho_0 V(\rho_{j+1}) - a\rho_j v_j, \tag{5}$$

where j denotes the site on the one-dimensional lattice, ρ_j and v_j denote the local density and the local average velocity on site j at time t, respectively. Equation (4) is the lattice version of continuity equation which relates the local density $\rho_j(t)$ of traffic to the local average speed $v_j(t)$.

2.2. Modified model

On the basis of lattice model proposed by Nagatani,[21,22] we present a multiple looking lattice hydrodynamic model. The evolution equation is

$$\begin{aligned}\partial_t(\rho_j v_j) =\ & a\rho_0 V(\rho_{j+1}) - a\rho_j v_j \\ & + \kappa\sum_{l=1}^{n}\beta_l \Delta Q_{j+l-1}, \end{aligned} \tag{6}$$

where κ is the strength coefficient of flow difference effect. As $\kappa = 0$, the new model is Nagatani's model. Naturally, we set the value of κ smaller than that of a. It means that the first and the second terms on the right part of Eq. (6) play a dominant role in this system. n denotes the number of lattices ahead considered; $\Delta Q_{j+l-1} = Q_{j+l} - Q_{j+l-1} = \rho_{j+l}v_{j+l} - \rho_{j+l-1}v_{j+l-1}$ $(l = 1, 2, \ldots, n,\ l \ll n)$; Q_j is the flow difference between $(j+1)$-th and j-th lattices. β_l is the weighted function of ΔQ, which means the intensity of reaction to the preceding lattice l and conforms the condition $\beta_l > \beta_{l+1}$. The function of β_l takes the following form[23]

$$\beta_l = \begin{cases} \dfrac{2}{3^l}, & l \neq n, \\ \dfrac{1}{3^{n-1}}, & l = n. \end{cases} \tag{7}$$

By eliminating speed v in Eqs. (4) and (6), the following density equation is obtained

$$\begin{aligned}&\partial_t^2\rho_j + a\rho_0^2(V(\rho_{j+1}) - V(\rho_j)) + a\partial_t\rho_j \\ &- \kappa\sum_{l=1}^{n}\beta_l\partial_t(\rho_{j+l}) + \kappa\sum_{l=1}^{n}\beta_l\partial_t(\rho_{j+l-1}) = 0. \end{aligned} \tag{8}$$

The optimal velocity is chosen as follows:

$$\begin{aligned}V(\rho) =\ & (v_{\max}/2)[\tanh(2/\rho_0 - \rho/\rho_0^2 - 1/\rho_c) \\ & + \tanh(1/\rho_c)]. \end{aligned} \tag{9}$$

This function has the turning point at $\rho = \rho_c$, when $\rho_0 = \rho_c$. For simplicity, we set $v_{\max} = 2$. The optimal velocity function has the following properties: it is a monotonically decreasing function, it has an upper bound (maximal velocity), and it has a turning point at the safety distance.

3. Linear stability analysis

The linear stability analysis is made to the extended model. The steady state is the uniform traffic flow with a constant density ρ_0 and optimal velocity $V(\rho_0)$. Hence, the steady-state solution is given by

$$\rho_j(t) = \rho_0, \quad v_j(t) = V(\rho_0). \tag{10}$$

Suppose that $y_j(t)$ is a small deviation from the steady-state density of the lattice j: $\rho_j(t) = \rho_0 + y_j(t)$, and $y_j(t) = \exp(\mathrm{i}\,kj + zt)$. Therefore, we obtain

$$V(\rho_j) = V(\rho_0) + V'(\rho_0)y_j(t),$$

$$V(\rho_{j+1}) = V(\rho_0) + V'(\rho_0)y_{j+1}(t), \tag{11}$$

$$y_{j+l}(t) = y_j(t)\exp(\mathrm{i}\,kl), \quad \partial_t\rho_j = zy_j(t), \tag{12}$$

$$\partial_t\rho_{j+l} = zy_{j+l}(t), \quad \partial_t^2\rho_j = z^2y_j(t), \tag{13}$$

where

$$V'(\rho_0) = \left.\frac{\mathrm{d}V(\rho)}{\mathrm{d}\rho}\right|_{\rho=\rho_0}.$$

