Stochastic bounded consensus tracking of leader–follower multi-agent systems with measurement noises and sampled-data*

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(Received 27 April 2012; revised manuscript received 5 June 2012)

This paper is concerned with the stochastic bounded consensus tracking problems of leader–follower multi-agent systems, where the control input of an agent can only use the information measured at the sampling instants from its neighbours or the virtual leader with a time-varying reference state, and the measurements are corrupted by random noises. The probability limit theory and the algebra graph theory are employed to derive the necessary and sufficient conditions guaranteeing the mean square bounded consensus tracking. It is shown that the maximum allowable upper boundary of the sampling period simultaneously depends on the constant feedback gains and the network topology. Furthermore, the effects of the sampling period on the tracking performance are analysed. It turns out that from the view point of the sampling period, there is a trade-off between the tracking speed and the static tracking error. Simulations are provided to demonstrate the effectiveness of the theoretical results.

Keywords: leader–follower multi-agent systems, stochastic bounded consensus tracking, measurement noises, sampled-data

PACS: 89.75.–k, 05.65.+b, 02.30.Yy, 05.40.Ca **DOI:** 10.1088/1674-1056/21/12/128902

1. Introduction

In recent years, consensus of multi-agent systems, due to their broad applications, including swarming, flocking, rendezvous, formation control, distributed sensor networks, and so on, has attracted a great deal of attention. For most of the existing consensus protocols,[1−5] the final common value to be achieved is a function of initial states of all the agents, and it is inherently a prior unknown constant. This is the so-called χ-consensus.[6] However, in real applications, it is often required that all the agents converge to a desired common value, which evolves with time, as occurs in the consensus tracking, where all the agents exchange their own state information with their neighborhood agents with the common goal of tracking a time-varying reference state that is available to only a portion of agents. Thus, the χ-consensus protocols are ineffective to directly deal with the consensus tracking of a desired time-varying reference state. To solve this problem, consensus protocols of leader–follower multi-agent systems are proposed, and up to now, significant attention has been paid to the consensus tracking of leader–follower multi-agent systems.[7−17] Jadbabaie *et al.*[7] considered the nearest neighbourhood principle and proved that if all the agents are jointly connected with their leader, then their states will converge to the state of the leader as time goes on. Ren and Beard[8] extended the results of Ref. [7] to the directed topology case and gave some more relaxed topology conditions. In Ref. [9], Ren proved that proportional-like continuous-time consensus protocols cannot guarantee that all the agents track a virtual leader with a time-varying reference state that is available to only a subset of agents, whereas proportional and derivative-like continuous-time consensus

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61203147, 60973095, 60804013, and 61104092), the Fundamental Research Funds for the Central Universities, China (Grant Nos. JUSRP111A44, JUSRP21011, and JUSRP11233), the Foundation of State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology (HUST), China (Grant No. DMETKF2010008), and the Humanities and Social Sciences Youth Funds of the Ministry of Education, China (Grant No. 12YJCZH218).

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protocols can do.

