Topology identification for a class of complex dynamical networks using output variables*

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A problem of topology identification for complex dynamical networks is investigated in this paper. An adaptive observer is proposed to identify the topology of a complex dynamical networks based on the Lyapunov stability theory. Here the output of the network and the states of the observer are used to construct the updating law of the topology such that the communication resources from the network to its observer are saved. Some convergent criteria of the adaptive observer are derived in the form of linear inequality matrices. Several numerical examples are shown to demonstrate the effectiveness of the proposed observer.

Keywords: complex dynamical networks, topology identification, adaptive observer, output variables

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1. Introduction

A complex dynamical network is a network that involves a large number of dynamical systems interacting with each other, where the vertices represent the dynamical systems and the edges represent the relationships between them. For example, the World Wide Web is a set of interacting routers, a social network is a set of interacting people, the power grid is a set of interacting electrical equipments, and so on. All of these networks exhibit complicated behaviours due to the connections of simple dynamics. Meanwhile, in the real world, the connections are often uncertain or unknown, so it is difficult to explain the complex dynamical networks or control the complex behaviours in networks.[1−4]

Recently, the problem of topology identification has been studied in several fields. For a network coupled with linear dynamical systems under an unknown tree topology, the unweighted pair group method with an arithmetic mean is used to reveal the unknown topology and can correctly identify the tree topology.[5] This method only needs the observation of the networks to reconstruct the link structure. For a topology characterized by a sparse matrix, a network filtering system that combines Lasso regression is used to reveal the connections between the nodes.[6] For a sparse network, the stable response dynamics of a network is used to reconstruct its connections under the assumption of manipulating the input of every single node and performing many experiments to detect the adjacent links.[7] However, such a hypothesis is neither feasible in most situations nor practical for a large scale network. For a sparse network in the presence of noise, when the data are limited, constrained optimization techniques based on the L_1 vector norm are used to identify the topology of network, but it is not guaranteed that an exact identification is provided.[8] In Refs. [5]–[8], the identified topologies are specially shaped, such as tree topology and sparse topology.

For general topology complex dynamical networks, the topology identification based on the observer approach has been reported. For a complex network coupled with time delay, Wu[9] proposed an adaptive observer based on outer synchronization to identify the topology. Liu *et al.*[10] investigated an uncertain complex network based on the adaptive observer. In Refs. [9] and [10], all states of the network needed to reconstruct the topology. To monitor and identify a biological neural network, an adaptive observer was proposed in Ref. [2], where the first state of each neuron is coupled with the other neurons. In Ref. [11], the adaptive law of topology is con-

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structed using each node's output and the observer's state for a complex dynamical network coupled with each node's output. For a time-delayed complex dynamical network, the problem of topology fault diagnosis was investigated, where each node's output was used to reconstruct the topology.[12] However, the accurate topology of complex network was not identified.

Although tree topology and sparse topology can describe a lot of complex networks, the other shapes of topology exist in the real world such as cyclic link structure, star network, and so on. Therefore, it is necessary to investigate the topology identification for general topology of complex dynamical networks. On the other hand, it is quite common that only the output of the network can be measured which usually does not contain all states of nodes, so that only partial states of nodes are used to construct the topology of networks.[13]

In the present paper, a theoretical frame of topology identification for general complex dynamical networks is constructed. The observations from a network to its observer are used to reconstruct the topology of the complex dynamical networks, which is realizable technologically due to the fact that only partial states are often measurable. The observer method and adaptive control techniques are employed to propose an adaptive observer for a complex network coupled with each node's states or each node's outputs.

The rest of the present paper is organized as follows. In Section 2, the problem to be considered is described. An adaptive scheme is given in Section 3, and several numerical examples are presented in Section 4. Some conclusions are drawn in Section 5.

2. Problem formulation

Consider a complex dynamical network with N identical nodes which can be characterized as

$$\begin{aligned} \dot{x}_i &= \boldsymbol{A}x_i + f(x_i) + \sum_{j=1}^{N} c_{ij}\boldsymbol{\Gamma}x_j, \\ y_i &= \boldsymbol{H}x_i, \end{aligned} \tag{1}$$

where $1 \leq i \leq N$, $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^{\mathrm{T}} \in R^n$ is the state vector of i-th node, $y_i \in R$ is the output vector of the i-th node, $\boldsymbol{A} \in R^{n\times n}$ is a known constant matrix, $f : R^n \to R^n$ is a nonlinear smooth vector field, $\boldsymbol{H} \in R^{m\times n}$ is the output matrix of each node, node dynamics is $\dot{x} = f(x)$, $\boldsymbol{\Gamma} \in R^{n\times n}$ is the inner-coupling matrix, and $\boldsymbol{C} = (c_{ij})_{N\times N} \in R^{N\times N}$ is the coupling configuration matrix. If there is a link from node i to node j $(i \neq j)$, then $c_{ij} = 1$; otherwise $c_{ij} = 0$. Assume that $\boldsymbol{C}$ is a diffusive matrix satisfying, $\sum_{j=1,j\neq i}^{N} c_{ij} = -c_{ii}$. It is noted that the configuration matrix $\boldsymbol{C}$ is not required to be irreducible and symmetric. In this paper, the configuration matrix $\boldsymbol{C}$ is not known and need to be identified later.

