Critical properties of mixed spin-1 and spin-5/2 with equal and unequal crystal fields*

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The effects of assuming equal or unequal crystal fields (CF) on the phase diagrams of a mixed spin-1 and spin-5/2 system are investigated in terms of the recursion relations on the Bethe lattice (BL). The equal CF case was considered for the coordination numbers $q = 3, 4$, and 6, while for $q = 3$ the unequal CF case was also studied. It was found that for the equal CF case, the model exhibits second-order phase transitions and two compensation temperatures for all q, the reentrant behavior for $q = 4$ and first-order phase transitions and tricritical point (TCP) for $q = 6$. In the unequal CF case for $q = 3$, the system yields first- and second-order phase transitions, TCP's, and three compensation temperatures. In addition, the TCP's in a very short range are classified as the stable and unstable ones depending on their free energies.

Keywords: mixed spin, Bethe lattice, compensation, ferrimagnet

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1. Introduction

The Ising model with high and mixed spins, i.e., the extended Ising model, attracts a great deal of attention because of its observed interesting critical behaviors. The two-sublattice mixed Ising ferrimagnetic systems with different magnitudes of spins have been considered for not only purely theoretical purposes but also experimentally as possible models to describe certain types of molecular-based magnetic materials.[1−5] In addition, the mixed-spin Ising models in comparison with the regular ones, i.e., with only one type of spins, present less translational symmetry. As a result, a new type of critical temperature is found and it is called as the compensation temperature ($T_{\rm comp}$) at which the net magnetization vanishes below the critical temperature ($T_{\rm c}$). Therefore, its existence provides interesting possibilities in technological applications such as in the thermomagnetic recording and magneto–optical readout applications.[6]

Having more than a few numbers of integer (I) and half-integer (HI) spins in nature gives us an opportunity of mixing them in vast number of ways as I–I, HI–HI, and I–HI to simulate different mixed spin models. Therefore, many possibilities of the mixed spin systems have already been considered so far. But the mixed spin-1 and spin-5/2 model has only been considered by using the effective field theory (EFT) with correlations,[7] where they have obtained the phase diagram of the model for only the equal CF case when $q = 3$ and 4. We should also note that the molecular mixed spin ferromagnetic material $MnNi(NO_2)_4$(ethylenediamine) is known as quasi-one-dimensional material which consists of spin-1 and spin-5/2.[8]

Hence, with this motivation, we have studied the mixed spin-1 and spin-5/2 model to investigate their critical behaviors for different coordination numbers for the cases with equal and unequal CFs on the Bethe lattice (BL). Therefore, we have studied the thermal variations of the magnetizations to obtain the critical temperatures of the model in addition to the compensation temperatures. The phase diagrams are obtained on the $(D/J, kT/J)$ planes for $q = 3, 4$, and 6, on the $(D_{\rm A}/J, kT/J)$ and $(D_{\rm B}/J, kT/J)$ planes for given values of $D_{\rm B}/J$ and $D_{\rm A}/J$, respectively, for $q = 3$ only.

The rest of the paper is arranged as follows: The BL formulation of the mixed spin-1 and spin-5/2 model in terms of the recursion relations is given in Section 2. The next section includes the thermal variations of the magnetizations at the first- and second-order phase transitions and at the compensation points. The phase diagrams of the model are presented in Section 4. The final section consists of a brief summary and discussion.

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2. The Bethe lattice and recursion relations

Before giving the formulation of the model in terms of the recursion relations, we should discuss the BL. It is an infinite Cayley or regular tree, i.e., a connected graph without circuits, and historically gets its name from the fact that its partition function is exactly that of an Ising model in the Bethe approximation.[9,10] It also provides exact solutions and the results of which are qualitatively better approximations for the regular lattices than solutions obtained by the conventional mean-field approximations (MFA).[11,12] The cluster variation method in the pair approximation studies on regular lattices yield results that are exact for the same model on the BL.[13–16] The BL considerations have also some limitations; it predicts a transition temperature which is higher than that for a regular lattice and it is not reliable for predicting critical exponents.[17] Thus, the BL approach may be considered as a powerful tool when compared with some of the approaches carried out on regular lattices.

The Hamiltonian of the mixed spin-1 and spin-5/2 model including only nearest-neighbor (NN) bilinear interaction J and CF interactions, $D_{\rm A}$ and $D_{\rm B}$, may be written as

$$\mathscr{H} = -J\sum_{\langle i,j\rangle}\sigma_i s_j - D_{\rm A}\sum_i \sigma_i^2 - D_{\rm B}\sum_j s_j^2, \quad (1)$$

where each σ_i located at site i is a spin with spin-1 (±1, 0) and each s_j located at site j is a spin with spin-5/2 ($\pm5/2$, $\pm3/2$, $\pm1/2$). For the mixed-spin system, we set up the BL in such a way that the central spin is spin-1, σ_0, with q number of NN spins of the other type, i.e. spin-5/2, which form the second generation. Each of these second generation spin-5/2, s_1, are joined with $(q-1)$ NN spins of spin-1, σ_2, therefore, in total the second generation has $q(q-1)$ NN spins which form the third generation and so on to infinity (see Fig. 1).