Substituting Eqs. (11)–(13) into Eq. (6), we have

$$\begin{aligned}&z^2 + a\rho_0^2V'(\rho_0)(\mathrm{e}^{\mathrm{i}\,k} - 1) + az \\ &-\kappa z\sum_{l=1}^{n}\beta_l\mathrm{e}^{\mathrm{i}\,kl}(1 - \mathrm{e}^{-\mathrm{i}\,k}) = 0. \end{aligned} \tag{14}$$

Let $z = u\,\mathrm{i}$, and substituting it into Eq. (14), with u being a real number, and making its real part and imaginary part equal zero, respectively. We obtain

$$\begin{aligned}&-u^2 + a\rho_0^2V'(\rho_0)(\cos k - 1) \\ &+ \kappa u\sum_{l=1}^{n}\beta_l(\sin kl - \sin k(l-1)) = 0, \end{aligned} \tag{15}$$

$$\begin{aligned}&a\rho_0^2V'(\rho_0)\sin k + au \\ &-\kappa u\sum_{l=1}^{n}\beta_l(\cos kl - \cos k(l-1)) = 0. \end{aligned} \tag{16}$$

From Eq. (16), we can have

$$u = -\frac{\rho_0^2V'(\rho_0)\sin k}{a - \kappa\sum_{l=1}^{n}\beta_l\cos kl + \kappa\sum_{l=1}^{n}\beta_l\cos k(l-1)}. \tag{17}$$

Substituting Eq. (17) into Eq. (15), and when $k \to 0$, we then obtain the stability condition

$$a > -2V'(\rho_0)\rho_0^2 - 2\kappa\sum_{l=1}^{n}\beta_l. \tag{18}$$

When $\kappa = 0$, the stable condition of Nagatani' model is obtained:

$$a > -2V'(\rho_0)\rho_0^2. \tag{19}$$

Compared Eq. (18) with Eq. (19), one can see that the stability condition can be satisfied in the new model more easier than that in the existing models, which indicate that the traffic stability could be enhanced further by introducing the effect of flux difference of multiple lattices ahead.

4. Nonlinear stability analysis

We apply the reductive perturbation method to Eq. (8) and the mKdV equation is derived just below the critical point. We introduce slow scales for space variable j and time variable t, and define the slow variables X and T for $0 < \varepsilon \ll 1$

$$X = \varepsilon(j + bt), \quad T = \varepsilon^3 t, \tag{20}$$

where b is a constant determined later, we set the density $\rho_j(t)$ as

$$\rho_j(t) = \rho_c + \varepsilon R(X, T). \tag{21}$$

According to Eqs. (20) and (21), expanding each term in Eq. (8) to the fifth order of ε leads to the following expression:

$$\begin{aligned}
&\varepsilon^2(a\rho_c^2 V'(\rho_c) + ab)\partial_X R \\
&+ \varepsilon^3\left(b^2 + \frac{a\rho_c^2 V'(\rho_c)}{2} - b\sum_{l=1}^{n}\beta_l\right)\partial_X^2 R \\
&+ \varepsilon^4\left(\left(\frac{a\rho_c^2 V'(\rho_c)}{6} + \frac{b}{2}\sum(1-2l)\beta_l\right)\partial_X^3 R\right. \\
&\left.+ \frac{a\rho_c^2 V'''(\rho_c)}{6}\partial_X R^3 + a\partial_T R\right) \\
&+ \varepsilon^5\left((2b - \sum\beta_l)\partial_T\partial_X R\right. \\
&+ \left(\frac{a\rho_c^2 V'(\rho_c)}{24} + \frac{b}{6}\sum((l-1)^3 - l^3)\beta_l\right)\partial_X^4 R \\
&\left.+ \frac{a\rho_c^2 V'''(\rho_c)}{12}\partial_X^2 R^3\right) = 0.
\end{aligned} \tag{22}$$

