Feedback control in a coupled Brownian ratchet*

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On the basis of the double-well ratchet potential which can be calculated theoretically and implemented experimentally, the influences of the time delay, the coupling constant, and the asymmetric parameter of the potential on the performance of a delayed feedback ratchet consisting of two Brownian particles coupled mutually with a linear elastic force are investigated. The centre-of-mass velocity of two coupled Brownian particles, the average effective diffusion coefficient, and the *Pe* number are calculated. It is found that the parameters are affected by not only the time delay and coupling constant but also the asymmetric parameter of the double-well ratchet potential. It is also found that the enhancement of the current may be obtained by varying the coupling constant of the system for the weak coupling case. It is expected that the results obtained here may be observed in some physical and biological systems.

Keywords: delayed feedback ratchet, double-well ratchet potential, coupling constant, center-of-mass velocity

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1. Introduction

Cells are always in a nonequilibrium state with a continuous energy transduction. The devices responsible for this transduction are the molecular motors, which can have many different functions.[1] For example, some of them can transport matter along the cell, acting as the kinesin protein motor, or can propel the cell through the extracellular medium, acting as the bacterial flagellar motor. A Brownian ratchet is such a nonequilibrium system, which rectifies thermal fluctuations in order to achieve directed transport of particles.[2−4] Net transport in an environment, dominated by thermal noise, is important for a variety of applications,[5] such as separation of polyelectrolytes,[6] moving vortices in superconductors,[7] and light-driven molecular switches.[8] Various biological processes, such as actin polymerization[9] and ion transport through membrane channels,[10] have been modeled as Brownian ratchets that can rectify thermal motion.

In most studies of Brownian ratchets, the potential is switched periodically or randomly, without regard to the particle distribution. Recently, a flashing ratchet with feedback control was introduced, where the external potential was dependent on the state of the system.[11] The instantaneous centre-of-mass velocity was maximized by turning the potential on only when the ensemble averaged force exerted by the potential was positive.

Feedback control could be implemented in systems where particles are monitored, as occurs in some experimental setups with colloidal particles.[12] Control theory is also of extreme relevance in biology,[13] and most protein motors probably operate as control systems.[14] Finally, controlled ratchets are relevant from a theoretical point of view. Delayed feedback has been demonstrated to produce complicated dynamics in chaotic, inertial ratchets,[15−17] stochastic systems that display a Hopf bifurcation in the absence of delay,[18,19] and biological systems,[20,21] and some analytical methods have recently been developed to study time delay in stochastic systems.[22−26] Here, we will study the influence of time delay on the effectiveness of the feedback control strategy introduced in Ref. [11].

One particular motor protein, kinesin, has attracted considerable attention, motivated by experimental results in which the dynamical details of its motion can be measured.[27] Kinesin is a protein with

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two heads that "walks" on the microtubules inside cells. Motivated by these experimental results, several authors[28−32] have introduced diverse models in order to understand the particular walking of kinesin. In this paper we will introduce a model inspired by the walking of motor proteins, such as kinesin on microtubules, but it is not restricted to the walking of motor proteins. The model can be used to describe the walking of macroscopic objects in the presence of fluctuations too. It consists of two particles coupled through a linear elastic force and subjected to independent white noises. Meanwhile, the system of two coupled particles is acted upon by an ensemble averaged force, due to the presence of a ratchet potential. However, in most studies of flashing ratchets, the piecewise linear sawtooth potential is used, which is one of the simplest realizations of Brownian motors in general.[3] In the present paper we will adopt the double-well ratchet potential[33,34] which can be implemented experimentally to investigate how the structure of the potential affects the current transportation of two coupled Brownian particles in the control of delayed feedback. We are interested in analysing the trajectories of the walker following in detail the motion of the two coupled particles, and we are also interested in the current or noise induced transport for this system.

The main concern of the present paper is to analyse the changes introduced in the transport property of the system by the presence of such interactions. For simplicity, the particles are assumed to lie on a line and interact through a harmonic potential with its two nearest neighbours. It is interesting to find that for an elastically coupled model, the enhancement of the center-of-mass velocity may be obtained by varying the coupling constant, and the current enhancement may also be obtained by changing the structure of the potential.