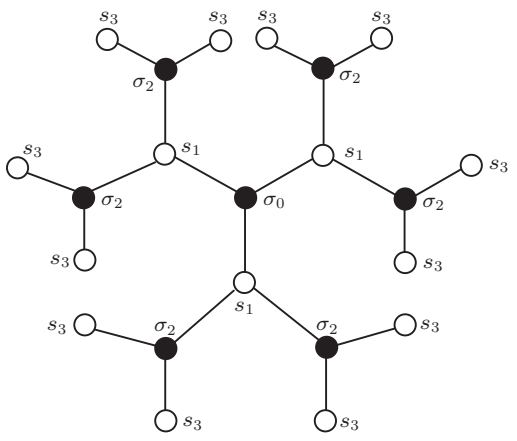


Fig. 1. The Bethe lattice, or regular tree, of $q = 3$ for the mixed spin-σ and spin-s Ising model.

In order to obtain a formulation in terms of the recursion relations, the partition function is needed and it is given from the definition as

$$Z = \sum_{\text{all config.}} \mathrm{e}^{-\beta\mathscr{H}} = \sum_{\rm Spc} P(\rm Spc), \quad (2)$$

where $P(\rm Spc)$ is an unnormalized probability distribution over the spin configurations, $(\rm Spc) = \{\sigma, s\}$, etc., and σ_i and s_j refer to the spin values at site i and j, respectively. If the BL is cut in some central point deep inside with a spin σ_0, then it splits up into q identical branches whose number depends on the number of NNs or the coordination numbers. Each of these branches is a rooted tree at the central spin σ_0. This implies that $P(\{\sigma_0\})$, $(Spc) = \{\sigma_0\}$, the spin configuration with the spin value σ_0 at the central site, may be written as

$$P(\{\sigma_0\}) = \exp[\beta D_{\rm A}\sigma_0^2] \times \prod_{k=1}^{q} Q_n(\sigma_0|\{s_1^{(k)}\}), \quad (3)$$

since the central-spin is spin-1, when BL is cut at that site the central spin σ_0 can interact only with the $D_{\rm A}$ and where $\{s_1^{(k)}\}$ denotes the spin-configurations of the k-th branch of the BL starting with $s_1^{(k)}$ other than the central spin σ_0, the suffix n expresses the fact that the sub-tree has n-shells, i.e., the number of steps from the root to the boundary sites. In Eq. (3) the function Q_n in the product is given as

$$Q_n(\sigma_0|\{s_1^{(k)}\}) = \exp\left[\beta\left(J\sigma_0 s_1^{(k)} + J\sum_{\langle i,j\rangle}\sigma_i\, s_j + D_{\rm A}\sum_i \sigma_i^2 + D_{\rm B}\sum_j s_j^2\right)\right], \quad (4)$$

where σ_i and s_j refer to the spins of the sites i and j of the sub-tree (other than the central spin σ_0, which is a spin-1). Site 1 with spin-5/2 is the site next to the central cite σ_0. The first summation in Eq. (4) is over all edges of the sub-tree other than the edge (0, 1) and the summation over i is over all sites with spin-1 other than the central site. Now if the sub-tree, e.g., the upper sub-tree, is cut at the site 1 next to 0, then it also decomposes into q pieces: one being "trunk"

(0, 1) and the rest are the identical branches. Each of these branches is a sub-tree like the original, but with $n-1$ shells and $q-1$ neighbors. Thus

$$Q_n(\sigma_0|\{s_1^{(k)}\}) = \exp[\beta\{J\sigma_0 s_1^{(k)} + D_{\rm B}(s_1^{(k)})^2\}] \times \prod_{l=1}^{q-1} Q_{n-1}(s_1^{(k)}|\{\tau_2^{(l)}\}), \quad (5)$$

where $\{\tau_2^{(l)}\}$ denotes the spin-configurations (other than $s_1^{(k)}$) on the l-th branch of the sub-tree. If now the upper sub-tree is cut at site 2 again with spin-1 next to site 1, then it again decomposes into q pieces: one again being the next trunk (1, 2) and the rest are the identical branches, again with $n-1$ shells and $q-1$ neighbors. Therefore

$$Q_{n-1}(s_1^{(k)}|\{\tau_2^{(l)}\}) = \exp[\beta(Js_1\sigma_2^{(l)} + D_{\rm A}(\sigma_2^{(l)})^2] \times \prod_{m=1}^{q-1} Q_{n-2}(\tau_2^{(l)}|\{r_3^{(m)}\}), \quad (6)$$