Hypothesis 1 (H1) Suppose that $f(x)$ is Lipschitz-continuous. That is, there exists a positive number constant α satisfying $\|f(\hat{x}) - f(x)\| \leq \alpha\|\hat{x} - x\|$ for $x \in R^n$ and $\hat{x} \in R^n$, where $\|\cdot\|$ represents the Euclid normal.

Hypothesis 2 (H2) Suppose that the constant matrix $\boldsymbol{A}$ is stable.

Hypothesis 3 (H3) Suppose that the network (1) is not synchronized.[14]

Hypothesis 4 (H4) Suppose that the topology matrix $\boldsymbol{C}$ is bounded with norm, i.e., there is a positive constant β such that $\|\boldsymbol{C}\|_2 \leq \beta$, where $\|\cdot\|_2$ represents the 2-norm of a matrix.

Hypothesis 5 (H5) Suppose that there exists matrix $\boldsymbol{B}$ such that $\boldsymbol{\Gamma} = H^{\mathrm{T}}\boldsymbol{B}$.

In this paper, an adaptive observer will be designed later to identify the topology of network (1) with Hypotheses 1–5.

Remark 1 For most complex dynamical networks, Hypotheses 1–5 can be satisfied. Hypotheses 1 and 2 represent the node character, which can be satisfied due to most nonlinear system being represented by an affine system. Hypothesis 3 is equivalent to the persistent excitation which is a necessary condition in parameter identification. Hypothesis 4 can be satisfied in a network whose node number is limited. And Hypothesis 5 is needed in terms of mathematic requirement.

3. Adaptive observer of complex dynamical networks

In order to identify the topology of network (1), an adaptive observer is constructed as

$$\begin{aligned} \dot{\hat{x}}_i &= \boldsymbol{A}\hat{x}_i + f(\hat{x}_i) + \sum_{j=1}^{N} \hat{c}_{ij}\boldsymbol{\Gamma}\hat{x}_j + \boldsymbol{K}(\hat{y}_i - y_i), \\ \hat{y}_i &= \boldsymbol{H}\hat{x}_i, \\ \dot{\hat{c}}_{ij} &= -(\hat{y}_i - y_i)_i^{\mathrm{T}}\boldsymbol{B}\hat{x}_j, \end{aligned} \tag{2}$$

where $1 \leq i \leq N$, $\hat{x}_i \in R^n$ is the state vector, $\hat{y}_i \in R^m$ is the output vector and $\boldsymbol{K_i}$ are the observer gains to be designed later.

Remark 2 The adaptive law of topology is constructed with the outputs of network (1) and network (2), which is different from the results in Refs. [9] and [10]. The states of network and its observer were used to identify the topology of network in Refs. [9] and [10], so that a large number of data are needed to transform from the network to its observer, which is not practical and not economical in engineering either. Meanwhile for the adaptive law derived as observer (2), only partial states need to be transmitted, thereby saving communication resources in the real network.

Let $\dot{e}_i = \hat{x}_i - x_i$, $e_{yi} = \hat{y}_i - y_i$, and $\tilde{c}_{ij} = \hat{c}_{ij} - c_{ij}$, then the error state between network (1) and its observer (2) can be written as

$$\begin{aligned}\dot{e}_i &= \boldsymbol{A}e_i + f(\hat{x}_i) - f(x_i) + \sum_{j=1}^{N} c_{ij}\boldsymbol{\Gamma}e_j \\ &\quad + \sum_{j=1}^{N} \tilde{c}_{ij}\boldsymbol{\Gamma}\hat{x}_j + \boldsymbol{K}e_{yi}, \\ e_{yi} &= \boldsymbol{H}e_i, \\ \dot{\tilde{c}}_{ij} &= \dot{\hat{c}}_{ij} = -e_{yi}^{\mathrm{T}}\boldsymbol{B}\hat{x}_j. \end{aligned} \tag{3}$$

To make the states in error system (3) approach zeros as time goes to infinity, i.e., $\lim\limits_{t\to\infty} \|e_i(t)\| = 0$, the criteria of choosing the observer gain $\boldsymbol{K}$ will be derived in the following theorem.

Theorem 1 Suppose that Hypotheses 1–3 hold. If there exist the matrix $\boldsymbol{K}$ and the constant $\varsigma > 0$ satisfying the following inequalities:

$$\begin{aligned}&\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}\boldsymbol{H} + \boldsymbol{H}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} < -\varsigma\boldsymbol{I_n}, \\ &\varsigma > 2\alpha + 2\beta\left\|\boldsymbol{\Gamma}\right\|_2, \end{aligned} \tag{4}$$

then adaptive observer (2) can estimate the states of network (1) and identify the topology of network (1).