Most researches on the consensus tracking of leader–follower multi-agent systems in the above literature assume that each agent measures the states of its neighbours and the leader accurately. Obviously, this assumption is only an ideal approximation for real communication channels, since real networks are often in uncertain communication environments with various source noises, channel noises, and sink noises. Thus, it is necessary to study the effects of measurement noises on the consensus tracking of leader–follower multi-agent systems. Recently, the consensus of leaderless multi-agent systems with measurement noises has been widely studied in Refs. [18]–[24]. Ren *et al.*[18] and Kingston *et al.*[19] introduced the time-varying consensus gains, designed the consensus protocols based on a Kalman filter structure, and proved that when there is no noise, the protocols designed can ensure consensus to be achieved asymptotically. Xiao *et al.*[20] considered the average consensus of first-order discrete-time multi-agent systems with additive input noises and designed the optimal weighted adjacent matrix to minimize the static mean square error. Since the consensus gain and the adjacent matrix are both time-invariant, the average state will diverge with the probability state as time goes on, even if the noises are bounded. In Ref. [21], the stochastic approximation type protocols with the decreasing consensus gains were employed to guarantee the mean square or almost sure consensus of first-order discrete-time multi-agent systems with measurement noises. In Ref. [22], Li and Zhang considered the average consensus of first-order discrete-time multi-agent systems with measurement noises and time-varying topologies, and proved that if the network switches between jointly-containing-spanning-tree, instantaneously balanced graphs, then the designed protocol can guarantee that each individual state converges, both almost surely and in mean square, to a common random variable, whose expectation is the average of the initial states of all the agents. In Ref. [23], Li and Zhang investigated the average consensus of first-order continuous-time multi-agent systems with measurement noises and obtained the convergence conditions guaranteeing the mean square consensus. However, compared with the consensus of leaderless multi-agent systems with measurement noises, the consensus tracking of leader–follower multi-agent systems with measurement noises has attracted only a little attention.[25−28] Huang and Hanton[25] considered the consensus of discrete-time leader–follower multi-agent systems with measurement noises, and derived the sufficient conditions guaranteeing the stochastic approximation type protocols with the decreasing consensus gains to reach the mean square consensus by using the stochastic Lyapunov analysis. Ma *et al.*[26] investigated the consensus of continuous-time leader–follower multi-agent systems with measurement noises, and obtained sufficient conditions for the mean square consensus by employing the stochastic analysis and the algebra graph theory. In Ref. [27], Hu and Feng developed a distributed tracking control scheme with distributed estimators based on a novel velocity decomposition technology for a continuous-time leader–follower multi-agent system with measurement noises, proving that the closed loop tracking control system is stochastically stable in a mean square and the estimation errors converge to zero in a mean square as well.

For most of the work in the above literature, continuous-time consensus protocols were used for guaranteeing the consensus of multi-agent systems with continuous-time dynamics, and discrete-time consensus protocols were used for guaranteeing the consensus of multi-agent systems with discrete-time dynamics. However, due to the application of digital sensors and controllers, in many cases, though the system itself is a continuous process, only sampled-data at discrete sampling instants are available for the synthesis of control laws. Compared with a full-state information transmission, which might result in high communication traffic, the transmission of the state information at the sampling instants can effectively save the bandwidth of networks and communication cost. Considering the broad applications of digital communication and control in distributed systems, it is of great significance to study the consensus of multi-agent systems based on sampled-data. Up to now, considerable efforts have been devoted to the sampled-data consensus of leaderless multi-agent systems without measurement noises,[29−32] and only a little attention has been paid to the sampled-data consensus of leaderless multi-agent systems with measurement noises,[33] and the sampled-data consensus of leader–follower multi-agent systems without measurement noises.[34] To the best of our knowledge, there is no published report on the sampled-data consensus tracking of leader–follower multi-agent systems with measurement noises.

Based on the above considerations, it is necessary to study the consensus tracking problems of leader–follower multi-agent systems with measurement noises based on sampled-data. To this end, in this paper, we investigate the stochastic bounded consensus track-

ing problems of leader–follower multi-agent systems, where the control input of an agent can only use the information measured at the sampling instants from its neighbours and the virtual leader with a time-varying reference state, and the measurements are corrupted by random noises.

The remainder of this paper is organized as follows. In Section 2, some preliminaries are provided and the problem is formulated. Convergence analysis of the mean square bounded consensus tracking protocols is investigated in Section 3. Numerical simulations are provided in Section 4. Finally, concluding remarks are stated in Section 5.