where $\{r_3^{(m)}\}$ indicates all the spins (other than $\tau_2^{(l)}$) on the next m-th branch of the upper subsub-tree. As a result, one takes n steps from root to the boundary on the BL and in the thermodynamic limit $n \to \infty$ should be taken. Adding the shells in this manner, the BL is set up in such a way that the central-spin is spin-1 with q-neighbors of spin-5/2, the next generation with spin-5/2 has $q-1$ neighbors with spin-1, and the next generation with spin-1 has $q-1$ neighbors of spin-5/2, and so on to infinity. It should be mentioned that the choice of the central spin has no effect on the system. Now we define,

$$g_n(\{\sigma_o\}) = \sum_{\{s_1\}} Q_n(\sigma_o|\{s_1\}), \quad (7)$$

which may be assumed as the partition function of the followed branch and where the summation over s_1, i.e., spin-5/2 with spin values $\pm 5/2, \pm 3/2$, and $\pm 1/2$, means that there is a certain probability for the spin σ_0 to be in a certain spin value, it is a spin-1 with spin values ± 1 and 0, as a result of its interaction with the external agencies and with its q NNs of s_1.

Now we are ready to obtain the exact recursion relations. If equation (7) is used for Eq. (5), a general formula for the partial partition functions $g_n(\sigma_0)$ of spin-1 is obtained as

$$g_n(\sigma_0) = \sum_{s_1} \exp[\beta(J\sigma_0 s_1 + D_{\rm B}s_1^2)][g_{n-1}(s_1)]^{q-1}, \quad (8)$$

where the summation over s_1, which is a spin-5/2, is carried out for the values $\pm 5/2, \pm 3/2$, and $\pm 1/2$ for each given values of σ_0, i.e., spin-1 with ± 1 and 0. Therefore, the two recursion relations are obtained by dividing any of these $g_n(\sigma_0)$ to the rest of the two for spin-1 as

$$A_n = \frac{g_n(+1)}{g_n(0)}, \quad B_n = \frac{g_n(-1)}{g_n(0)}. \quad (9)$$

Similar procedure may be employed for Eqs. (6) and (7), so the partial partition function for spin-5/2 is found as

$$g_{n-1}(s_1) = \sum_{\sigma_2} \exp[\beta(J\sigma_2 s_1 + D_{\rm A}\sigma_2^2)] \times [g_{n-2}(\sigma_2)]^{q-1}, \quad (10)$$

where the summation is for overall values of σ_2 with ± 1 and 0, since it is a spin-1. Thus, now for each given values of s_1, one obtains six different partial partition functions for each spin values of $\pm 5/2, \pm 3/2$, and $\pm 1/2$. Thus five recursion relations are obtained for spin-5/2 as

$$C_{n-1} = \frac{g_{n-1}(+5/2)}{g_{n-1}(-1/2)}, \quad D_{n-1} = \frac{g_{n-1}(+3/2)}{g_{n-1}(-1/2)},$$
$$E_{n-1} = \frac{g_{n-1}(+1/2)}{g_{n-1}(-1/2)}, \quad F_{n-1} = \frac{g_{n-1}(-5/2)}{g_{n-1}(-1/2)},$$
$$G_{n-1} = \frac{g_{n-1}(-3/2)}{g_{n-1}(-1/2)}. \quad (11)$$

Here, we did not give the explicit expressions since they are too long and their calculations are straightforward.

As it is always the case, in order to obtain the critical behaviors one needs the order-parameters and the free energy. The order-parameters are the dipolar and the quadrupolar moments. The M_1 and Q_1, i.e., the magnetization and quadrupolar moment, for spin-1 is obtained as

$$M_1 = \langle\sigma_i\rangle = \frac{{\rm e}^{\beta' d_{\rm A}}(A_n^q - B_n^q)}{{\rm e}^{\beta' d_{\rm A}}(A_n^q + B_n^q) + 1}, \quad (12)$$

$$Q_1 = \langle\sigma_i^2\rangle = \frac{{\rm e}^{\beta' d_{\rm A}}(A_n^q + B_n^q)}{{\rm e}^{\beta' d_{\rm A}}(A_n^q + B_n^q) + 1}, \quad (13)$$

where $\langle\cdots\rangle$ corresponds to the usual thermal average, $\beta' = \beta J$ and $d_{\rm A} = D_{\rm A}/J$. The $M_{5/2}$ and $Q_{5/2}$ for spin-5/2 are calculated similarly as