Proof Construct the Lyapunov candidate as

$$V = \frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}e_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}^2, \tag{5}$$

then the time derivative of V will be obtained as

$$\begin{aligned}\dot{V} &= \frac{1}{2}\sum_{i=1}^{N}(\dot{e}_i^{\mathrm{T}}e_i + e_i^{\mathrm{T}}\dot{e}_i) + \sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}\dot{\tilde{c}}_{ij} \\ &= \sum_{i=1}^{N}\left(e_i^{\mathrm{T}}(\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}})e_i + 2e_i^{\mathrm{T}}(f(\hat{x}_i) - f(x_i))\right) \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\left(e_i^{\mathrm{T}}c_{ij}\boldsymbol{\Gamma}e_j + c_{ij}e_j^{\mathrm{T}}\boldsymbol{\Gamma}^{\mathrm{T}}e_i\right) \\ &\quad + 2\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}e_i^{\mathrm{T}}\boldsymbol{\Gamma}\hat{x}_j \\ &\quad + \sum_{i=1}^{N} e_i^{\mathrm{T}}(\boldsymbol{K}\boldsymbol{H} + \boldsymbol{H}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}})e_i \\ &\quad + 2\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}\dot{\hat{c}}_{ij}. \end{aligned} \tag{6}$$

Using H1 and H3, one can obtain the following equation:

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N}\left(e_i^{\mathrm{T}}(\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}})e_i + 2e_i^{\mathrm{T}}(f(\hat{x}_i) - f(x_i))\right) \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\left(e_i^{\mathrm{T}}c_{ij}\boldsymbol{\Gamma}e_j + c_{ij}e_j^{\mathrm{T}}\boldsymbol{\Gamma}^{\mathrm{T}}e_i\right) \\ &\quad + \sum_{i=1}^{N} e_i^{\mathrm{T}}(\boldsymbol{K}\boldsymbol{H} + \boldsymbol{H}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}})e_i \\ &\le \sum_{i=1}^{N} e_i^{\mathrm{T}}(\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}\boldsymbol{H} + \boldsymbol{H}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} + 2\alpha\boldsymbol{I_n})e_i \\ &\quad + e^{\mathrm{T}}(\boldsymbol{C}\otimes\boldsymbol{\Gamma} + \boldsymbol{C}^{\mathrm{T}}\otimes\boldsymbol{\Gamma}^{\mathrm{T}})e \\ &\le \sum_{i=1}^{N} e_i^{\mathrm{T}}(\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}\boldsymbol{H} + \boldsymbol{H}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} + 2\alpha\boldsymbol{I_n})e_i \\ &\quad + 2\left\|\boldsymbol{C}\right\|_2\left\|\boldsymbol{\Gamma}\right\|_2 e^{\mathrm{T}}e \\ &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{K}\boldsymbol{H} + \boldsymbol{H}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} \\ &\quad + 2\alpha\boldsymbol{I_n} + 2\left\|\boldsymbol{C}\right\|_2\left\|\boldsymbol{\Gamma}\right\|_2\boldsymbol{I_n})e_i, \end{aligned} \tag{7}$$

where $e = \left(e_1^{\mathrm{T}}, e_2^{\mathrm{T}}, \ldots, e_n^{\mathrm{T}}\right)^{\mathrm{T}}$. Due to H5, there exists a large enough positive constant ς such that inequality (4) holds. Therefore, one can obtain $\dot{V} < 0$. By using the Lyapunov stability theorem, one can obtain the error state $e_i \to 0$ as time $t \to +\infty$. Because hypothesis H2 holds, the topology error $c_{ij} \to 0$ as time $t \to +\infty$. The proof is completed.

Theorem 1 can be extended to a complex dynamical network coupled with the node outputs.[15]

Consider a general complex dynamical network consisting of N identical nodes coupled with the node outputs, which is described as

$$\dot{x}_i = \boldsymbol{A}x_i + f(x_i) + \sum_{j=1}^{N} c_{ij}\boldsymbol{L}y_j, \tag{8}$$

$$y_i = \boldsymbol{H}x_i, \tag{9}$$

where topology $\boldsymbol{C} = (c_{ij})_{N\times N} \in R^{N\times N}$ is unknown and validated later.

Design an adaptive observer for network (8) as

follows:

$$\begin{aligned}
&\dot{\hat{x}}_i = \boldsymbol{A}\hat{x}_i + f(\hat{x}_i) + \sum_{j=1}^{N} \hat{c}_{ij}\boldsymbol{L}\hat{y}_j + \boldsymbol{K}(\hat{y}_i - y_i),\\
&\hat{y}_i = \boldsymbol{H}\hat{x}_i,\\
&\dot{\hat{c}}_{ij} = -(\hat{y}_i - y_i)_i^{\mathrm{T}}\boldsymbol{L}\hat{y}_j. \qquad (10)
\end{aligned}$$

In order that observer (9) can identify the topology of network (8), the following hypothesis is needed.

Hypothesis 6 (H6) Suppose that there exists matrix $\boldsymbol{B}$ such that $\boldsymbol{LH} = \boldsymbol{H}^{\mathrm{T}}\boldsymbol{B}$.

The error dynamics between network (8) and adaptive observer (9) is

$$\begin{aligned}
\dot{e}_i =\ & \boldsymbol{A}e_i + f(\hat{x}_i) - f(x_i) + \sum_{j=1}^{N} c_{ij}\boldsymbol{L}e_{yj}\\
&+ \sum_{j=1}^{N} \tilde{c}_{ij}\boldsymbol{L}\hat{y}_j + \boldsymbol{K}e_{yi}, \qquad (11)\\
e_{yi} =\ & \boldsymbol{H}e_i,\\
\dot{\tilde{c}}_{ij} =\ & \dot{\hat{c}}_{ij} = -e_{yi}^{\mathrm{T}}\boldsymbol{B}\hat{x}_j. \qquad (12)
\end{aligned}$$

To obtain $\lim\limits_{t\to\infty} \|e_i(t)\| = 0$, the observer gain $\boldsymbol{K}$ is chosen in terms of the following theorem.