Near the critical point, we have $a_c = a(1 + \varepsilon^2)$. By taking $b = -\rho_c^2 V'(\rho_c)$ and eliminating the second and third order terms of Eq. (22), we obtain the simplified equation

$$\begin{aligned}
&\varepsilon^4[\partial_T R - g_1\partial_X^3 R + g_2\partial_X R^3] \\
&+ k\varepsilon^5[g_3\partial_X^2 R + g_4\partial_X^4 R + g_5\partial_X^2 R^3] = 0,
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
g_1 &= -\rho_c^2 V'(\rho_c)\left(\frac{1}{6} + \frac{\kappa}{2a_c}\sum(2l-1)\beta_l\right), \\
g_2 &= \frac{\rho_c^2 V'''(\rho_c)}{6}, \quad g_3 = \frac{-\rho_c^2 V'(\rho_c)}{2}, \\
g_4 &= \frac{\rho_c^2 V'(\rho_c)}{a_c}(2\rho_c^2 V'(\rho_c) \\
&\quad + \kappa\sum\beta_l)\left(\frac{1}{6} + \frac{\kappa}{2a_c}\sum(2l-1)\beta_l\right) \\
&\quad + \frac{\rho_c^2 V'(\rho_c)}{a_c}\left(\frac{a_c}{24} + \frac{\kappa}{6}\sum(l^3 - (l-1)^3)\beta_l\right), \\
g_5 &= \frac{\rho_c^2 V'''(\rho_c)}{12} + \frac{1}{6a_c}\left(2\rho_c^2 V'(\rho_c)\right. \\
&\quad \left.+ \kappa\sum\beta_l\right)\rho_c^2 V'''(\rho_c).
\end{aligned}$$

In order to derive the standard mKdV equation, we make the following transformations: $T' = g_1 T$, $R = \sqrt{g_1/g_2}R'$. Hence, equation (23) turns into

$$\partial_{T'} R' - \partial_X^3 R' + \partial_X R'^3 + \varepsilon M[R'] = 0, \tag{24}$$

where

$$M[R'] = \frac{1}{g_1}\left(g_3\partial_X^2 R' + \frac{g_1 g_5}{g_2}\partial_X^2 R'^3 + g_4\partial_X^4 R'\right).$$

Equation (24) is the mKdV equation with an $O(\varepsilon)$ correction term. We ignore the $O(\varepsilon)$ term and obtain the mKdV equation with the kink–antikink solution

$$R_0'(X, T') = \sqrt{c}\tanh\left[\sqrt{\frac{c}{2}}(X - cT')\right]. \tag{25}$$

Supposing $R'(X, T') = R_0'(X, T') + \varepsilon R_1'(X, T')$, in order to determine the value of the propagation velocity for the kink–antikink soliton solution, it is necessary to satisfy the following condition

$$(R_0', M[R_0']) \equiv \int_{-\infty}^{+\infty} \mathrm{d}X R_0' M[R_0'] = 0 \tag{26}$$

with $M[R_0'] = M[R']$. By solving Eq. (26), we obtain the selected value c:[31]

$$c = \frac{5g_2 g_3}{2g_2 g_4 - 3g_1 g_5}. \tag{27}$$

Hence, we obtain the kink–antikink soliton solution ($-\rho_c^2 V'(\rho_c) = 1, \rho_c^6 V'''(\rho_c) = 2$)

$$\rho_j = \rho_c + \varepsilon\sqrt{\frac{g_1 c}{g_2}}\tanh\left(\sqrt{\frac{c}{2}}(X - cg_1 T)\right) \tag{28}$$

with $\varepsilon^2 = a_c/a - 1$. The amplitude A of the solution is

$$A = \sqrt{\frac{g_1}{g_2}\varepsilon^2 c}. \tag{29}$$

The kink–antikink solution represents the coexisting phase, which includes both freely moving phase (within low density) and jamming phase (within high density). The densities of the freely moving phase are given by $\rho = \rho_c - A$, the densities of the jamming

phase are given by $\rho = \rho_c + A$. Then, we obtain the coexisting curve in the density-sensitivity space.