$$M_{5/2} = \langle s_j\rangle = \frac{5\,{\rm e}^{25\beta' d_{\rm B}/4}(C_l^q - F_l^q) + 3\,{\rm e}^{9\beta' d_{\rm B}/4}(D_l^q - G_l^q) + {\rm e}^{\beta' d_{\rm B}/4}(E_l^q - 1)}{2[\,{\rm e}^{25\beta' d_{\rm B}/4}(C_l^q + F_l^q) + {\rm e}^{9\beta' d_{\rm B}/4}(D_l^q + G_l^q) + {\rm e}^{\beta' d_{\rm B}/4}(E_l^q + 1)]},$$

$$Q_{5/2} = \langle s_j^2 \rangle = \frac{25\,\mathrm{e}^{25\beta' d_\mathrm{B}/4}(C_l^q + F_l^q) + 9\,\mathrm{e}^{9\beta' d_\mathrm{B}/4}(D_l^q + G_l^q) + \mathrm{e}^{\beta' d_\mathrm{B}/4}(E_l^q + 1)}{4[\,\mathrm{e}^{25\beta' d_\mathrm{B}/4}(C_l^q + F_l^q) + \mathrm{e}^{9\beta' d_\mathrm{B}/4}(D_l^q + G_l^q) + \mathrm{e}^{\beta' d_\mathrm{B}/4}(E_l^q + 1)]}, \tag{14}$$

where $d_\mathrm{B} = D_\mathrm{B}/J$ and $l = n - 1$.

Before going to the next section, we give brief definitions of second- and first-order phase transition and compensation temperatures: (i) the Curie temperature or the second-order phase transition temperature, T_c, is the temperature when the sublattice magnetizations go to zero continuously separating the ordered phases, i.e. ferrimagnetic (F) phase, from the disordered one, i.e. paramagnetic (P) phase. (ii) In order to calculate the first-order phase transition temperature T_t, at which the discontinuity in the order-parameters occur, one needs the free energy expression. It is usually found by using the definition $F = -kT \ln Z$ in thermodynamic limit. Thus, the free energy expression is found in terms of the recursion relations as

$$\begin{aligned} F/J = -\frac{1}{\beta'}\Bigg\{ & \frac{q-1}{2-q}\ln[\,\mathrm{e}^{\beta'(-1/2+d_\mathrm{A})}A_n^{q-1} \\ & + \mathrm{e}^{\beta'(1/2+d_\mathrm{A})}B_n^{q-1} + 1] \\ & + \ln[\,\mathrm{e}^{\beta' d_\mathrm{A}}(A_n^q + B_n^q) + 1] \\ & + \frac{1}{2-q}\ln[\,\mathrm{e}^{25\beta' d_\mathrm{B}/4}C_{n-1}^{q-1} \\ & + \mathrm{e}^{9\beta' d_\mathrm{B}/4}D_{n-1}^{q-1} + \mathrm{e}^{\beta' d_\mathrm{B}/4}E_{n-1}^{q-1} \\ & + \mathrm{e}^{25\beta' d_\mathrm{B}/4}F_{n-1}^{q-1} \\ & + \mathrm{e}^{9\beta' d_\mathrm{B}/4}G_{n-1}^{q-1} + \mathrm{e}^{\beta' d_\mathrm{B}/4}]\Bigg\}, \end{aligned} \tag{15}$$

where it is clear that $\ln Z$ is equal to $\{\cdots\}$ in the above equation. The T_t's are determined from a free energy analysis. It should be mentioned that in solving the recursion relations by iteration, one has to assign an initial value for each of the recursion relations. Therefore, varying the initial values may result in different solutions for all the thermodynamic functions including the free energy. As a result, the temperature at which the free energy values are equal to each other is the T_t. (iii) The compensation temperature, T_comp, can be located by finding the crossing points between the absolute values of the sublattice magnetizations, i.e., $|M_1(T_\mathrm{comp})| = |M_{5/2}(T_\mathrm{comp})|$, or when the net magnetization goes to zero $M_\mathrm{NET} = |M_1(T_\mathrm{comp})| - |M_{5/2}(T_\mathrm{comp})| = 0$ with the conditions $\mathrm{sign}[M_1(T_\mathrm{comp})] = -\mathrm{sign}[M_{5/2}(T_\mathrm{comp})]$ with $T_\mathrm{comp} < T_\mathrm{c}$. These conditions ensure that at T_comp the two sublattice magnetizations cancel each other, whereas at the critical temperature, T_c, both sublattice magnetizations also the net magnetization go to zero.