Theorem 2 Suppose that Hypotheses 1–3 and Hypothesis 6 hold. If there exist the matrix $\boldsymbol{K}$ and the constant $\varsigma > 0$ satisfying the following inequalities:

$$\begin{aligned}
&\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{KH} + \boldsymbol{H}^{\mathrm{T}}\boldsymbol{K}^{\mathrm{T}} < -\varsigma\boldsymbol{I_n},\\
&\varsigma > 2\alpha + 2\beta\left\|\boldsymbol{LH}\right\|_2, \qquad (13)
\end{aligned}$$

then adaptive observer (9) can estimate the states of network (8) and identify the topology of network (8).

The proof of Theorem 2 is similar to that of Theorem 1, so that it is omitted here.

4. Numerical simulation

In this section, several numerical examples are given to demonstrate the effectiveness of the topology identification of network (1) using adaptive observer (2). In the networks, chaotic system will be taken as the node dynamics.

It is well known that Lorenz, Chen, and Lü systems are several typical benchmark chaotic systems.[16–19] Lorenz chaotic system is described as[19]

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = \begin{pmatrix} -a & a & 0 \\ 0 & -1 & 0 \\ 0 & 0 & c \end{pmatrix} \begin{pmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{pmatrix} + \begin{pmatrix} 0 \\ bx_{i1} - x_{i1}x_{i3} \\ x_{i1}x_{i2} \end{pmatrix}$$

$$= \boldsymbol{A}x_i + f(x_i), \qquad (14)$$

where a, b, and c are parameters. When $a = 10$, $b = 28$, and $c = -8/3$, system (12) is chaotic. For any two state vectors x_i and x_j of the Lorenz system, since chaotic attractor is bounded in a certain region, there exists a constant θ satisfying $\|x_{ik}\| \le \theta$ and $\|x_{jk}\| \le \theta$ for $k = 1, 2, 3$. Then one obtains the following inequality

$$\begin{aligned}
&\|f(x_i) - f(x_j)\|\\
&= \sqrt{(x_{j1}x_{j3} - x_{i1}x_{i3})^2 + (28e_{i1} + x_{i1}x_{i2} - x_{j1}x_{j2})^2}\\
&\le (2\theta + 28)\left\|x_i - x_j\right\| = \alpha\left\|x_i - x_j\right\|. \qquad (15)
\end{aligned}$$

Thus the Lorenz system satisfies Hypothesis H1. Since the egienvalues of matrix $\boldsymbol{A}$ locate on the left half plane of complex horizon, the Lorenz system satisfies H5.

In the simulation, the parameters are given as follows:

$$\begin{aligned}
&\boldsymbol{K} = \begin{bmatrix} -125.2621 \\ -128.2621 \\ -126.0398 \end{bmatrix}, \quad \boldsymbol{\Gamma} = \begin{bmatrix} 0.02 & 0.02 & 0.02 \\ 0.02 & 0.02 & 0.02 \\ 0.02 & 0.02 & 0.02 \end{bmatrix},\\
&\boldsymbol{H} = [\,2\ 2\ 2\,], \quad \boldsymbol{B} = [\,0.01\ 0.01\ 0.01\,]^{\mathrm{T}}. \qquad (16)
\end{aligned}$$

The initial values of the network states and their observers' states are randomly chosen in the interval [0, 1] and the initial values of the topology matrix are chosen as 1.

Firstly, a star network containing seven nodes is constructed, whose topology is

$$\boldsymbol{C} = \begin{bmatrix} -6 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}. \qquad (17)$$

Since Hypotheses H1–H5 hold, according to Theorem 1, adaptive observer (2) can identify the topology of network (1) using the adaptive law $\dot{\hat{c}}_{ij} = -e_{yi}^{\mathrm{T}}\boldsymbol{B}\hat{x}_j$ and the error states converge to zeros as time $t \to +\infty$. The error states are shown in Fig. 1 and the identified topology is shown in Fig. 2. From Fig. 2, one can see that the identified topology by using adaptive observer (2) can converge to the original topology of network (1).

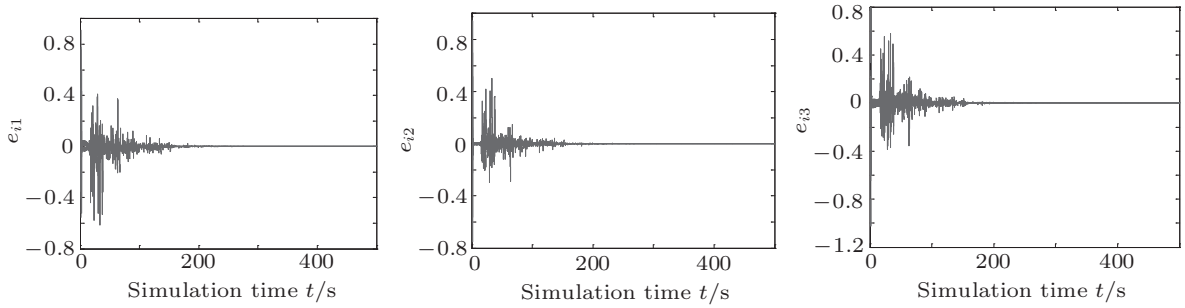


Fig. 1. State errors e_{i1}, e_{i2}, and e_{i3}, for $i = 1, 2, \ldots, 7$ between star network (1) and its observer (2).

