Experimental observation of a multirhythmic pattern in chains of Rossler circuits*

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(Received 19 July 2012; revised manuscript received 1 September 2012)

The influence of parameter mismatches on multirhythmic patterns in chains of coupled Rossler circuits are explored experimentally. The parameter mismatches in coupled chaotic oscillators are found to help form a kind of multirhythmic pattern as reported in chains of biological coupled oscillators [*Phys. Rev. Lett.* **92** 228102]. Moreover, a new type of multirhythmic pattern based on the envelope of time series is observed.

Keywords: multirhythmic pattern, multirhythmic pattern based on envelope, chaotic oscillators, Rossler circuit

PACS: 87.18.Hf, 05.45.Xt **DOI:** 10.1088/1674-1056/21/12/128701

1. Introduction

Recently, investigations on coupled oscillators in a variety of scientific fields ranging from physical science to biological and social sciences have attracted a great attention. Lots of complex phenomena such as synchronization,[1−5] spatiotemporal patterns,[6−8] and hysteresis[9] have been found. Among these, pattern formation[10] in the coupled oscillators, which is considered to be related to information processing[11,12] in biological organism and various types of quadrupedal gaits (such as a pace, a trot, and a gallop[13,14]), has become one of the focuses in nonlinear science. Pattern formations were experimentally investigated in electric circuits,[15] chemical oscillator systems,[16] and chains[17] of or rings[18] of biological coupled periodical oscillators where rich patterns with various symmetries are observed. In rings of three biological coupled oscillator systems constructed with living cells of plasmodium of the true slime mold, physarum polycephalum. Three types of oscillation modes, i.e., a rotation, a partial in-phase, and a half period oscillations are observed. In a rotation, the phase differences between neighboring oscillators are all about $2\pi/3$. In a partial in-phase, two oscillators are in-phase and the other one is antiphase against the first two. In a half period oscillation (named as a multirhythmic pattern in Ref. [17]), described as "a novel oscillation mode", where two oscillators are in antiphase and the third one oscillates twice while the first two oscillated once, namely, the third oscillator has a half period against those of the first two. The antiphase pairs and the third oscillator are locked at 1:2 frequencies (multirhythmic pattern). Among these, the first two types of patterns are generally observed as desynchronization patterns by various bifurcations in coupled identical periodical and chaotic oscillators. For example, in rings of coupled Rossler oscillators, various desynchronous spatiotemporal patterns as rotating wave and partial in-phase regimes are explored extensively by Zhang *et al.*,[19,20] which is useful to the research on the coupling caused order. However, the third pattern has not been reported in coupled chaotic oscillators, though it was predicted by symmetric Hopf bifurcation theory based on group theory[21] and was observed in coupled periodical oscillators in electron circuits.[22−24]

It is natural to raise the question that whether it is possible to observe the half period oscillation pattern in chains of coupled chaotic oscillators. To answer this question, we set up a chain of three coupled Rossler circuits and check the patterns with the increment of coupling constant. By properly introducing parameter mismatches on the central circuit of the coupled chaotic oscillators, a multirhythmic pattern

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11262006 and 11062002) and the Science and Technology Project of Jiangxi Province, China (Grant Nos. GJJ12330 and 2010GQW0021).

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and other patterns (including partial in-phase, complete in-phase, and antiphase patterns) are observed for proper parameters. Moreover, a kind of multirhythmic pattern based on the envelope of time series is firstly observed in the coupled chaotic circuit.

The rest of this paper is organized as follows. In Section 2, we present the experimental setup. In Section 3, we exhibit the experimental results and discuss the regimes of the multirhythmic pattern. Finally, a brief conclusion is presented in Section 4.

2. Experimental setup

To investigate the effects of parameter mismatches on the multirhythmic patterns of coupled chaotic oscillators, Carroll's model[25] of electronic circuit units as shown in Fig. 1(a) are set up. This model represents a small modification of the Rossler system by replacing the nonlinear part with a piecewise linear function. With parameters presented in Fig. 1(a), the circuit has a single screw attractor, whose main frequency is exponentially decreasing with the increment of parameter R_5 as shown in Fig. 1(b).