3. Thermal variations of the order parameters

In this section, some characteristic behaviors of the order-parameters are studied to give a clear understanding of the nature of the phase transitions which may occur in mixed spin-1 and spin-5/2 model with CF. The definitions and the obtaining of the second- and first-order phase transitions and compensation temperatures are given briefly in the previous section. Now let us show the behavior of the order-parameters at the critical temperatures:

(I) Figure 2(a) was obtained for $D_\mathrm{A}/J = D_\mathrm{B}/J = 3.0$ and $q = 3$, and it illustrates the behaviors of the order parameters when system goes to a second-order phase transition. The order parameters M_1 and $M_{5/2}$ start from ± 1 and $\pm 5/2$ at zero temperature, respectively, and as the temperature increases both M_1 and $M_{5/2}$ decrease to go zero to combine each other at a temperature called as the Curie temperature which is indicated with T_c. In addition, the quadrupolar order parameters Q_1 and $Q_{5/2}$ start from $+1$ and $+25/4$, respectively, at zero temperature and as the temperature increases they decrease and little cusps are presented at the T_c.

(II) Figure 2(b) was calculated for unequal CFs when $D_\mathrm{A}/J = -7.0$, $D_\mathrm{B}/J = 0.0$, and $q = 3$, and it illustrates the behavior of the order parameters in the case of first-order phase transitions. Again the order parameters start from their ground state values at zero T. As T increases they decrease and at the T_t they all present jump discontinuities.

(III) The second- and first-order phase transitions are presented in Fig. 2(c) for $D_\mathrm{A}/J = -3.0$, $D_\mathrm{B}/J = -0.65$, and $q = 3$. $M_1 = Q_1 = M_{5/2} = 0$ and $Q_{5/2} = 0.25$ at zero T. Now as T increases they also increase very slowly and at the T_t they jump to higher values. They again decrease with the further increase of $T > T_\mathrm{t}$ and eventually M_1 and $M_{5/2}$ combine at T_c and where the Q's make little kinks.

(IV) In order to show the existence of the compensation temperature, the thermal behaviors of the net magnetization and order parameters are presented in Fig. 2(d) and which was obtained for $D_\mathrm{A}/J = 0.0$ and $D_\mathrm{B}/J = -1.499$ when $q = 3$. The behavior of the order parameters are the same as those in

Fig. 2(a). The details of the net magnetization curve was given in two insets since they are seen at very low values. As is seen, the net magnetization curve goes to zero thrice before the T_c. The first one is shown in the first inset (i) where the compensation temperature $kT_{comp1}/J = 0.416$. The next two are shown in the inset (ii) at the $kT_{comp2}/J = 1.079$ and $kT_{comp3}/J = 1.211$. It should be mentioned that for the equal CF case only two compensation temperatures are obtained for all q.

(V) The last figure was presented in Fig. 2(e) and obtained for $D_A/J = D_B/J = -2.002$ and $q = 4$. Now, M_1 and $M_{5/2}$ start from zero at zero temperature and as the temperature increases they split up at the first Curie temperature indicated with T_{c1}. Then, they first increase with the further increase of T and then start decreasing. Eventually, the magnetizations combine at the second T_{c2}. The existence of two T_c's are actually an indication of the reentrant behavior which will be explained in the next section.

After studying thermal variations of the order parameters and net magnetization, we are now ready to obtain the distinct phase diagrams of the model which will be done in the next section.