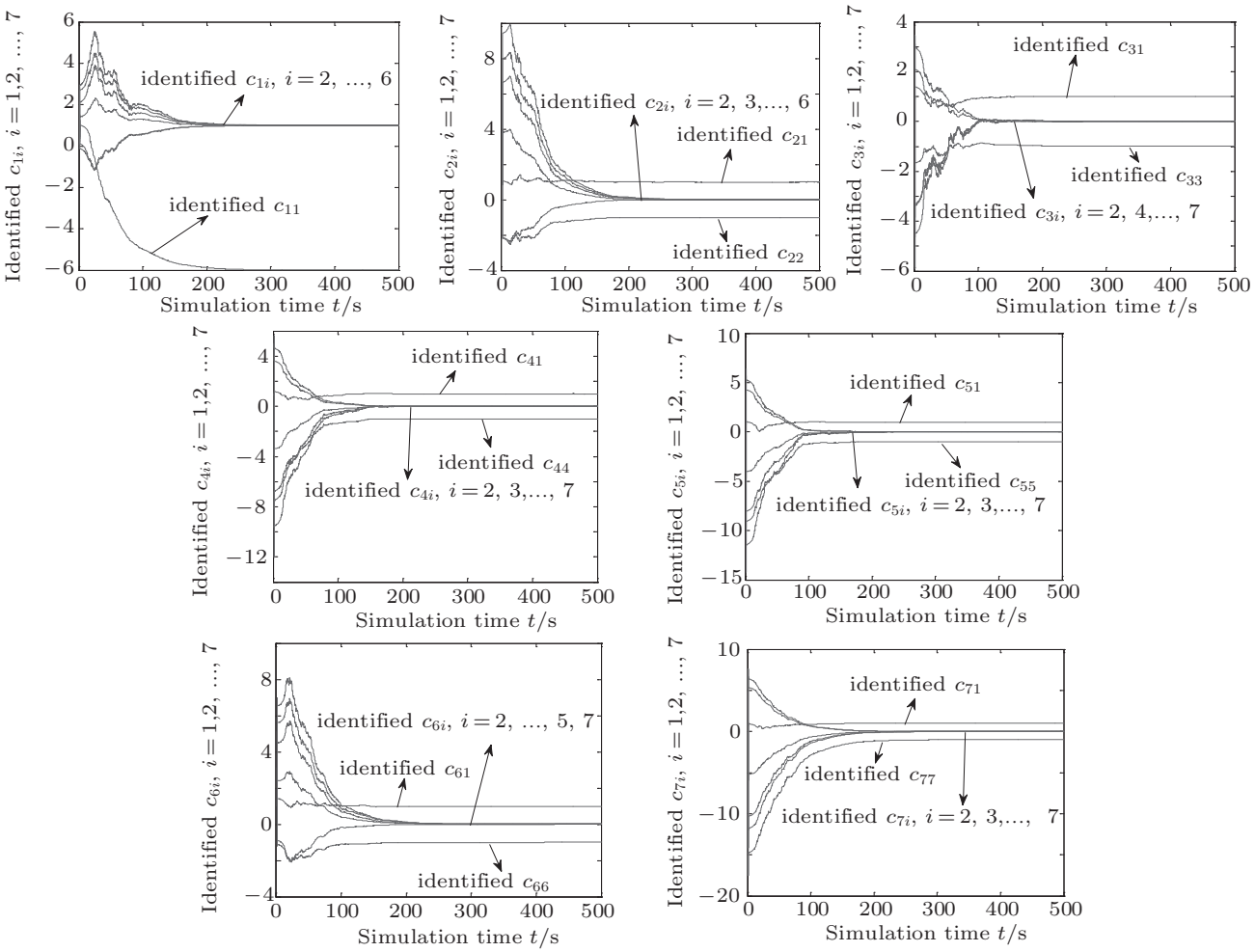


Fig. 2. Identified topologies of a star complex dynamical network.

Secondly, a tree-shape network containing seven nodes is constructed, whose topology is as follows:

$$\boldsymbol{C} = \begin{bmatrix} -2 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -3 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & -3 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}. \quad (18)$$

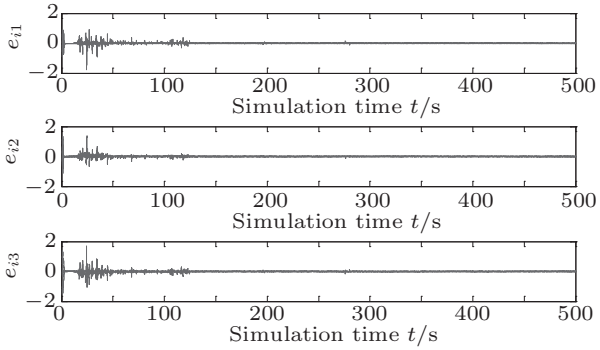


Fig. 3. Errors e_{i1}, e_{i2}, and e_{i3}, for $i = 1, 2, \ldots, 7$ between tree-shape network (1) and its observer (2).