2. Preliminaries and problem statement

2.1. Algebra graph theory and some notations

Let $G = (V, E, A)$ be a weighted undirected graph with a set of nodes $V = \{v_1, v_2, \ldots, v_N\}$, a set of edges $E \subseteq V \times V$, and the weighted adjacent matrix $A = [a_{ij}]$ with nonnegative adjacent elements a_{ij}. The node indexes of G belong to a finite index set $I = \{1, 2, \ldots, N\}$. An edge of G is denoted by $e_{ij} = (v_i, v_j)$. The adjacent elements associated with the edges are positive, i.e., $e_{ij} \in E \Leftrightarrow a_{ij} > 0$. Moreover, we assume $a_{ii} = 0$ for all $i \in I$. Assume that the considered graph is undirected, which means $e_{ij} \in E \Leftrightarrow e_{ji} \in E$. Thus, the adjacent matrix A is symmetric. The set of neighbours of node v_i is denoted by $N_i = \{v_j \in V : e_{ij} \in E\}$. The degree of node v_i is defined as $d_i = \sum_{j \in N_i} a_{ij}$. The Laplacian matrix of G is defined as $L = D - A$, where $D = \mathrm{Diag}(d_1, d_2, \ldots, d_N)$ is the degree matrix of G with diagonal elements d_i and zero off-diagonal elements. An important fact of L is that all the row sums are zero and thus L has a right eigenvector 1_N associated with the zero eiginvalue, where 1_N denotes the N-dimensional column vector with all ones. A path between two distinct nodes v_i and v_j is meant by a sequence of distinct edges of the form $(v_i, v_{k_1}), (v_{k_1}, v_{k_2}), \ldots, (v_{k_l}, v_j)$. A graph is called "connected" if there is a path between any two distinct nodes of the graph. An important property of the Laplacian matrix, which is instrumental in the convergence analysis of the consensus protocols, is that the graph G is connected if and only if $\mathrm{rank}(L) = N - 1$. Thus, for a connected graph G, L has one and only one zero eigenvalue and all the other eigenvalues of L are positive and real.

The following notations will be used throughout this paper. 0_N denotes the N-dimensional column vector with all zeros. I_N and $0_{N\times N}$ denote the N-dimensional identity matrix and the N-dimensional matrix with all zeros, respectively. For a given vector or matrix M, M^{T} and $\|M\|_2$ denote its transpose and 2-norm, respectively. For a given matrix M, $\det(M)$ and $\lambda_i(M), i \in I$ denote its determinant and N eigenvalues. For a given random variable X, $E(X)$ denotes its mathematical expectation.

2.2. Problem statement

In a multi-agent system with N agents, an agent and an available information flow between two agents are considered as a node and an edge in an undirected graph, respectively. Consider the system of dynamic agents described by

$$\dot{x}_i(t) = u_i(t),\ i \in I, \tag{1}$$

where $x_i(t) \in R$ and $u_i(t) \in R$ represent, respectively, the state of the i-th agent and the associated control input called the consensus protocol.

In order to guarantee that all the agents track the virtual leader with the desired reference state $x^*(t)$, in Ref. [26] Ma *et al.* proposed the following continuous-time consensus tracking protocol

$$u_i(t) = a(t)\Big[-\alpha_i(x_i(t) - x_i^*(t)) + \sum_{j\in N_i} a_{ij}(x_{ij}(t) - x_i(t))\Big],\ i \in I, \tag{2}$$

where $a(\cdot) : [0, +\infty) \to [0, +\infty)$ is a piecewise continuous function called a time-varying consensus gain, the constant feedback gain α_i is positive if the i-th agent has access to the virtual leader and 0 otherwise, $x_i^*(t) = x^*(t) + \rho_i\eta_i(t)$ and $x_{ij}(t) = x_j(t) + \sigma_{ij}\omega_{ij}(t)$, $j \in N_i$ denote, respectively, the state measurements that the i-th agent receives from the virtual leader and the j-th agent, $\{\eta_i(t) : i \in I\}$ and $\{\omega_{ij}(t) : j \in N_i, i \in I\}$ are two mutually independent standard Gaussian white noises, and $\rho_i > 0$ and $\sigma_{ij} > 0$ represent, respectively, the intensities of $\eta_i(t)$ and $\omega_{ij}(t)$.