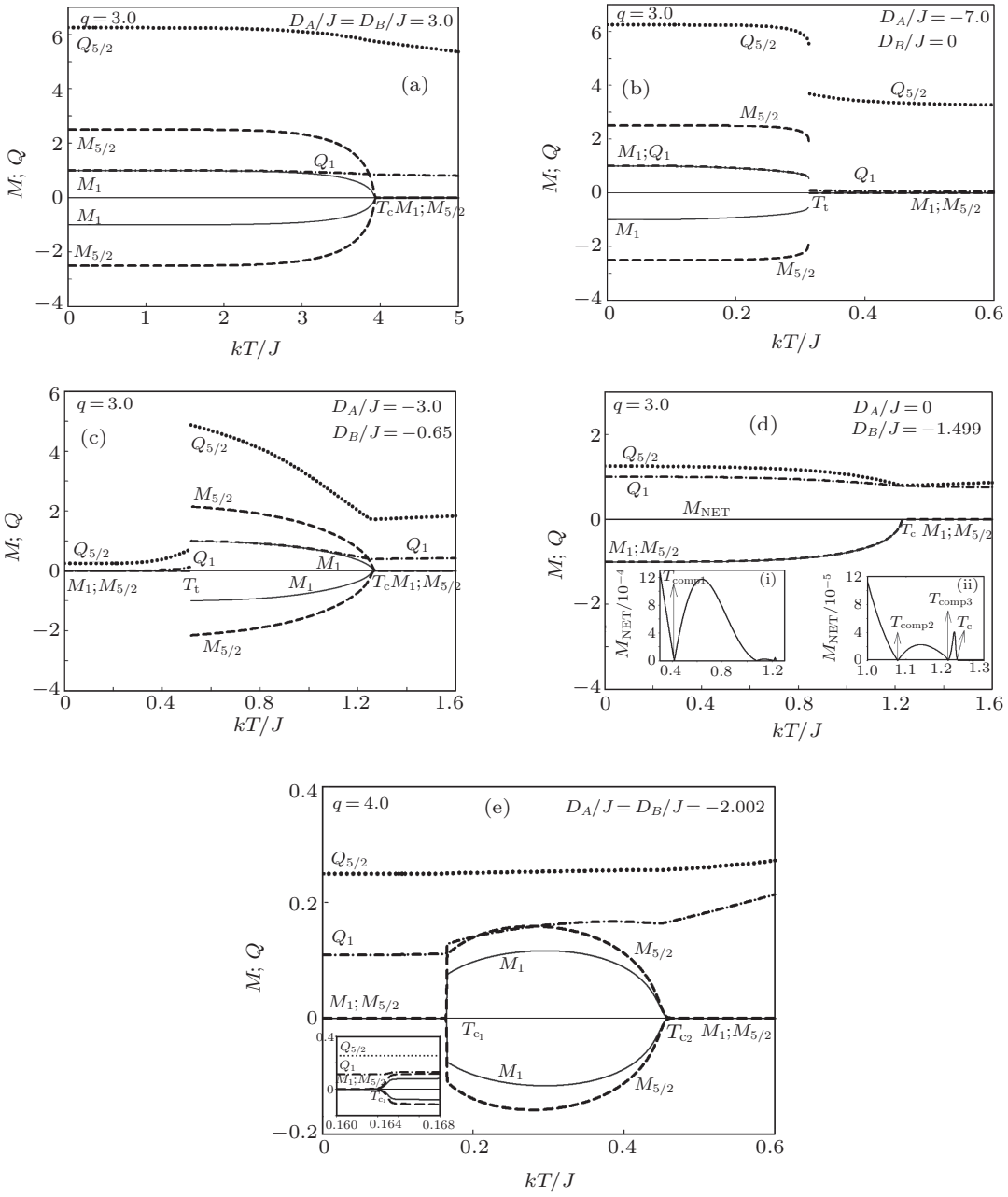


Fig. 2. Thermal variations of the order parameters for $q = 3$: (a) $D_A/J = D_B/J = 3.0$; (b) $D_A/J = -7.0$, and $D_B/J = 0.0$; (c) $D_A/J = -3.0$ and $D_B/J = -0.65$; (d) $D_A/J = 0.0$, $D_B/J = -1.499$, and for $q = 4$; (e) $D_A/J = D_B/J = -2.002$.

4. The phase diagrams

In the phase diagrams, the second- and first-order phase transition lines are indicated with solid and dashed lines, respectively, the compensation lines are indicated with gray-dotted lines and the TCP's are pointed out by the use of triangles.

The first phase diagrams, Figs. 3(a)–3(c), of the mixed spin-1 and spin-5/2 Ising model are obtained for the equal CF case ($D_{\rm A} = D_{\rm B} = D$) and on the $(kT/J, D/J)$ planes for the coordination numbers $q = 3$, 4, and 6. The ferrimagnetic (F) phase is separated from the paramagnetic (P) phase by the $T_{\rm c}$-lines for $q = 3$ and $q = 4$, and they are separated from each other by the $T_{\rm c}$-line at high temperatures and $T_{\rm t}$-line at low temperatures which combined at the TCP for $q = 6$ only. The reentrant behavior was only exhibited for $q = 4$ as shown in the inset of Fig. 3(b) and seen in the short range $-2.0004 \le D/J \le -2.0033$. In all the figures, the $T_{\rm comp}$ lines make half-closed loops which are unclosed and obtained from the existence of two compensation temperatures for each q.

