Synchronization of noise-perturbed optical systems with multiple time delays*

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In this paper, complete synchronization and generalized synchronization between two unidirectionally coupled optical systems with multiple time delays and noise perturbation are investigated. Sufficient conditions for both complete synchronization and generalized synchronization are rigorously established. Numerical simulations fully support the theoretical results. The effect of parameter mismatch on the quality of synchronization is also explored.

Keywords: synchronization, noise, time delays

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1. Introduction

Synchronization, in particular chaos synchronization, is deemed to have a great potential in a large number of application areas, ranging from secure communication to modeling brain activity, and even to the optimization of nonlinear system performance.[1] During the past decades, many methods and experimental techniques have been presented to realize the synchronization of master-slave chaotic systems, such as the variable structure method, the impulsive control method, the fuzzy control method, the active control method, the nonlinear control method, the linear feedback control method, and so on.[2−12]

Time-delayed systems are ubiquitous in nature, technology, and society because of finite signal transmission times, switching speeds, and memory effects.[13] The dynamics of a multi-feedback system is a representative example of the multi-delay systems. Prominent examples of such dynamics can be found in biological and biomedical systems, laser physics, and integrated communications. In laser physics such a situation arises in lasers subject to two or more optical or electro-optical feedback. The second optical feedback can be useful to stabilize laser intensity.[14] A semiconductor laser with two delay times can offer a higher complexity of dynamics than that achievable in a more conventional single delay time system.[15] The high complexity of such a multiple time delay system can provide new architecture for enhancing message security in a chaos based encryption system. Recently, the synchronization of laser systems has attracted a great deal of attention.[16−21] Complete synchronization,[17] phase synchronization,[18] anticipating synchronization,[19] and generalized synchronization[20−24] have been considered in time-delayed coupled systems. According to the Krasovskii–Lyapunov theory, some sufficient conditions for different types of synchronization have been given to ensure synchronization in the coupled systems with single and multiple delays.[16,19,21,23−25]

Since noise is ubiquitous in nature and man-made systems, and in the processes of applications, synchronization is unavoidably influenced by noise. Therefore, the synchronization of a system with noise perturbation has become a very interesting field recently.[25−31] From recent studies,[26−28] the LaSalle-type invariance principle for stochastic differential equations is useful for discussing the synchronization of noise-perturbed chaotic systems. According to this theory, some sufficient conditions are given to ensure the synchronization of noise-perturbed systems. However, to the best of our knowledge, chaotic synchronization between the noise-perturbed multi-feedback system has not been investigated yet and the so-called LaSelle-type invariance principle is not valid for this case. Therefore, it is necessary and important to propose the synchronization conditions for this case.

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In this paper, we investigate two types of synchronization, namely, complete synchronization and generalized synchronization, between two noise perturbed optical systems with multiple time delays. According to the Lyapunov functional and combining with the linear matrix inequality (LMI) approach, we derive sufficient conditions for different synchronization regimes. We support the analytical results with numerical simulations. Furthermore, we generalize the above approach to a wide class of nonlinear time-delayed systems.

The rest of this paper is organized as follows. In Section 2, unidirectionally coupled Ikeda systems with multiple delays and noise perturbation are presented, and corresponding sufficient conditions for the complete synchronization are rigorously established. The generalized synchronization between two unidirectionally coupled non-identical Ikeda systems is investigated in Section 3. The simulation results are presented in Section 4. Finally, some conclusions are given in Section 5.

2. Complete synchronization between multi-feedback Ikeda systems

In this section, we first consider the synchronization between the double-feedback Ikeda systems:

$$\dot{x} = -\alpha x + m_1 \sin x_{\tau_1} + m_2 \sin x_{\tau_2}, \tag{1}$$

$$\dot{y} = -\alpha y + m_3 \sin y_{\tau_1} + m_4 \sin y_{\tau_2} + c(x-y) + \sigma(e, e_{\tau_1}, e_{\tau_2})\dot{W}, \tag{2}$$