Figure 1 shows the coexisting curves and the neutral stability curves in the density-sensitivity space. For each pair of curves, the space is divided into three regions, i.e. the stable region, the metastable region, and the unstable region. From Fig. 1, we can find the critical points, the neutral stability lines, and the coexisting curve decrease with the growing "forward looking", which means the stability of the uniform traffic flow has been strengthened further by considering the effect of multiple flux difference ahead.

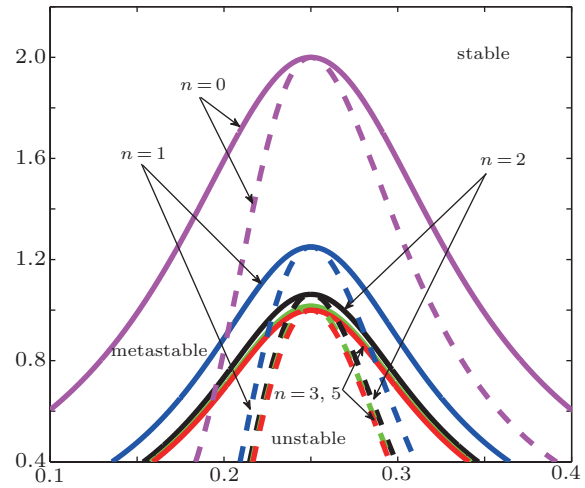


Fig. 1. (colour online) The phase diagram on (ρ, a) plane. The solid curves and the dotted lines indicate the coexisting curves and the neutral stability lines, respectively, for $n = 0$, 1, 2, 3, 5.

5. Simulation

For the convenience of simulation, we rewrite Eq. (6) into the difference form

$$\begin{aligned}\rho_j(t+2\tau) &- \rho_j(t+\tau) + \tau\rho_0^2(V(\rho_{j+1}(t)) - V(\rho_j(t))) \\ &- \kappa\tau\sum\beta_l(\rho_{j+l}(t+\tau) - \rho_{j+l}(t) \\ &- \rho_{j+l-1}(t+\tau) + \rho_{j+l-1}(t)) = 0. \qquad (30)\end{aligned}$$

The initial conditions of the numerical simulation are set as follows: The lattice number $N = 100$, and the boundary condition is periodical. The density is assumed to be the step function at the average density

$$\rho_j(0) = \rho_0 = 0.25, \quad j \in [1, N], \qquad (31)$$

$$\rho_j(1) = \begin{cases} \rho_0, & 1 \le j < n_1,\ n_2 < j \le N, \\ \rho_0 + \Delta\rho, & j = n_1, \\ \rho_0 - \Delta\rho, & j = n_2, \end{cases} \qquad (32)$$

where $n_1 = 49$, $n_2 = 50$, $\Delta\rho$ is the initial disturbance.