Three Rossler units are diffusively coupled with a coupling circuit as shown in Fig. 2. The middle unit is numbered with circuit 0 and the other two are numbered with circuits 1 and 2 respectively.

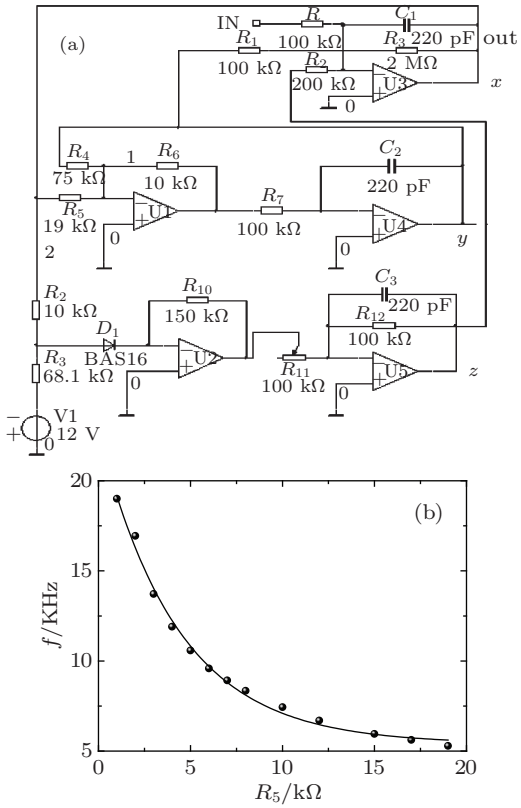


Fig. 1. (a) Rossler circuit unit and (b) the main frequency of single Rossler circuit versus parameter R_5.