In this paper, in a way different from that in Ref. [26], we use the consensus tracking protocol (2) without the time-varying consensus gain $a(t)$,

$$u_i(t) = -\alpha_i(x_i(t) - x_i^*(t)) + \sum_{j\in N_i} a_{ij}(x_{ij}(t) - x_i(t)),\ i \in I. \tag{3}$$

Using the period sampling technology and zero-order hold circuit, the following consensus tracking

protocol based on sampled-data is induced from the continuous-time consensus tracking protocol (3)

$$\begin{aligned} u_i(t) = & -\alpha_i(x_i(kh) - x_i^*(kh)) \\ & + \sum_{j\in N_i} a_{ij}(x_{ij}(kh) - x_i(kh)), \\ & t \in [kh, kh+h). \end{aligned} \tag{4}$$

Definition 1 The system (1) under the protocol (4) is said to achieve the mean square bounded consensus tracking, if the system (1), using protocol (4), has the following property

$$\lim_{t\to\infty} E((x_i(t) - x^*(t))^2) \le C < \infty, \forall i \in I, \tag{5}$$

where C is a bounded positive constant that is independent of t.

In order to carry out the convergence analysis of the consensus tracking protocol (4), we need to provide the following lemma.

Lemma 1 If the network topology composed of N agents is fixed, undirected and connected, and at least one agent has the access to the virtual leader, then the matrix $(\Delta + L)$ must be positive-definite, where $\Delta = \mathrm{Diag}(\alpha_1, \ldots, \alpha_N)$.[35]

3. Convergence analysis

In this section, employing the probability limit theory and the algebra graph theory, we derive the necessary and sufficient conditions that guarantee that the multi-agent system (1), using the consensus tracking protocol (4), reaches the mean square bounded consensus tracking. The main results are shown in the following theorem.

Theorem 1 Assume that the communication network topology G composed of N agents is fixed, undirected and connected, at least one agent has the access to the virtual leader, and the time-varying reference state of the virtual leader satisfies

$$\Big| \frac{x^*(kh+h) - x^*(kh)}{h} \Big| \le \xi^* < \infty, \quad \forall h > 0,\ k = 0, 1, 2, \ldots,$$

i.e., the changing rate of $x^*(kh)$ is bounded, then the multi-agent system (1), using the consensus tracking protocol (4), will achieve the mean square bounded consensus tracking if and only if

$$0 < h < \frac{2}{\max_{i\in I}\{\lambda_i(\Delta + L)\}}. \tag{6}$$

Proof Using the period sampling technology and zero-order hold circuit, the discrete-time model of system (1) becomes

$$\begin{aligned} x_i(kh+h) = x_i(kh) + hu_i(kh), \\ k = 0, 1, 2, \ldots;\ i \in I. \end{aligned} \tag{7}$$

Substituting protocol (4) into system (7) yields the dynamics equation at the sampling instants:

$$\begin{aligned} x_i(kh+h) = & x_i(kh) - h\alpha_i(x_i(kh) - x_i^*(kh)) \\ & + h\sum_{j\in N_i} a_{ij}(x_{ij}(kh) - x_i(kh)), \\ & k = 0, 1, 2, \ldots;\ i \in I. \end{aligned} \tag{8}$$

Notice $L1_N = 0_N$ and denote

$$\begin{aligned} x(kh) &= (x_1(kh), x_2(kh), \ldots, x_N(kh))^{\mathrm{T}}, \\ n(kh) &= (n_1(kh), n_2(kh), \ldots, n_N(kh))^{\mathrm{T}}, \end{aligned}$$

and

$$n_i(kh) = \alpha_i\rho_i\eta_i(kh) + \sum_{j\in N_i} a_{ij}\sigma_{ij}\omega_{ij}(kh),\ i \in I,$$

then we can rewrite the dynamics equation (8) in a compact form

$$\begin{aligned} x(kh+h) = & x(kh) - h(\Delta + L)(x(kh) - x^*(kh)1_N) \\ & + hn(kh),\ k = 0, 1, 2, \ldots. \end{aligned} \tag{9}$$