Firstly, we perform the simulation to test the improvement of traffic flow stability by multiple flux difference ahead. Here, we take $a = 1.5, n = 4, \kappa = 0, 0.1, 0.2, 0.3$, respectively. Figure 2 shows the spatiotemporal evolution of density waves after a sufficient long time $t = 10^4$ s. In panels (a), (b), and (c), the traffic flow is unstable because the stability condition Eq. (18) is not satisfied. When small disturbance are added to the uniform traffic flow, they are amplified and finally evolve into inhomogeneous flow. One can see that the kink–antikink waves appear and propagate backwards in panels (a), (b), and (c) in Fig. 2. While the disturbance in panel (d) will dissolve and the traffic flow is uniform flow over the whole space after long enough time, which means that the traffic flow can be stabilized by the growing effect of flux difference. Figure 3 shows the density profile obtained at $t = 10^4$ s corresponding to Fig. 2. In panels (a), (b), and (c), although the traffic flow is not uniform flow, the amplitudes of the density waves decreases, which indicate the positive effect of flux difference on the traffic flow. When the effect of flux difference is large enough, the stability condition equation (18) can be satisfied and the traffic with small disturbances will evolve into uniform flow as shown in panel (d). Therefore, considering the multiple information of flow difference is necessary. Figure 3 shows the density profile obtained at $t = 10^4$ s corresponding to Fig. 2.

Secondly, we investigate how many lattices are useful and should be considered. To illustrate the effect of flux difference clearly, we set the initial condition in unstable region, i.e., $a = 1$, $\kappa = 0.4$, $n = 1, 2, 3, 4$, respectively. Figure 4 shows the space–time evolution of the density waves for different lattices after $t = 10^4$ s. The kink–antikink solution appears as traffic jams and the density waves propagate backward. From Fig. 4, we can see that, the amplitude of density wave decreases as the considered number of lattices ahead increases. But the influence of lattices in front is very little after $n = 3$, which is consistent with the description in Fig. 1. This just matches the actual situation, that is to say, when the car is farther with the considered one's distance, the influence on the considered car is smaller. Therefore, the simulation results are in good agreement with theoretical analysis. Figure 5 shows the density profile obtained at $t = 10^4$ s corresponding to Fig. 4.