The state errors converge to zeros and the identified topology matrix converges to its real value within 200 s. The state errors between tree network (1) and its observer (2) are shown is Fig. 3 and the identified connections from node 1 to the others are shown in Fig. 4. For concision, the other connections between nodes are omitted here. From Figs. 3 and 4, it follows that the adaptive observer (2) can identify the topology of network (1) according to Theorem 1.

Next, a ring network containing seven nodes is constructed. The state errors between ring network (1) and its observer are shown in Fig. 5. The connections from node 1 to the others are shown in Fig. 6 and the other connections are omitted.

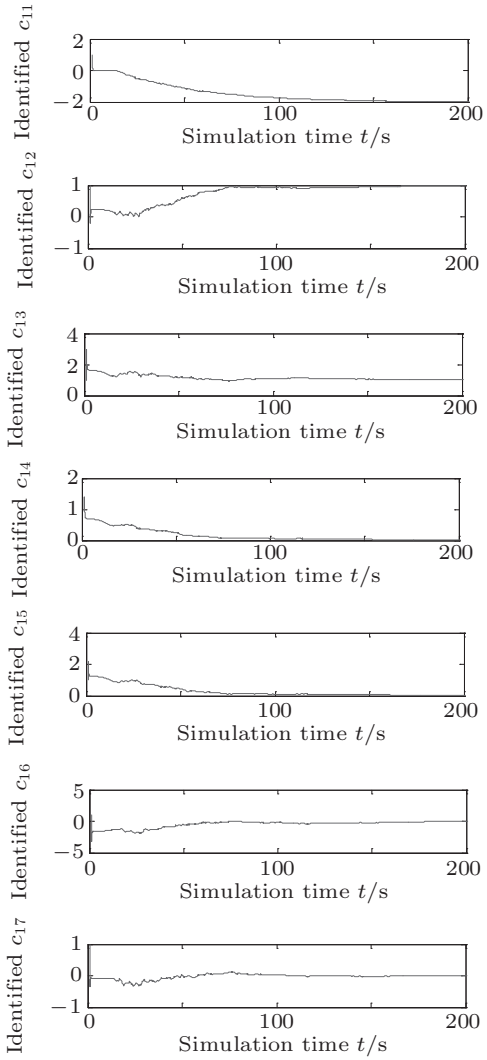


Fig. 4. Identified connections from node 1 to the others.

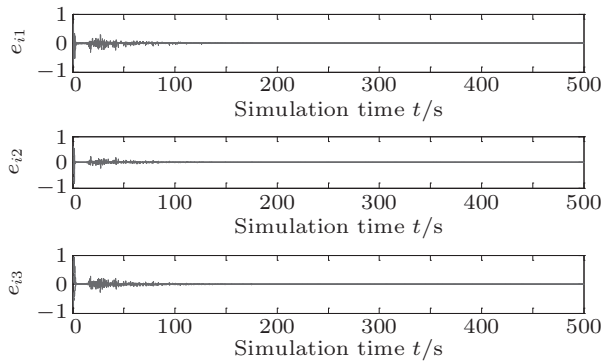


Fig. 5. State errors e_{i1}, e_{i2}, and e_{i3}, for $i = 1, 2, \ldots, 7$ between ring network (1) and its observer.

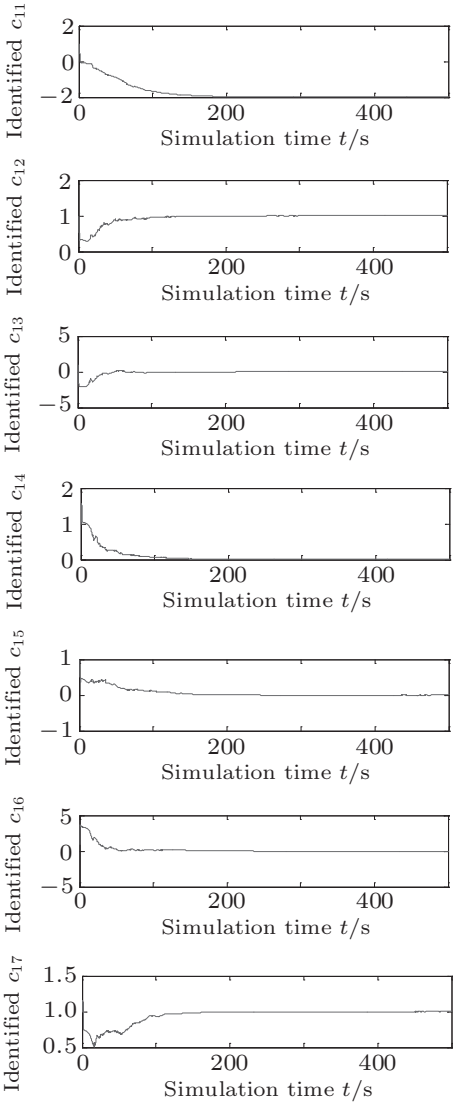


Fig. 6. Identified connections from node 1 to the others.