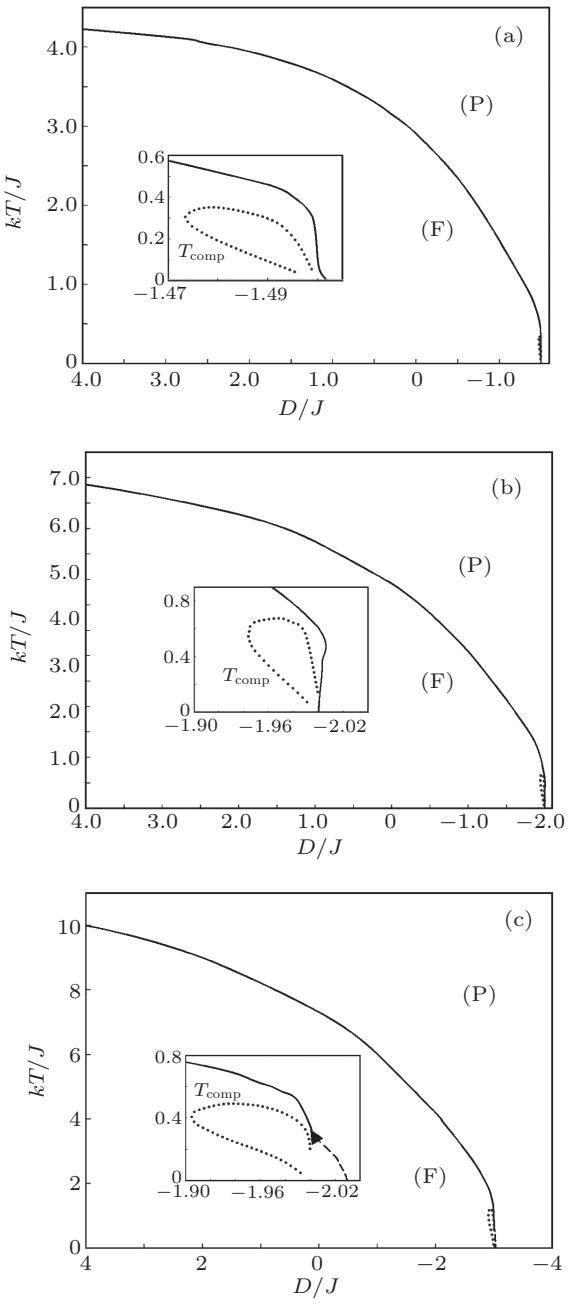


Fig. 3. The phase diagrams with equal CF strengths on the $(kT/J, D/J)$ planes: (a) for $q = 3$, (b) for $q = 4$, and (c) for $q = 6$.

The rest of the phase diagrams are calculated for the case with unequal CFs when $q = 3$. They are obtained on the $(kT/J, D_{\rm A}/J)$ and $(kT/J, D_{\rm B}/J)$ planes for selected values of $D_{\rm B}/J$ and $D_{\rm A}/J$, respectively. Figure 4 was obtained on the $(kT/J, D_{\rm A}/J)$ plane for given values of $D_{\rm B}/J$. The $T_{\rm c}$-lines are seen at higher T's for higher $D_{\rm B}/J$ values. Actually, the model gives only $T_{\rm c}$-lines for our selected values $0.07 \le D_{\rm B}/J \le 4.0$. The TCP's are seen in the range $0.01 \le D_{\rm B}/J \le -1.49$ and the combinations of which make a half-circular shaped line. Then again only the $T_{\rm c}$-lines are seen for higher negative values, i.e., $D_{\rm B}/J \le -1.5$. Our further calculation has shown the existence of the stable and unstable triciritical points which are indicated with filled and open triangles and are only found in the range $0.02 \le D_{\rm B}/J \le 0.06$ as seen in the inset. They are classified by using a free energy analysis, i.e., the ones with lower free energies are stable and the others are unstable for the given system parameters. Note that the similar behavior was also found for the mixed-spin with spin-1 and spin-3/2 case.[19,20] It is very interesting to note also that the compensation temperatures was only found when $D_{\rm B}/J = -1.5$.

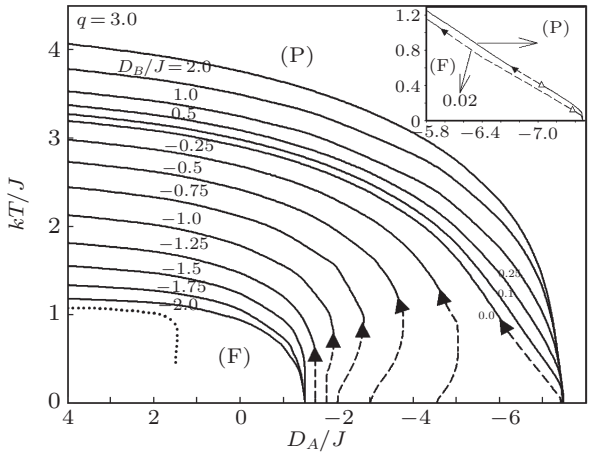


Fig. 4. The phase diagrams with unequal CF on the $(kT/J,\ D_{\rm A}/J)$ planes for values of $D_{\rm B}/J$, and stable and unstable TCP's in the inset.

The next phase diagrams are built on the $(kT/J, D_{\rm B}/J)$ planes for given values of $D_{\rm A}/J$ and illustrated in Figs. 5(a) and 5(b). In the figures, the $T_{\rm c}$-lines start from constant values at higher temperatures for higher positive values of $D_{\rm A}/J$ and as $D_{\rm B}/J$ is increased their temperatures gradually decrease and they become constant again at lower temperatures for lower $D_{\rm B}/J$'s. This behavior is seen when $D_{\rm A}/J \geq -1.48$. Below this value of $D_{\rm A}/J$ we again see a combination of the $T_{\rm c}$ and $T_{\rm t}$-lines combining at the TCP's and found only in the range about $-7.4 \leq D_{\rm A}/J \leq -1.51$. By the way, the compensation temperature lines are now constructed from three compensation temperatures in comparison to the equal CF case where only two of them are found.