where $x_\tau \equiv x(t-\tau)$, c is the coupling strength between the driving system x and the response system y. $\sigma(e, e_{\tau_1}, e_{\tau_2})$ is called the noisy intensity matrix, where $\sigma : R \times R \times R \to R^{1\times m}$. Moreover, σ is locally Lipschitz continuous and satisfies the linear growth condition with $\sigma(0,0,0) \equiv 0$. $W = (w_1, \ldots, w_m)^{\mathrm{T}}$ is an m-dimensional Brownian motion defined in a complete probability space $(\Omega, \mathscr{F}, P)$ with a natural filtration $\{\mathscr{F}_t\}_{t\geq 0}$. Accordingly, $\dot{W}$ is an m-dimensional white noise. The kind of noise term in system (2) is mostly utilized to describe the coupling process influenced by environmental fluctuation, inaccurate design of coupling strength, etc.[26,27] Since the speed of the environmental fluctuation is far less than the change rate of a concrete system, we also assume that

$$\mathrm{trace}(\sigma^{\mathrm{T}}\sigma) \leq pe^2 + q_1 e_{\tau_1}^2 + q_2 e_{\tau_2}^2, \tag{3}$$

where p, q_1, q_2 are three positive numbers.

This investigation is of considerable practical importance, as the equations of class B lasers with feedback can be reduced to an equation of the Ikeda type.[32] The Ikeda model is introduced to describe the dynamics of an optical bistable resonator, plays an important role in electronics and physiological studies, and is well known for the delay-induced chaotic behaviour.[33] Physically x is the phase lag of the electric field across the resonator; α is the relaxation coefficient for the driving x and the driven y dynamical variables; $m_{1,2}$ and $m_{3,4}$ are the laser intensities injected into the driving and the driven systems, respectively; $\tau_{1,2}$ are the feedback delay times in the coupled systems. For a sufficiently large laser intensity and feedback delay time the system is hyperchaotic.

To study the complete synchronization between system (1) and system (2) it is convenient to analyse the dynamics of error $e = x - y$. Thus under the condition $m_1 = m_3$, $m_2 = m_4$, the following error dynamics are obtained:

$$\begin{aligned}\dot{e} = {} & -(\alpha + c)e + 2m_1 \cos\frac{x_{\tau_1} + y_{\tau_1}}{2} \sin\frac{e_{\tau_1}}{2} \\ & + 2m_2 \cos\frac{x_{\tau_2} + y_{\tau_2}}{2} \sin\frac{e_{\tau_2}}{2} \\ & - \sigma(e, e_{\tau_1}, e_{\tau_2})\dot{W}.\end{aligned} \tag{4}$$

Therefore, it follows from the theory of stochastic differential equation[34] (see also the Appendix A) that the error dynamics (4) possess a unique global solution on $t \geq 0$, denoted by $e(t, \phi)$ for any initial data $\phi \in C^{\mu}_{\mathscr{F}_0}([-\tau, 0], R)$. $e(t, 0) \equiv 0$ is a trivial solution of the error dynamics (4). Obviously, if this trivial solution is globally asymptotically stable in a physical sense, the complete synchronization between system (1) and system (2) can be realized.

To this end, construct a function

$$V = e^2 + k_1 \int_{t-\tau_1}^{t} e^2(s)\,\mathrm{d}s + k_2 \int_{t-\tau_2}^{t} e^2(s)\,\mathrm{d}s, \tag{5}$$

where k_1 and k_2 are both selectively positive numbers. Thus, according to formula (A2) in Appendix A, the operator $\mathscr{L}$ acting on the function V along with error system (4) gives

$$\begin{aligned}& \mathscr{L}V(e, e_\tau, t) \\ = {} & [-2(\alpha + c) + k_1 + k_2]e^2 \\ & + 4m_1 e \cos\frac{x_{\tau_1} + y_{\tau_1}}{2} \sin\frac{e_{\tau_1}}{2} \\ & + 4m_2 e \cos\frac{x_{\tau_2} + y_{\tau_2}}{2} \sin\frac{e_{\tau_2}}{2} - k_1 e_{\tau_1}^2 - k_2 e_{\tau_2}^2 \\ & + \mathrm{trace}(\sigma^{\mathrm{T}}(e, e_{\tau_1}, e_{\tau_2})\sigma(e, e_{\tau_1}, e_{\tau_2})).\end{aligned} \tag{6}$$

Note that an estimation for any $\varepsilon_i > 0$

$$4m_i e \cos\frac{x_{\tau_i}+y_{\tau_i}}{2}\sin\frac{e_{\tau_i}}{2} \le \varepsilon_i^{-1}m_i e^2 + \varepsilon_i m_i e_{\tau_i}^2, \quad (7)$$

where $i = 1, 2$.