Finally, the topology of a general complex dynamical network is identified according to Theorem 1, whose topology is as follows:

$$\boldsymbol{C} = \begin{bmatrix} -6 & 2 & 2 & 0 & 0 & 2 & 0 \\ 2 & -5 & 0 & 0 & 3 & 0 & 0 \\ 2 & 3 & -9 & 0 & 0 & 0 & 4 \\ 2 & 0 & 0 & -6 & 0 & 4 & 0 \\ 2 & 1 & 1 & 0 & -4 & 0 & 0 \\ 1 & 2 & 1 & 1 & 1 & -5 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}. \quad (19)$$

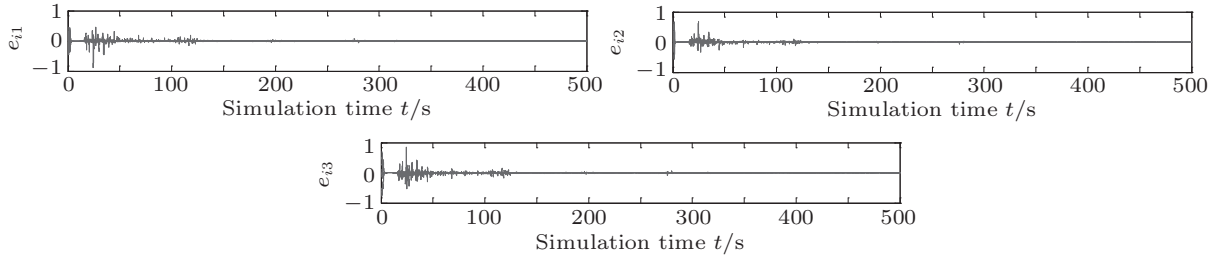
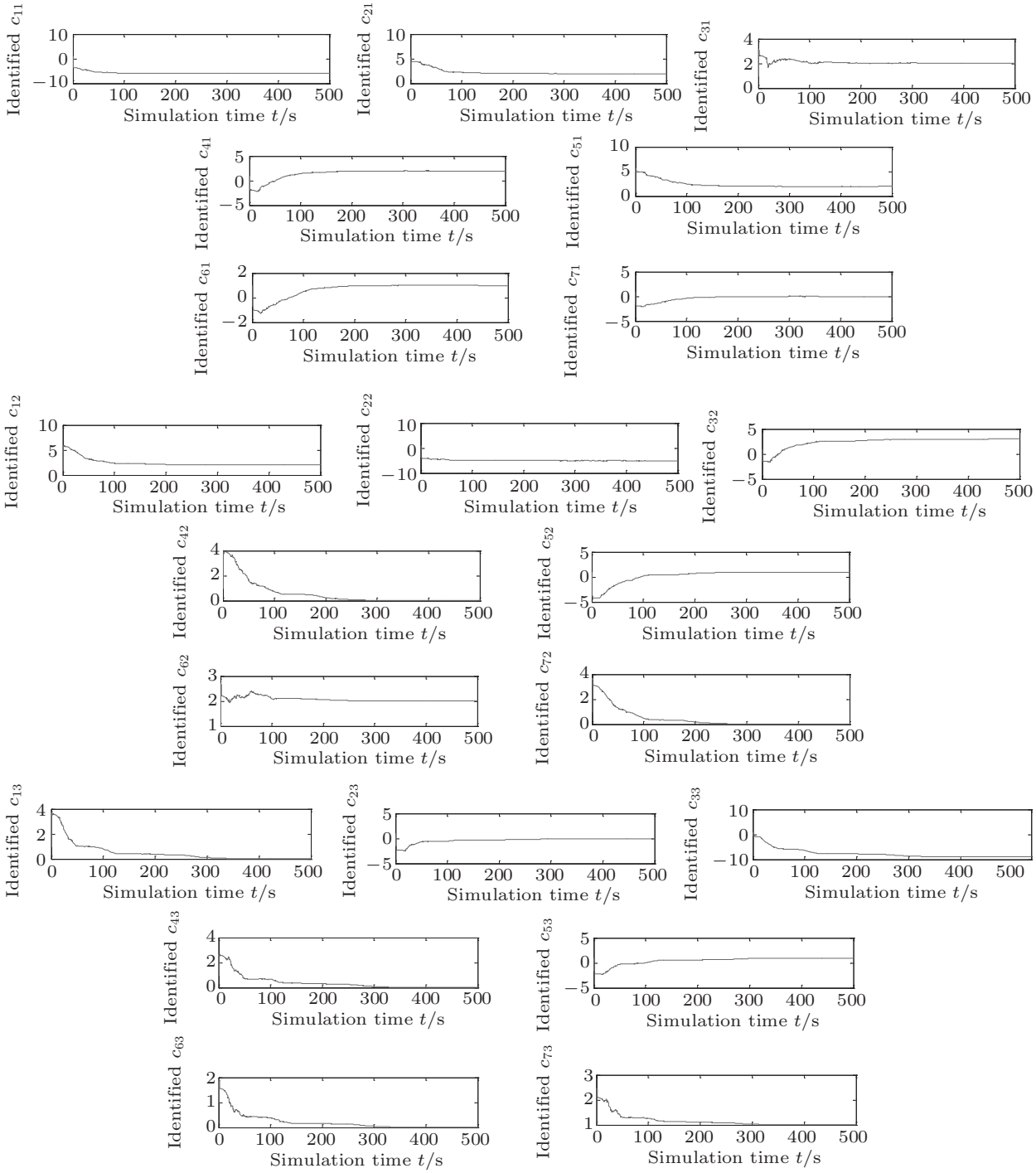


Fig. 7. State errors e_{i1}, e_{i2}, and e_{i3}, for $i = 1, 2, \ldots, 7$ between a general complex network and its observer.