Denote $\tilde{x}(kh) = x(kh) - x^*(kh)1_N$ and $P = I - h(\Delta + L)$. From Eq. (9) we obtain the dynamics equation about the tracking error vector $\tilde{x}(kh)$ at the sampling instants

$$\begin{aligned} \tilde{x}(kh+h) = & P\tilde{x}(kh) - (x^*(kh+h) - x^*(kh))1_N \\ & + hn(kh). \end{aligned} \tag{10}$$

From Eq. (10), we have

$$\begin{aligned} \tilde{x}(nh) = & P^n\tilde{x}(0) + \sum_{k=0}^{n-1} P^{n-k-1}\big[-(x^*(kh+h) \\ & - x^*(kh))1_N + hn(kh)\big]. \end{aligned} \tag{11}$$

It follows from Eq. (11) that

$$\begin{aligned} E(\|\tilde{x}(nh)\|_2^2) &\le 2\|P\|_2^{2n}\|\tilde{x}(0)\|_2^2 + 2E\left(\left\|\sum_{k=0}^{n-1} P^{n-k-1}\Big[-(x^*(kh+h) - x^*(kh))1_N + hn(kh)\Big]\right\|_2^2\right) \\ &\le 2\|P\|_2^{2n}\|\tilde{x}(0)\|_2^2 + 4\left\|\sum_{k=0}^{n-1} P^{n-k-1}(x^*(kh+h) - x^*(kh))1_N\right\|_2^2 + 4h^2E\left(\left\|\sum_{k=0}^{n-1} P^{n-k-1}n(kh)\right\|_2^2\right). \end{aligned} \tag{12}$$

From Lemma 1, on the assumption that the network topology composed of N agents is fixed, undirected and

connected, and that at least one agent has access to the virtual leader, we have

$$\lambda_i(\Delta + L) > 0,\ \forall i \in I. \tag{13}$$

Notice that the matrix P is symmetric, thus

$$\|P\|_2 = \max_{i\in I}\{|1 - h\lambda_i(\Delta + L)|\}, \tag{14}$$

which together with Eq. (13) further yields that

$$\|P\|_2 < 1, \tag{15}$$

which is equivalent to $-1 < 1 - h\lambda_i(\Delta + L) < 1, \forall i \in I$, i.e., condition (6). Thus, under the given assumption and conditions we have the following estimations

$$\begin{aligned}\Big\|\sum_{k=0}^{n-1} P^{n-k-1}(x^*(kh+h) - x^*(kh))1_N\Big\|_2^2 &\le \left(\sum_{k=0}^{n-1}\|P\|_2^{n-k-1}\Big\|(x^*(kh+h) - x^*(kh))1_N\Big\|_2\right)^2\\ &\le Nh^2{\xi^*}^2\left(\frac{1-\|P\|_2^{n-1}}{1-\|P\|_2}\right)^2\end{aligned} \tag{16}$$

and

$$E\left(\left\|\sum_{k=0}^{n-1} P^{n-k-1}n(kh)\right\|_2^2\right) \le \sum_{k=0}^{n-1}\|P\|_2^{2(n-k-1)}E(\|n(nh)\|_2^2) \le \sum_{i\in I}\Big(\alpha_i^2\rho_i^2 + \sum_{j\in N_i}a_{ij}^2\sigma_{ij}^2\Big)\frac{1-\|P\|_2^{2(n-1)}}{1-\|P\|_2^2}. \tag{17}$$