Then the substitution of inequalities (3) and (7) into Eq. (6) produces the following estimation of $\mathscr{L}V(e, e_\tau, t)$:

$$\begin{aligned}\mathscr{L}V(e,e_\tau,t) &\le \kappa e^2 + (\varepsilon_1 m_1 + q_1 - k_1)e_{\tau_1}^2 \\ &\quad + (\varepsilon_2 m_2 + q_2 - k_2)e_{\tau_2}^2 \\ &= (e, e_{\tau_1}, e_{\tau_2})\Theta(e, e_{\tau_1}, e_{\tau_2})^{\mathrm{T}}, \quad (8)\end{aligned}$$

where $\kappa = -2(\alpha + c) + \varepsilon_1^{-1}m_1 + \varepsilon_2^{-1}m_2 + p + k_1 + k_2$

$$\Theta = \begin{pmatrix} \kappa & 0 & 0 \\ 0 & \varepsilon_1 m_1 + q_1 - k_1 & 0 \\ 0 & 0 & \varepsilon_2 m_2 + q_2 - k_2 \end{pmatrix}.$$

If

$$2(\alpha + c) > \varepsilon_1^{-1}m_1 + \varepsilon_2^{-1}m_2 + p + k_1 + k_2, \quad (9a)$$

$$k_i > \varepsilon_i m_i + q_i \quad (i = 1, 2), \quad (9b)$$

then matrix Θ is definitely negative and its maximal eigenvalue, denoted by $-\lambda$, is negative. Therefore, with inequality (8), we have an estimation as follows:

$$\mathscr{L}V(e, e_\tau, t) \le -\lambda(e^2 + e_{\tau_1} + e_{\tau_2}). \quad (10)$$

Then taking expectation on both sides of inequality (10), we have

$$\mathbb{E}(\mathscr{L}V) \le -\lambda\mathbb{E}e^2.$$

From this, the mean-square exponential stability can be obtained by the standard technique. That is, by the Itô formula and using expression (5), a positive constant γ is obtained such that

$$\begin{aligned}\gamma\mathbb{E}|e(t)|^2 &\le \mathbb{E}(V(t)) \\ &= \mathbb{E}(V(t_0)) + \mathbb{E}\int_{t_0}^{t}\mathscr{L}V(s)\mathrm{d}s \\ &\le \mathbb{E}(V(t_0)) - \lambda\int_{t_0}^{t}\mathbb{E}e^2(s)\mathrm{d}s.\end{aligned}$$

Then, from the Gronwall inequality, we have

$$\mathbb{E}(e^2(t)) \le \frac{\mathbb{E}(V(t_0))}{\gamma}\exp\left[-\frac{\lambda}{\gamma}(t - t_0)\right],$$

which yields the mean-square exponential stability of system (4). Then, summarizing the above argument, we obtain the following result on the sufficient condition of complete synchronization between system (1) and system (2).

Theorem 1 If there exist positive numbers $\alpha, c, p, m_i, q_i, k_i, \varepsilon_i$ $(i = 1, 2)$ such that inequalities (9a) and (9b) are satisfied, then the trivial solution of system (4) is mean-square asymptotically stable, that is, systems (1) and (2) are asymptotically synchronized for almost each of initial data.

Remark 1 Let $\varepsilon_1 = \varepsilon_2 = 1, k_1 = m_1 + q_1 + 1$, $k_2 = m_2 + q_2 + 1$, then from inequalities (9a) and (9b) we will have

$$\alpha + c > m_1 + m_2 + \frac{p + q_1 + q_2}{2} + 1. \quad (11)$$

Clearly, inequality (11) shows that both the noise and the laser intensities $m_{1,2}$ act destructively on synchronization, while the relaxation coefficient α and the coupling strength c each play a constructive role.