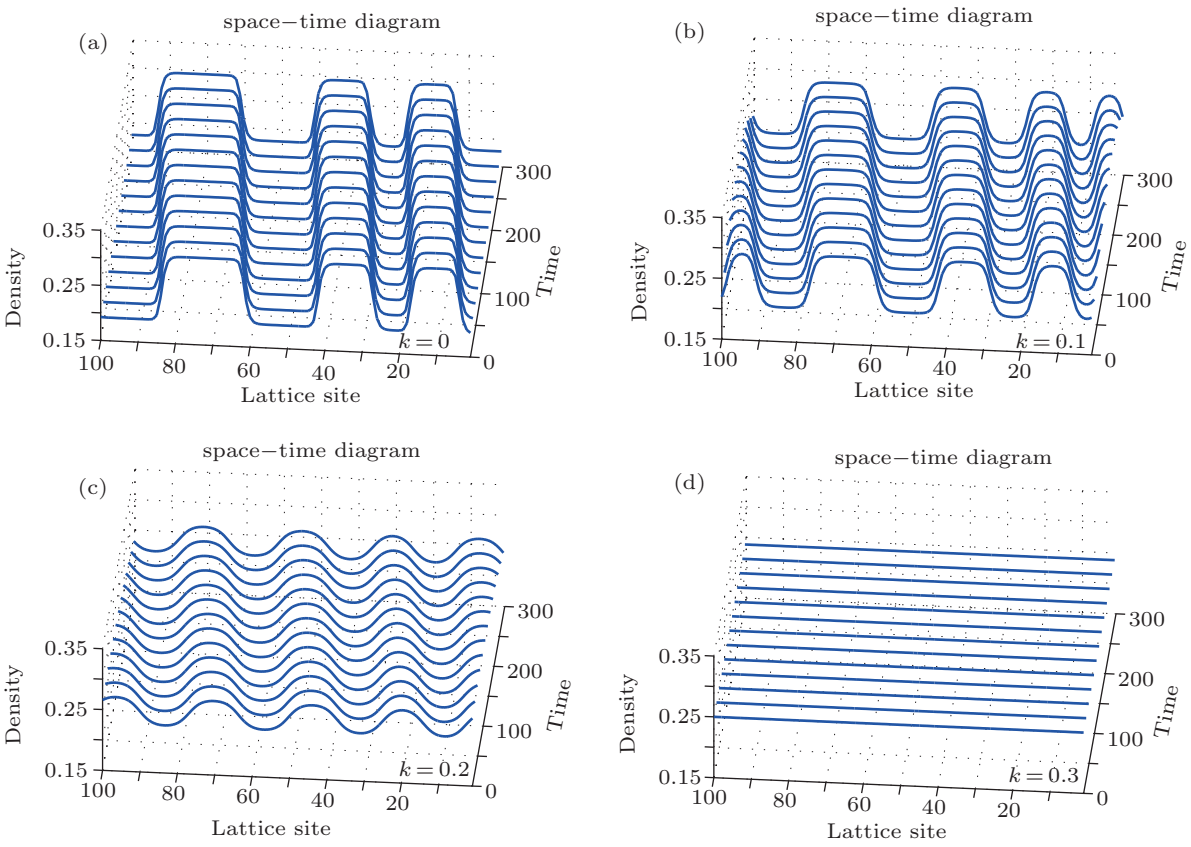


Fig. 2. (colour online) Spatiotemporal evolution of density wave for different parameters $a = 1.5$, $\kappa = 0$, 0.1, 0.2, 0.3 after $t = 10^4$ s.

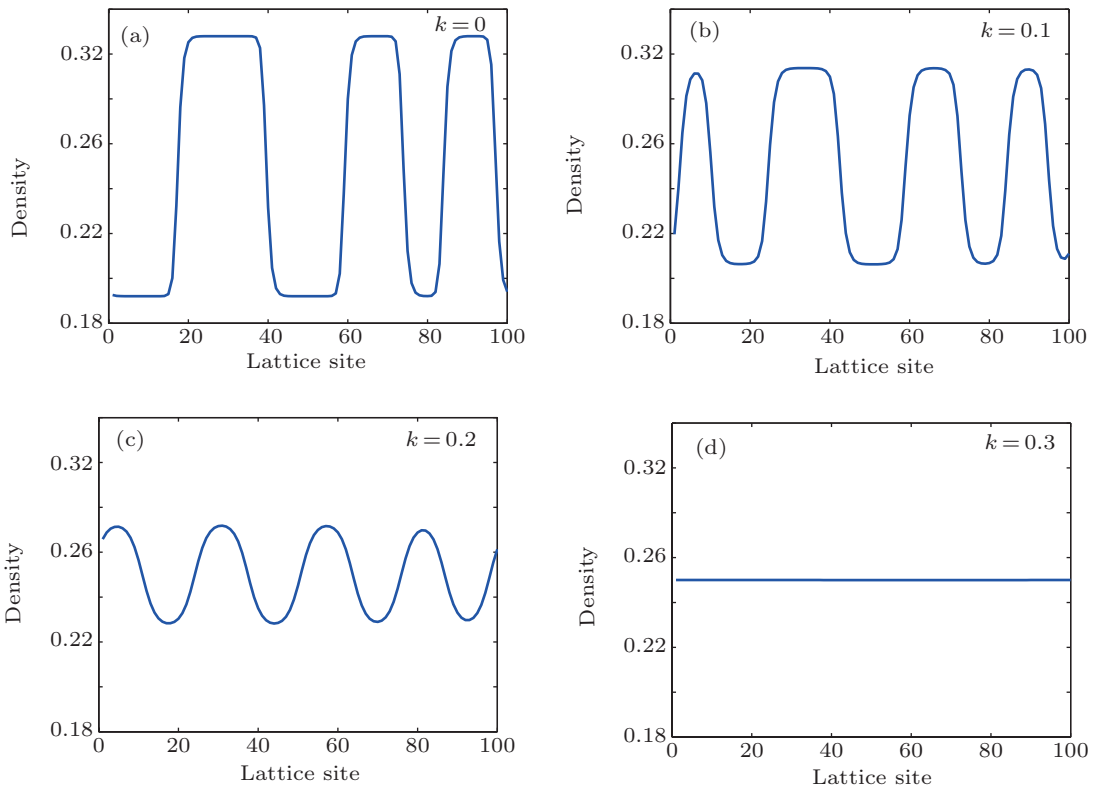


Fig. 3. (colour online) The density profile at $t = 10^4$ s for different parameters $a = 1.5$, $\kappa = 0$, 0.1, 0.2, 0.3.