2. A delayed feedback ratchet

The delayed feedback ratchet considered here consists of two Brownian particles at temperature T_0 in a periodic potential $U(x)$. One can choose different periodic potentials according to different aims. Now, the double-well ratchet potential[33,34]

$$U(x) = -U_1 \mathrm{e}^{-\sin^2(\pi x)/2\sin^2(\pi R)} - U_2 \mathrm{e}^{-\sin^2(\pi(x-d))/2\sin^2(\pi R)} \quad (1)$$

is used to replace the piecewise linear sawtooth potential often used in the literature and its schematic diagram is shown in Fig. 1. In Eq. (1) U_1 and U_2 are the depths of the stronger and the weaker wells, respectively, which are separated by a distance $d = 0.36$, and each well has a width $R = 0.15$. The asymmetry parameter of the potential is $\beta = U_1/U_2 = 10/9$, x is the position, and the ratchet period $L = 1$. The parameters U_1 and U_2 have the same energy unit, and d, R, x, and L all have the same length unit, and the ratio of one of them to another is always a dimensionless parameter.

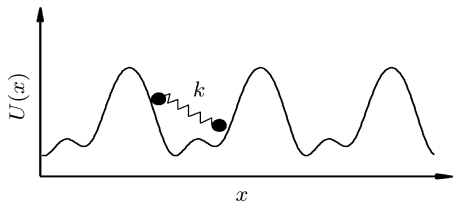


Fig. 1. Schematic diagram of the double-well ratchet potential $U(x)$.

The force acting on the particles is $F(x) = -U'(x)$, where the prime denotes the spatial derivative. We neglect the inertia effect here. Thus, the state of this system can be described by the positions $x_i(t)$ $(i = 1, 2)$ of the two coupled particles satisfying the overdamped Langevin equations. Since the boundary condition of particles is free, the Langevin equations are written as

$$\gamma \dot{x}_1(t) = \alpha(t) F(x_1(t)) + k(x_2 - x_1 - l) + \xi_1(t), \quad (2)$$

$$\gamma \dot{x}_2(t) = \alpha(t) F(x_2(t)) - k(x_2 - x_1 - l) + \xi_2(t), \quad (3)$$

and the average force acting on each particle is

$$f(t) = \frac{1}{2}\sum_{i=1}^{2} F(x_i(t)) = \frac{1}{2}[F(x_1(t)) + F(x_2(t))], \quad (4)$$

where γ is the friction coefficient (related to the diffusion coefficient D through Einstein's relation $D = k_\mathrm{B}T_0/\gamma$), $\xi_i(t)$ are Gaussian white noises of zero mean and variance $\langle \xi_i(t)\xi_j(t')\rangle = 2\gamma k_\mathrm{B}T_0\delta_{ij}\delta(t-t')$. The two particles interact through a harmonic potential, so k is the coupling constant and l is the natural length of the spring. The variable $\alpha(t)$ denotes the action of the controller. There is a controller in the system, which measures the sign of the average force and, after a time τ, switches the potential on $(\alpha = 1)$ if the ensemble average of the force is positive or switches the potential off $(\alpha = 0)$ if it is negative. Thus, the

delayed control protocol considered here may be expressed as

$$\alpha(t) = \begin{cases} \Theta(f(t-\tau)), & t \geq \tau, \\ 0, & \text{otherwise}, \end{cases} \tag{5}$$

where Θ is the Heaviside function [$\Theta(x) = 1$ if $x > 0$, else $\Theta(x) = 0$].

It is worthwhile to point out that the model adopted here is more general than those in Refs. [33] and [34], and the important results in these references can be directly derived from the case of the coupling constant $k = 0$ in the present paper. It is interesting to note that for the case of $k = 0$, although there are no explicit mechanical interactions between the particles, the use of information about the control of the system introduces a coupling between the particles, in that the force acting on each particle depends on the position of another particle. For this reason, for closed-loop strategies[35–38] the average velocity is dependent on the number of particles in the ensemble, in contrast to open-loop policies[39] which, for non-interacting particles, output the same velocity regardless of the ensemble size.[40]

The first basic quantity of interest in the system is the centre-of-mass velocity of two Brownian particles, and given by the relation[41]

$$V_{\rm cm} = \lim_{T\to\infty} \frac{1}{2T} \sum_{i=1}^{2} \int_0^T \dot{x}_i(t)\, {\rm d}t. \tag{6}$$

In order to quantify the transport coherence of the coupled Brownian particles, we introduce the Pe number,[32,42,43] i.e.,

$$Pe = \frac{|V_{\rm cm}|\, L}{D_{\rm eff}}, \tag{7}$$

where $D_{\rm eff}$ is the average effective diffusion coefficient determined by

$$D_{\rm eff} = \lim_{T\to\infty} \sum_{i=1}^{2} \frac{\left\langle x_i(t)^2 \right\rangle - \langle x_i(t) \rangle^2}{2 \cdot 2T}. \tag{8}$$

The Pe number describes the competition between the directional drift and the stochastic diffusion of the particle. The directional drift will increase with the increase of the Pe number. Thus, the larger Pe number means that the drift predominates over the diffusion and there is high transport coherence.