Remark 2 The same method developed above could be applied to the following n-tuple feedback Ikeda systems:

$$\begin{aligned}\dot{x} &= -\alpha x + m_1\sin x_{\tau_1} + m_2\sin x_{\tau_2} + \cdots + m_n\sin x_{\tau_n}, \\ \dot{y} &= -\alpha y + m_1\sin y_{\tau_1} + m_2\sin y_{\tau_2} + \cdots \\ &\quad + m_n\sin y_{\tau_n} + c(x - y) + \sigma(e, e_{\tau_1}, \ldots, e_{\tau_n})\dot{w},\end{aligned}$$

where the noise intensity matrix $\sigma(e, e_{\tau_1}, \ldots, e_{\tau_n})$ satisfies $\sigma(0, 0, \ldots, 0)$ and

$$\mathrm{trace}(\sigma^{\mathrm{T}}\sigma) \le pe^2 + \sum_{i=1}^{n} q_i e_{\tau_i}^2.$$

Taking the following Lyapunov function

$$V = e^2 + \sum_{i=1}^{n} k_i\int_{t-\tau_i}^{t} e^2(s)\mathrm{d}s,$$

one can find that the sufficient mean-square asymptotical stability conditions for the synchronization manifold $y = x$ are

$$2(\alpha + c) > p + \sum_{i=1}^{n}\varepsilon_i^{-1}m_i + \sum_{i=1}^{n} k_i, k_i > \varepsilon_i m_i + q_i,$$

where k_i, ε_i, q_i $(i = 1, \ldots, n)$ are positive numbers.

3. Generalized synchronization between the multi-feedback Ikeda systems

The above arguments show that complete synchronization may be achieved under the assumption of well-matched system parameters. Unfortunately, parameter mismatch is inevitable in practical implementations of chaos synchronization systems. From

the above analysis it is clear that $y = x$ is not the synchronization manifold for systems (1) and (2) when $m_1 \neq m_3$ and $m_2 \neq m_4$. Let $m_1 = m_2$, $m_3 = m_4$, and $r = m_1/m_3$. Now we attempt to understand the effect of the mismatch in parameters on the quality of complete synchronization by tuning r. The r is tuned from both sides of critical value ($r = r_c = 1$). The effect of such a perturbation on the quality of complete synchronization can be measured with a similarity function, defined as[35]

$$S = \frac{\langle [x(t) - y(t)]^2 \rangle}{[\langle x(t)^2 \rangle \langle y(t)^2 \rangle]^{1/2}}.$$

A global minimum of $S = S_{\min} = 0$ represents stable complete synchronization. For coupled Ikeda systems (1) and (2), r is varied from 0.5 to 1.5 for $\tau_1 = 1$, $\tau_2 = 2$, $\alpha = -5$, $m_1 = m_2 = 10$, and the corresponding S versus r plot is shown in Fig. 1, where S is calculated on the interval $[T/2, T]$ ($T = 10$ is the total length of the simulation). A sharp dipping into the minimum is observed at the critical value ($r = r_c = 1$), and the quality of synchronization decreases as $|r|$ is increased. The coupling strength c greater than the threshold value does not play any significant role in determining the quality of synchronization.

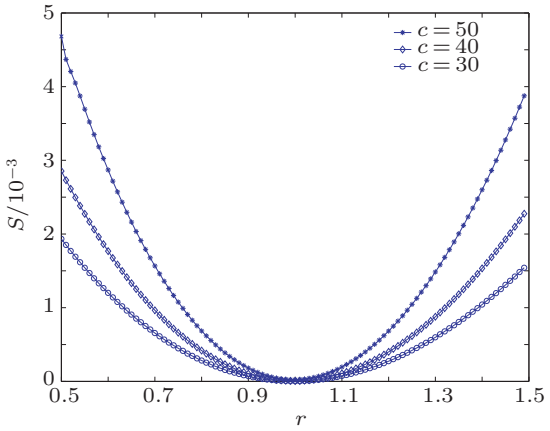


Fig. 1. (colour online) Curves for synchronization index S versus r for different values of c. The other parameters are $\alpha = 5$, $m_1 = m_2 = 10$, $\tau_1 = 1$, and $\tau_2 = 2$.