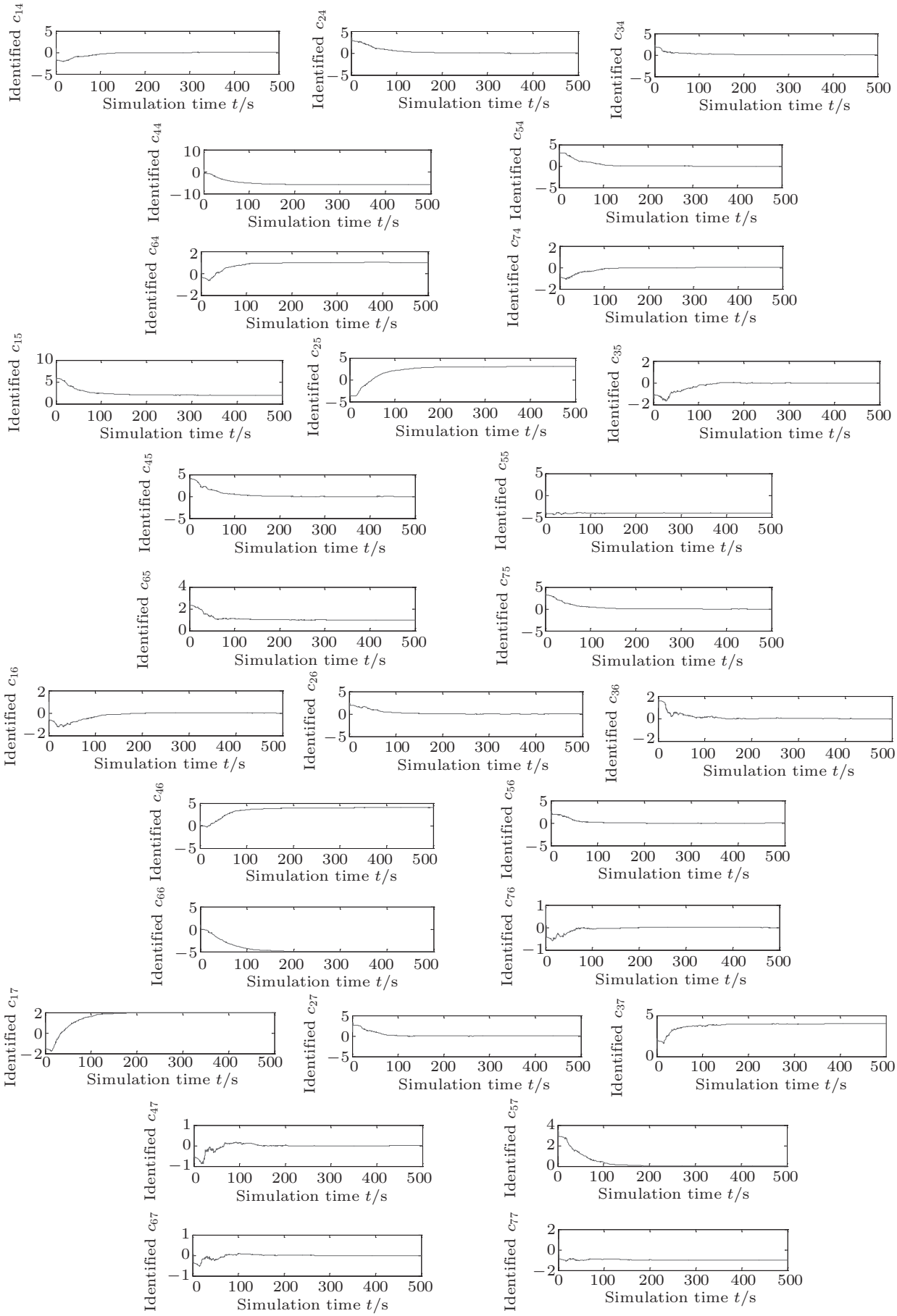


Fig. 8. Identified topology of a general complex network.

Obviously, this network is not symmetrical and weighted. The state errors between network (1) and its observer (2) are shown in Fig. 7 and the identified topology is shown in Fig. 8. It is inferred that the state errors can converge to zeros and the identified topology can converge to its real value as described in (17) as time goes to infinity.

5. Conclusion

A topology identification scheme of complex dynamical networks is proposed based on the adaptive observer approach in this paper, where only the output of the complex dynamical network is used to construct the controller in the adaptive observer and the complex network is coupled with the node states. The measurable output of a complex network and the measurable states of the observer are used to construct the adaptive law of the topology, which is practical in real engineering due to partial states being measurable in order to economize limited communication resources and reduce the number of sensors. Several numerical examples are given to demonstrate the effectiveness of the proposed adaptive observer including tree-shape topology, star-shape topology, ring topology, and the general topology.

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