Combining inequalities (12), (15), (16), and (17) gives

$$\begin{aligned}\limsup_{n\to\infty} E(\|\tilde{x}(nh)\|_2^2) &\le 2\|\tilde{x}(0)\|_2^2\lim_{n\to\infty}\|P\|_2^{2n} + 4Nh^2{\xi^*}^2\lim_{n\to\infty}\left(\frac{1-\|P\|_2^{n-1}}{1-\|P\|_2}\right)^2\\ &\quad + 4h^2\sum_{i\in I}\Big(\alpha_i^2\rho_i^2 + \sum_{j\in N_i}a_{ij}^2\sigma_{ij}^2\Big)\lim_{n\to\infty}\frac{1-\|P\|_2^{2(n-1)}}{1-\|P\|_2^2}\\ &= \frac{4Nh^2{\xi^*}^2}{(1-\|P\|_2)^2} + \frac{4h^2}{1-\|P\|_2^2}\sum_{i\in I}\Big(\alpha_i^2\rho_i^2 + \sum_{j\in N_i}a_{ij}^2\sigma_{ij}^2\Big).\end{aligned} \tag{18}$$

Substituting protocol (4) into system (1) yields the continuous-time dynamics equation

$$\dot{x}(t) = -(\Delta + L)\tilde{x}(kh) + n(kh),\ t \in [kh, kh+h). \tag{19}$$

Integrating both sides of Eq. (19) from kh to t results in

$$x(t) - x(kh) = (t-kh)[-(\Delta + L)\tilde{x}(kh) + n(kh)],\ t \in [kh, kh+h),$$

which further leads to

$$\begin{aligned}&\lim_{t\to\infty}\sup_{kh\le t<kh+h} E\big(\|x(t) - x(kh)\|_2^2\big)\\ &\le 2h^2\left\{\|\Delta + L\|_2^2\left[\frac{4Nh^2{\xi^*}^2}{(1-\|P\|_2)^2} + \frac{4h^2}{1-\|P\|_2^2}\sum_{i\in I}\Big(\alpha_i^2\rho_i^2 + \sum_{j\in N_i}a_{ij}^2\sigma_{ij}^2\Big)\right] + \sum_{i\in I}\Big(\alpha_i^2\rho_i^2 + \sum_{j\in N_i}a_{ij}^2\sigma_{ij}^2\Big)\right\}.\end{aligned} \tag{20}$$

Combining Eqs. (18) and (20) gives

$$\begin{aligned}&\limsup_{t\to\infty} E(\|x(t) - x^*(t)1_N\|_2^2)\\ &= \limsup_{t\to\infty} E(\|(x(t) - x(kh)) + \tilde{x}(kh) + (x^*(kh)1_N - x^*(t)1_N)\|_2^2)\\ &\le \limsup_{t\to\infty} E([\|(x(t) - x(kh)) + \tilde{x}(kh)\|_2 + \|x^*(kh)1_N - x^*(t)1_N\|_2]^2)\\ &= \limsup_{t\to\infty} E(\|(x(t) - x(kh)) + \tilde{x}(kh)\|_2^2)\\ &\le 2\limsup_{t\to\infty} E(\|x(t) - x(kh)\|_2^2) + 2\limsup_{t\to\infty} E(\|\tilde{x}(kh)\|_2^2)\\ &\le \frac{8Nh^2{\xi^*}^2(1+2h^2\|\Delta + L\|_2^2)}{(1-\|P\|_2)^2} + 4h^2\left[1 + \frac{2(1+2h^2\|\Delta + L\|_2^2)}{1-\|P\|_2^2}\right]\sum_{i\in I}\Big(\alpha_i^2\rho_i^2 + \sum_{j\in N_i}a_{ij}^2\sigma_{ij}^2\Big) \triangleq C,\end{aligned} \tag{21}$$

which further leads to expression (5).

According to Definition 1, the multi-agent system (1), using the sampled-data based consensus tracking protocol (4), achieves the mean square bounded consensus tracking. The proof is completed.