Although complete synchronization fails in the presence of parameter mismatch, we can analyse the condition for generalized synchronization according to the auxiliary system method[36] which gives another identical driven auxiliary system $z(t)$. The generalized synchronization between $x(t)$ and $y(t)$ is established with the achievement of complete synchronization between $y(t)$ and $z(t)$. In what follows we consider the complete synchronization between the following Ikeda systems:

$$\begin{aligned} \dot{y} &= -\alpha y + m_3 \sin y_{\tau_1} + m_4 \sin y_{\tau_2} \\ &\quad + c(x - y) + \sigma(e, e_{\tau_1}, e_{\tau_2})\dot{W}, \qquad (12) \\ \dot{z} &= -\alpha z + m_3 \sin z_{\tau_1} + m_4 \sin z_{\tau_2} \\ &\quad + c(x - z) + \sigma(\tilde{e}, \tilde{e}_{\tau_1}, \tilde{e}_{\tau_2})\dot{W}, \qquad (13) \end{aligned}$$

where $\tilde{e} = x - z$.

The dynamics of the error $\delta = y - z$ is given by

$$\begin{aligned} \dot{\delta} &= -(\alpha + c)\delta + 2m_1 \cos \frac{y_{\tau_1} + z_{\tau_1}}{2} \sin \frac{\delta_{\tau_1}}{2} \\ &\quad + 2m_2 \cos \frac{y_{\tau_2} + z_{\tau_2}}{2} \sin \frac{\delta_{\tau_2}}{2} \\ &\quad - \bar{\sigma}(\delta, \delta_{\tau_1}, \delta_{\tau_2})\dot{W}, \qquad (14) \end{aligned}$$

where $\bar{\sigma}(\delta, \delta_{\tau_1}, \delta_{\tau_2}) = \sigma(e, e_{\tau_1}, e_{\tau_2}) - \sigma(\tilde{e}, \tilde{e}_{\tau_1}, \tilde{e}_{\tau_2})$ is locally Lipschitz continuous and satisfies the linear growth condition with $\bar{\sigma}(0, 0, 0) \equiv 0$, and we also assume that $\mathrm{trace}(\bar{\sigma}^{\mathrm{T}}\bar{\sigma}) \leq p\delta^2 + q_1\delta_{\tau_1}^2 + q_2\delta_{\tau_2}^2$.

Applying the above approach for the case of complete synchronization, we obtain the following result for the generalized synchronization between coupled Ikeda systems (1) and (2) (or complete synchronization between systems (12) and (13)).

Theorem 2 Generalized synchronization between system (1) and system (2) could be achieved for almost each of initial data, provided that there exist positive numbers α, $m_{3,4}$, c, p, $q_{1,2}$, $k_{1,2}$, $\varepsilon_{1,2}$ such that $2(\alpha + c) > \varepsilon_1^{-1}m_3 + \varepsilon_2^{-1}m_4 + p + k_1 + k_2$, $\varepsilon_1 m_3 + q_1 < k_1$, and $\varepsilon_2 m_4 + q_2 < k_2$.

4. Simulation results

In this section, numerical simulations are provided to illustrate the effectiveness of the obtained results. Consider the parameters in Eqs. (1) and (2) as $\alpha = 5$, $m_1 = m_3 = 9$, $m_2 = m_4 = 10$, $c = 30$, $\tau_1 = 1$, $\tau_2 = 2$. Meanwhile, assume that $W = [w_1, w_2]^{\mathrm{T}}$ is a two-dimensional Brownian motion and the noise intensity matrix is in the form of $\sigma(u, v, z) = [0.3u + 0.4v, 0.5z]$. Thus,

$$\begin{aligned} &\mathrm{trace}(\sigma^{\mathrm{T}}(u, v, z)\sigma(u, v, z)) \\ &= (0.3u + 0.4v)^2 + 0.25z^2 \\ &\leq 0.18u^2 + 0.32v^2 + 0.25z^2. \end{aligned}$$