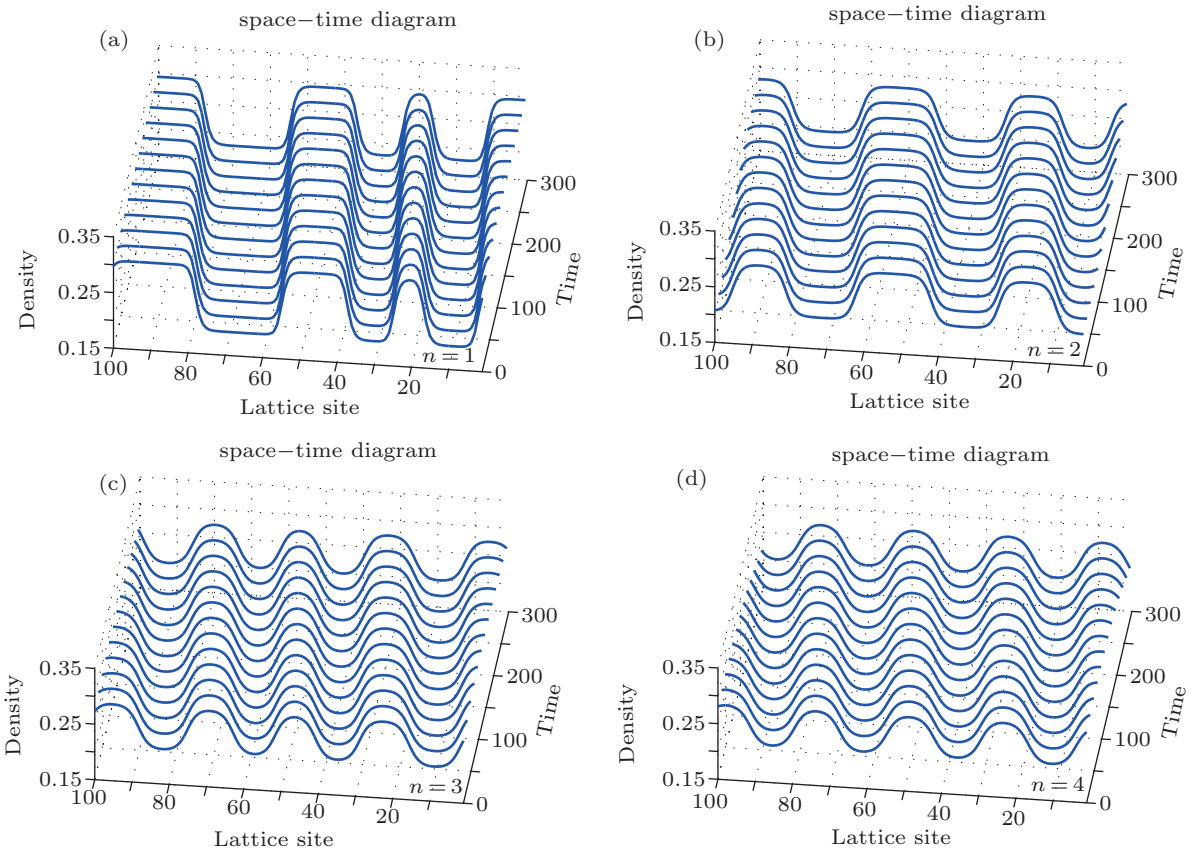


Fig. 4. (colour online) Spatiotemporal evolution of density wave for different parameters $a = 1$, $\kappa = 0.4$, $n = 1$, 2, 3, 4 after $t = 10^4$ s.