The numerical solution of Eqs. (2) and (3) is performed by using the stochastic Runge–Kutta algorithm. The transport processes of two coupled particles are simulated and each trajectory consists of 10^5 steps of an integral with the small time step of $h = 10^{-3}$. In the following calculations, the parameters are set to be $L = 1$, $d = 0.36$, $R = 0.15$, $U_1 = 3.2 k_{\rm B} T_0$, $\beta = U_1/U_2 = 10/9$, $\gamma = 1$, $D = 1$, $l = 0.01$, and $k_{\rm B} T_0 = 1$ if not mentioned, where the unit of energy is Joule.

3. Results and discussion

3.1. Influence of the delay time

Figure 2(a) shows the curves of $V_{\rm cm}$ varying with delay time τ for different coupling constants. It is seen that the centre-of-mass velocity of the system is a monotonic function of delay time τ but not a monotonic function of coupling constant for small τ. The correlation between the present sign of the average force and the measured sign used actually in the controller decreases with the increase of time delay. It can be understood that the decrease in the centre-of-mass velocity is a consequence of the loss of information about the present sign of the average force. Thus,

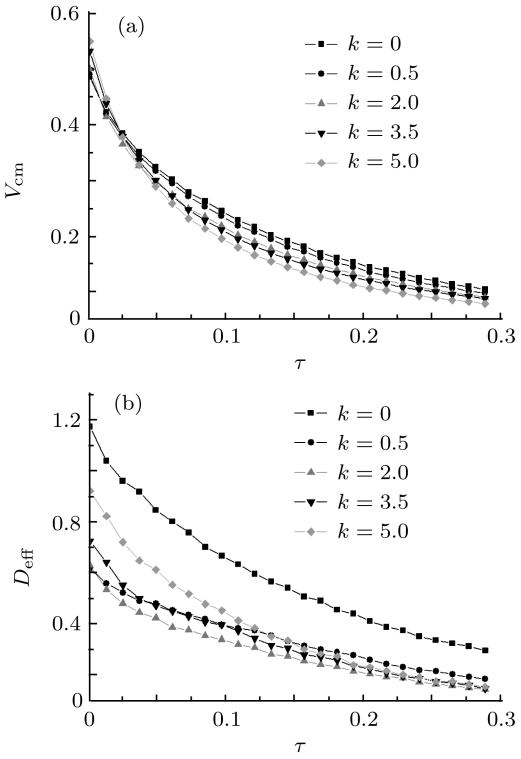


Fig. 2. Curves of (a) centre-of-mass velocity $V_{\rm cm}$ and (b) average effective diffusion coefficient $D_{\rm eff}$ varying with the time delay τ for different values of coupling constant k.

the controller action begins to be uncorrelated to the present state of the system and it effectively begins to act as an open-loop ratchet.[37] The result is compatible with that obtained in Refs. [37] and [38]. In addi-

tion, it can be clearly seen that the magnitude of the net current is dependent on the coupling constant for a small delay time. We will discuss the phenomenon in detail below.

Figure 2(b) shows the curves of effective diffusion coefficient $D_{\rm eff}$ of two coupled Brownian particles, as a function of delayed time τ, for different coupling constants. It is easily found that the value of effective diffusion coefficient $D_{\rm eff}$ of the Brownian particles decreases monotonically with delayed time τ. For the uncoupled case, particles are easier to diffuse, but for the coupling case, it is difficult for particles to diffuse, which means that the fluctuation of displacement of coupled particles decreases with the increase of coupling constant. However, it is found that $D_{\rm eff}$ is not a monotonic function of the coupling constant.

3.2. Influence of the coupling constant

Figure 3(a) shows the centre-of-mass velocity as a function of the coupling constant k for different values of delay time τ. With the increase of delay time, the centre-of-mass velocity decreases. This result is compatible with that obtained above. For the case of $k > 5$, it is easily found that the centre-of-mass velocity is almost independent of the spring constant and the velocity approaches a constant. This means that the influence of the spring is weak in comparison with that of the delay time for strong coupling. However, for the small coupling constant ($k < 5$), one can observe an enhancement of the centre-of-mass velocity. This means that a suitable spring constant can enhance the current for weak coupling. Recently, it has been revealed in various systems[44−48] that the behaviour of the coupling constant can facilitate directed transport.

In order to understand the phenomenon deeply, we calculate $D_{\rm eff}$ as a function of coupling constant k for different delay times, as shown in Fig. 3(b). It is interesting to find that for a small coupling constant, the effective diffusion coefficient $D_{\rm eff}$ exhibits a peak, and the height of the peak decreases with the increase of the delay time. This indicates once more that an optimal coupling constant may lead to the most efficient directed transport for the weak coupled case. When the coupling is strong, i.e., $k > 5$, the effective diffusion coefficient $D_{\rm eff}$ does not vary obviously with the increase of k.