Therefore, setting $\varepsilon_1 = \varepsilon_2 = 1, k_1 = 10.32, k_2 = 11.25$ and utilizing (11) we clearly see that inequalities (9a) and (9b) are satisfied, which, due to Theorem 1, is a sufficient condition of the complete synchronization between system (1) and system (2) with the above-mentioned parameters for almost each initial function $\phi \in C^{\alpha}_{\mathscr{F}_0}([-2, 0], R)$ of error dynamics (4). In Fig. 2,

we depict different sample paths of systems (1), (2), and (4), respectively. As clearly seen from Figs. 2(a)–2(c), a complete synchronization between system (1) and system (2) is achieved. Figure 3 shows the time series of the driver system x and the driven systems y and z for $m_1 = 14$, $m_2 = 4$, $m_3 = 4$, $m_4 = 15$, and all the other parameters have the same values as those in Fig. 2. Figure 3(a) shows the generalized synchronization between x and y. In Fig. 3(b) the complete synchronization between y and z is shown.

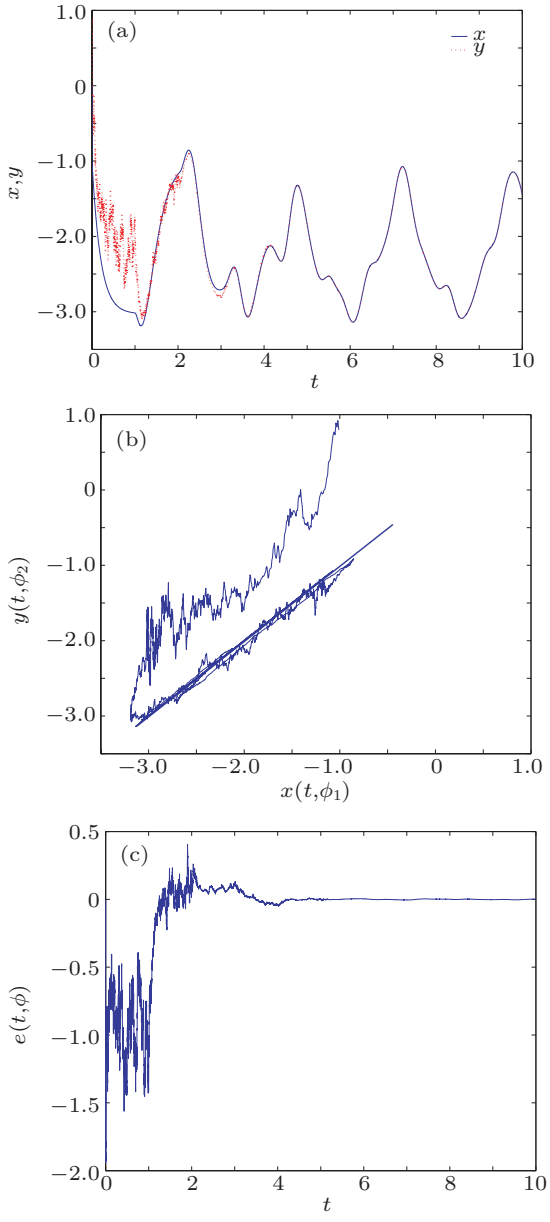


Fig. 2. (colour online) Complete synchronization between coupled Ikeda systems (1) and (2). (a) The time series of the driving system x (solid line) and the response system y (dotted line); (b) the phase plots in the x versus y plane; (c) the phase plots for the error system in the t versus e plane. The parameters are $\alpha = 5$, $m_1 = 9$, $m_2 = 10$, $c = 30$, $\tau_1 = 1$, and $\tau_2 = 2$. The initial functions are $\phi_1 = -1$, $\phi_2 = 1$, and $\phi = \phi_1 - \phi_2 = -2$.