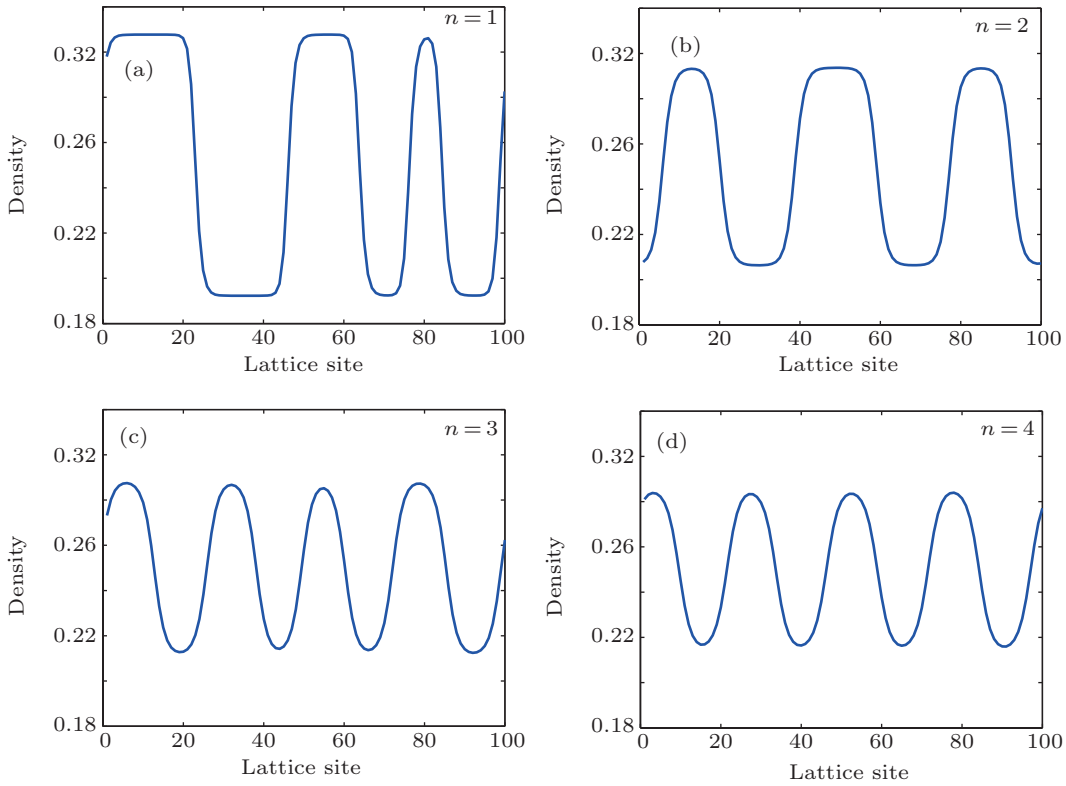


Fig. 5. (colour online) The density profile at $t = 10^4$ s for different parameters $a = 1$, $\kappa = 0.4$, $n = 1$, 2, 3, 4.

6. Conclusions

We have presented an extended lattice traffic model to improve the stability of traffic flow by taking into account the interaction of multiple lattices. The traffic behaviour has been analytically analysed by using the linear and nonlinear analyses. From the theoretical analysis, it is obvious that the multi-lattice consideration could further stabilize traffic flow. The traffic jam in the traffic flow with the "forward looking" behavior interaction is described by the modified KdV equation. Simulations were made to validate the theoretical results. The simulation results confirmed the stability analyses and find that the "most stable" state is $n = 3$. When $n > 3$, the influence on the considered car is small which can be omitted. So, only three of lattices' information need to be considered for cooperative driving, which is consistent with the theoretical analysis results.

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