Remark 1 Different from most of the existing consensus protocols of multi-agent systems with measurement noises, the consensus tracking protocol (4) dose not use the time-varying consensus gains to reduce the effects of measurement noises on the consensus convergence of closed-loop systems. It can also be seen from the proof of Theorem 1 that measurement noises indeed do not destroy the consensus convergence of closed-loop systems, which only affect the static tracking performance (tracking error). The main reason is that all the eigenvalues of $(\Delta + L)$ are positive on the assumed condition that the communication network topology G composed of N agents is fixed, undirected and connected, and at least one agent has access to the virtual leader.

Remark 2 It follows from Eq. (14) that if

$$0 < h < \frac{1}{\max_{i\in I}\{\lambda_i(\Delta + L)\}},$$

which satisfies inequality (6), then

$$\begin{aligned}\|P\|_2 &= 1 - h\min_{i\in I}\{\lambda_i(\Delta + L)\}\\ &< 1 - \frac{\min_{i\in I}\{\lambda_i(\Delta + L)\}}{\max_{i\in I}\{\lambda_i(\Delta + L)\}} < 1.\end{aligned}$$

In this case, $\|P\|_2$ decreases with the increase of h. This means that increasing the sampling period can accelerate the convergence speed of achieving the consensus tracking. However, from inequality (21) we obtain that if

$$0 < h < \frac{1}{\max_{i\in I}\{\lambda_i(\Delta + L)\}},$$

then

$$C = \frac{8N{\xi^*}^2(1 + 2h^2\|\Delta + L\|_2^2)}{(\min_{i\in I}\{\lambda_i(\Delta + L)\})^2} + \left[4h^2 + \frac{8h(1 + 2h^2\|\Delta + L\|_2^2)}{\min_{i\in I}\{\lambda_i(\Delta + L)\}(2 - h\min_{i\in I}\{\lambda_i(\Delta + L)\})}\right]\sum_{i\in I}\left(\alpha_i^2\rho_i^2 + \sum_{j\in N_i} a_{ij}^2\sigma_{ij}^2\right),$$

which increases with the increase of h. This means that increasing the sampling period enlarges the upper bound of the static tracking error. Therefore, from the viewpoint of choosing the sampling period, a trade-off exists between the tracking speed and the static tracking error.

4. Simulations

In this section, numerical simulations are provided to illustrate the effectiveness of the above theoretical results. Consider a multi-agent system composed of four agents with the network topology shown in Fig. 1, where agent 0 represents the virtual leader and without loss of generality all the weights of edges are assumed to be 1. The measurement noises are independent of white Gaussian sequences with an intensity equal to 1. The initial states of the four agents are randomly chosen.

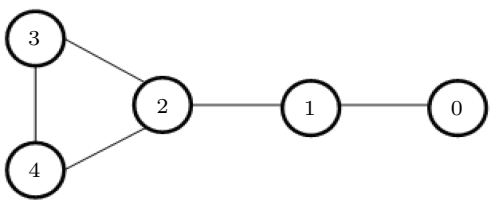


Fig. 1. Network topology composed of four agents and a virtual leader.

It follows from inequality (6) that the maximal allowable sampling period that guarantees system (1), using protocol (4), and achieves the mean square bounded consensus tracking, is 0.486. The numerical results are shown in Figs. 2 and 3, respectively. It can be seen from Fig. 2 that when $h = 0.4 < 0.486$, which satisfies inequality (6) and multi-agent system (1), using The consensus tracking protocol (4), achieves mean square bounded consensus tracking. Figure 3 shows that when $h = 0.5 > 0.486$, which does not satisfy inequality (6), or multi-agent system (1), using the consensus tracking protocol (4), cannot achieve the mean square bounded consensus tracking. Thus, the results of Theorem 1 are numerically verified.

In order to better investigate the effects of the sampling period on the tracking speed and the static tracking error, here we consider the virtual leader with the time-invariant state. The numerical results are shown in Figs. 4 and 5, respectively. By comparing Fig. 4 and Fig. 5, we find that increasing the sampling period accelerates the tracking speed but enlarges the static tracking error. Thus, the trade-off between the tracking speed and the static tracking error is numerically verified.