Figure 3(c) shows the curves for Pe number as a function of k for different delay times. It can be found from Fig. 3(c) that the Pe number is directly proportional to the value of $V_{\rm cm}$ since $D_{\rm eff}$ may be considered to be constant for a fixed delay. The results show that the Pe can obtain a maximal value for weak coupling, which means that the coupling between the two coupled particles can cause high transport coherence. It is also found that the maximal value of the Pe number depends on the maximal velocity of particles in the case of weak coupling. It indicates once again that the larger the delay time, the more difficult the diffusion of particles is, thus, the smaller the effective diffusion coefficient, the slower the decrease speed of the effective diffusion coefficient is.

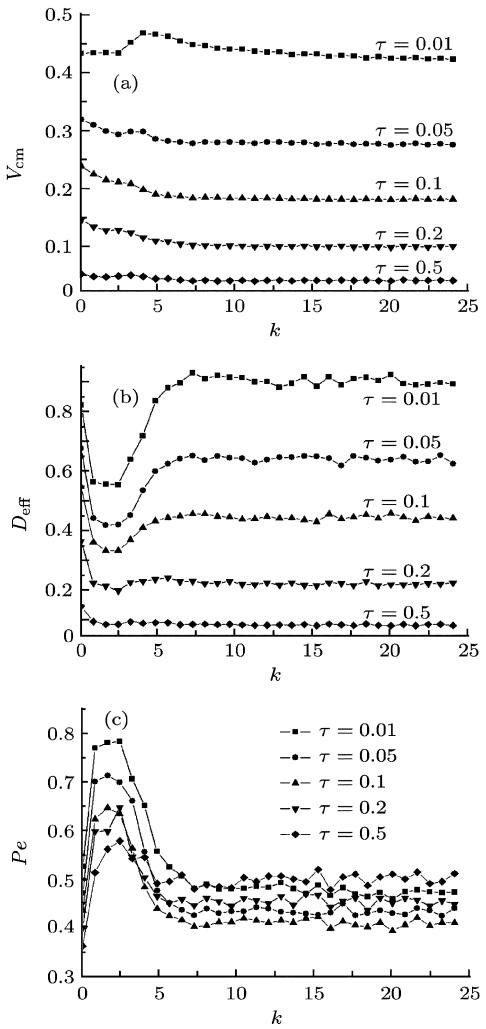


Fig. 3. Curves of (a) centre-of-mass velocity $V_{\rm cm}$, (b) average effective diffusion coefficient $D_{\rm eff}$, and (c) Pe number varying with the coupling constant k for different values of delay time τ.

3.3. Influence of the asymmetry parameter

Figure 4 shows the dependences of the centre-of-mass velocity on the asymmetry parameter of poten-

tial β for some given coupling constants. It is clearly seen that $V_{\rm cm}$ is not a monotonic function of β so that there is an optimized value of β at which the centre-of-mass velocity attains its maximum. When the asymmetry of the potential β is large, the centre-of-mass velocity decreases with the increase of β. It is seen that the current enhancement may also be obtained by changing the structure of the potential.

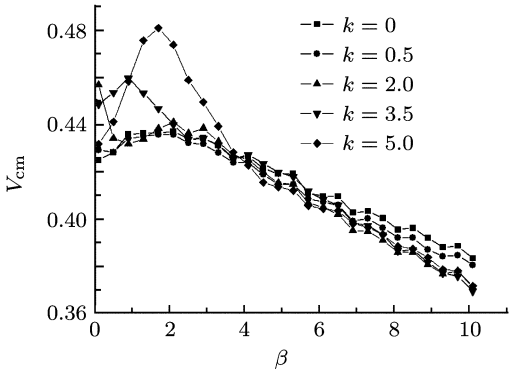


Fig. 4. Curves for centre-of-mass velocity $V_{\rm cm}$ as a function of asymmetry parameter of potential β for different values of coupling constant k.

4. Conclusion

We systematically investigated the performance of a feedback ratchet consisting of two coupled Brownian particles confined in the double-well ratchet potential and also discussed in detail the influences of the time delay, the coupling constant, and the asymmetric parameter of the potential on the centre-of-mass velocity of Brownian particles, average effective diffusion coefficient, and the Pe number of a delayed feedback ratchet. It is interesting to find that the enhancement of the centre-of-mass velocity may be obtained by varying the coupling constant of the system for the weak coupling case and that the current enhancement may also be obtained by changing the structure of the potential. The theoretical results obtained here are significant because they may be observed in some physical and biological systems by using the experimentally realizable double-well ratchet potential.

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