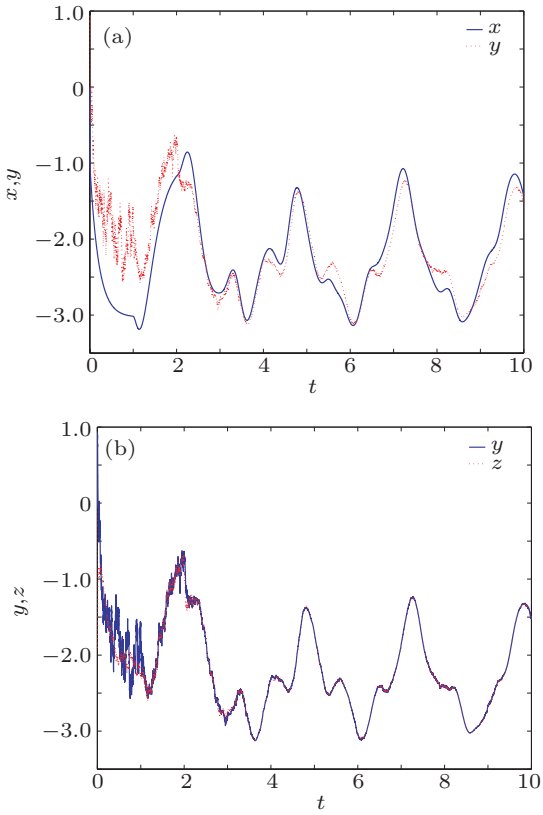


Fig. 3. Generalized synchronization between coupled Ikeda systems (1) and (2). (a) The time series of the driving system x (solid line) and the response system y (dotted line); (b) the time series of response systems y (solid line) and z (dotted line). The parameters are $\alpha = 5$, $m_1 = 14$, $m_2 = 4$, $m_3 = 4$, and $m_4 = 15$. All the other parameters have the same values as those in Fig. 2.

5. Conclusion

In this paper, we investigated complete synchronization and generalized synchronization between two unidirectionally coupled Ikeda systems with multiple time delays and noise perturbation. First we considered the complete synchronization between the double-feedback and the multi-feedback Ikeda systems. By using the Lyapunov functional, we showed that the complete synchronization can be realized. Then we studied the generalized synchronization between two unidirectionally coupled non-identical Ikeda systems and obtained the existence conditions for the generalized synchronization. Generalization of the approach to a wider class of coupled nonlinear double-feedback system is also presented. Numerical simulations of the coupled Ikeda systems fully verify our main results.

Appendix A

Consider the following n-dimensional stochastic differential delay equation:

$$\mathrm{d}x(t) = f(t, x, x_\tau)\mathrm{d}t + g(t, x, x_\tau)\mathrm{d}W, \qquad \text{(A1)}$$

on $t \geq 0$ with initial data $\{x(\theta) : -\tau \leq \theta \leq 0\} = \phi \in C^{\alpha}_{\mathscr{F}_0}([-\tau, 0], R^n)$, $f, g : R_+ \times R^n \times R^n \to R^n$ are locally Lipschitz continuous and satisfy linear growth condition. It is well known that equation (A1) has a unique solution which is denoted by $x(t, \phi)$. Furthermore, we always assume $f(t, 0, 0) = 0, g(t, 0, 0) = 0$, so equation (A1) admits a trivial solution $x(t, 0) \equiv 0$.

Let $C^{2,1}(R^n \times R_+, R_+)$ denote the family of all non-negative functions $V(x, t)$ on $R^n \times R_+$ which are continuously twice differentiable in x and once differentiable in t. For each $V \in C^{2,1}(R^n \times R_+, R_+)$, we define the operator $\mathscr{L}V$ related to Eq. (A1) from $R^n \times R^n \times R_+$ to R by

$$\mathscr{L}V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} \cdot f + \frac{1}{2}\mathrm{trace}\left[g^{\mathrm{T}} \frac{\partial^2 V}{\partial^2 x} \cdot g\right], \quad \text{(A2)}$$

where $\partial V/\partial x = (\partial V/\partial x_1, \ldots, \partial V/\partial x_n)$, and $\partial^2 V/\partial^2 x = (\partial^2 V/\partial x_i \partial x_j)_{n\times n}$.

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