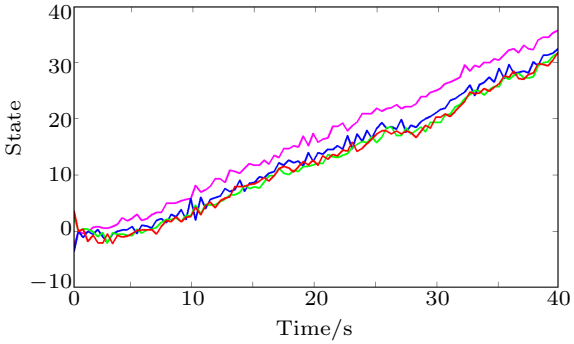


Fig. 2. Time evolutions of states of system (1), using protocol (4), for $x^*(t) = t$ and $h = 0.4$.

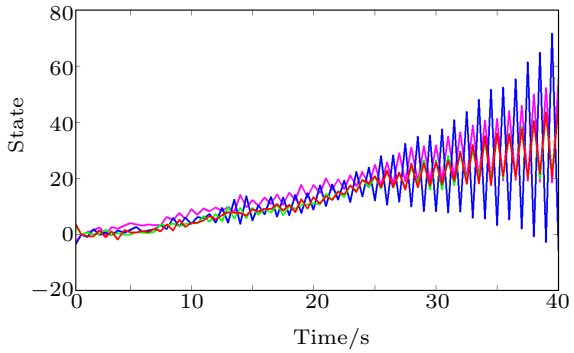


Fig. 3. Time evolutions of states of system (1), using protocol (4), for $x^*(t) = t$ and $h = 0.5$.

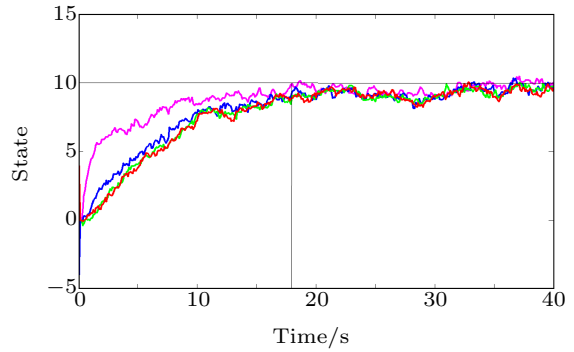


Fig. 4. Time evolutions of states of system (1), using the protocol (4) for $x^*(t) = 10$ and $h = 0.1$.

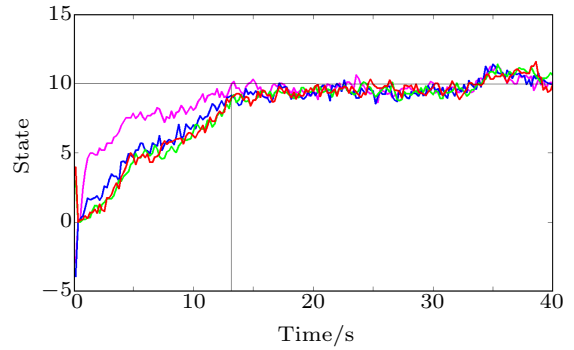


Fig. 5. Time evolutions of states of system (1), using protocol (4) for $x^*(t) = 10$ and $h = 0.2$.

5. Conclusions

In this paper, we investigated the stochastic bounded consensus tracking problems of leader–follower multi-agent systems with measurement noises and the virtual leader with the time-varying reference state that is only available to a portion of agents based on sampled-data. Applying the probability limit theory and the algebra graph theory, we obtained the necessary and sufficient conditions guaranteeing the mean square bounded consensus tracking. Moreover, the effects of the sampling period on tracking performance were analysed. Future work includes extending the results of this paper to second-order multi-agent systems.

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