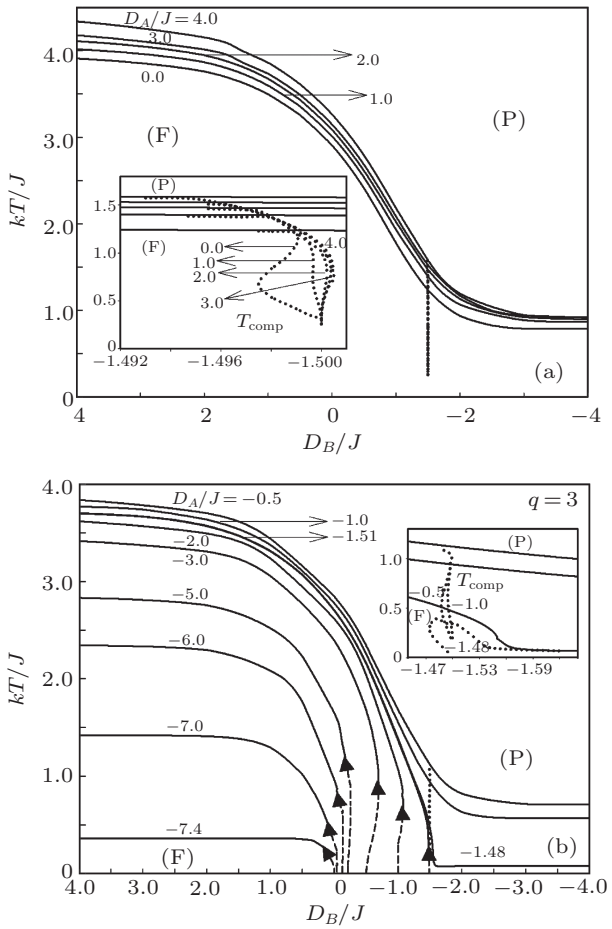


Fig. 5. The phase diagrams with unequal CF on the $(kT/J, D_{\rm B}/J)$ planes: (a) for positive values of $D_{\rm A}/J$, (b) for negative values of $D_{\rm A}/J$, and $(T_{\rm comp})$ in the insets.

5. Conclusion

The critical behaviors of the mixed spin-1 and spin-5/2 Ising model were investigated for equal and unequal CFs on the BL. The thermal variations of the order parameters were obtained to understand the nature of the phase transitions and then the temperature-dependent phase diagrams of the model are calculated. It is observed that the model undergoes second- and first-order phase transitions for equal and unequal CF's, therefore, TCP's. It is also found that the model exhibits compensation temperatures and reentrant behavior.

Now, we give a brief discussion of what we have found. In the case of equal CF case.

I) The model presents only second-order phase transitions for $q = 3$ (Fig. 3(a)) and $q = 4$ (Fig. 3(b)).

II) The reentrant behavior was observed for only $q = 4$ as seen in Fig. 3(b).

III) The ferrimagnetic (F) phase is separated from paramagnetic (P) phase with only the $T_{\rm c}$ lines when $q = 3$ and 4, but for $q = 6$ they are also separated with the $T_{\rm t}$ lines at low temperatures.

IV) Two-compensation temperatures for all q's and TCP's for only $q = 6$ were obtained.

V) The temperatures of the phase transition lines increase with the increase of q as expected.

VI) All the phase transition lines, i.e., $T_{\rm c}$ or $T_{\rm t}$ lines, start from $D/J \cong -1.5$, -2.0, and -3.0 for $q = 3, 4$, and 6, respectively.

For the unequal CF case our findings may be listed as follows.

1) The system always gives second-order phase transitions for higher positive values of $D_{\rm A}/J$ and $|D_{\rm B}/J|$.

2) TCP's are only observed for negative values of $D_{\rm A}/J$ on the $(kT/J, D_{\rm B}/J)$ planes.

3) The stable and unstable TCP's are only found in the range $0.02 \leq D_{\rm B}/J \leq 0.06$ for the $D_{\rm B}/J$ values in the $(kT/J, D_{\rm A}/J)$ planes.

4) All the $T_{\rm c}$, $T_{\rm t}$, and $T_{\rm comp}$ lines obtained in different planes are in total agreement with each other which indicates the reliability of our results.

Finally, we should mention that our equal CF results are in partial agreement with Ref. [7], unfortunately, there are no other works available for the case with unequal CF case in the literature. We hope that this work may inspire other researchers so that this model can be studied by using different techniques.

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