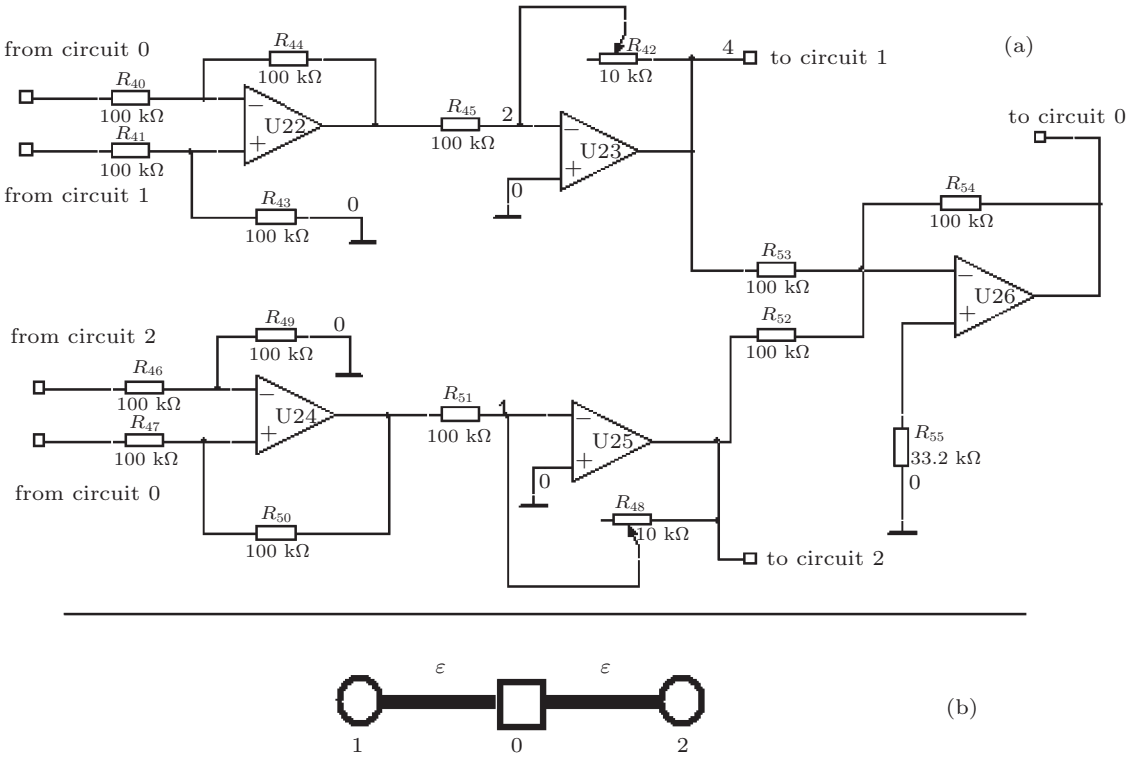


Fig. 2. (a) The coupling circuit unit, the input signals are from the x of the Rossler units (in Fig. 1(a)), while the output signals get to the IN pin of the Rossler units (in Fig. 1(a)). The coupling constant is changed through R_{42} and R_{48}. (b) The diagram of a chain of three coupled circuits. Circuit 0 is different from the other two identical circuits 1 and 2.

The coupled system is described as follows:

$$
\begin{aligned}
\dot{x}_0(t) &= -\alpha(\delta x_0(t) + y_0(t) + \beta z_0(t)) \\
&\quad + \varepsilon\Big(\sum (x_i(t) - x_0(t))\Big), \\
\dot{y}_0(t) &= \alpha\Big(\frac{R_6}{R_5} x_0(t) + \gamma y_0(t)\Big), \\
\dot{z}_0(t) &= \alpha(g(x_0(t) - z_0(t))), \\
g(x_0(t)) &= 0, \quad x_0(t) \le 2.1, \\
g(x_0(t)) &= \mu(x_0(t) - 2.1), \quad x_0(t) > 2.1, \\
\dot{x}_i(t) &= -\alpha(\delta x_i(t) + y_i(t) \\
&\quad + \beta z_i(t)) + \varepsilon(x_0(t) - x_i(t)), \\
\dot{y}_i(t) &= \alpha\Big(\frac{R_6}{R_5} x_i(t) + \gamma y_i(t)\Big), \\
\dot{z}_i(t) &= \alpha(g(x_i(t) - z_i(t))), \\
g(x_i(t)) &= 0, \quad x_i(t) \le 2.1, \\
g(x_i(t)) &= \mu(x_i(t) - 2.1), \quad x_i(t) > 2.1, i = 1, 2, \quad (1)
\end{aligned}
$$

where ε is the coupling constant (ε=R_{42}/R_{45}, R_{45} = 100 kΩ), $\mu = 15$, $\alpha = 1/(R_2 C_1)$, (C_1 = 220 pF), $\delta = 0.05$, $\beta = 0.5$, $\gamma = R_6/R_4 = 1.33$, $R_6 = 10$ kΩ. Initially, R_5s of three units are arbitrarily set to be the same value of 19 kΩ.

3. Experimental results

In this section, we investigate the effects of parameter mismatches between the middle circuit 0 and other two identical circuits 1 and 2 on the multirhythmic pattern in a chain of three coupled Rossler circuits. For three identical coupled oscillators, the increasing coupling may drive the system from an incoherent state to a partial in-phase/antiphase (circuit 0 and 1 are in the in-phase/antiphase state while circuit 2 is not) then finally complete synchronization. The transition processes, which involves increasing the coupling constant (by changing the value of R_{42} and R_{48} simultaneously) are different for various R_5 (in the range of 19 kΩ to 1 kΩ). When R_5 of circuit 0 is slightly away from 19 kΩ, circuits 1 and 2 transit from the incoherence state to the antiphase locking state (APS) then to the phase locking state (PS) (almost in-phase). Interestingly, when R_5 is around 16 kΩ, the coupled system transits from the incoherent state to a multirhythmic pattern based on the envelope of time series (MPE), where the envelopes of circuits 1 and 2 are in antiphase states (Figs. 3(b) and 3(c)), while that of circuit 0 (Fig. 3(a)) has a half period oscillation. This multirhythmic pattern based on the envelope of the time series is firstly observed in coupled chaotic oscillators.

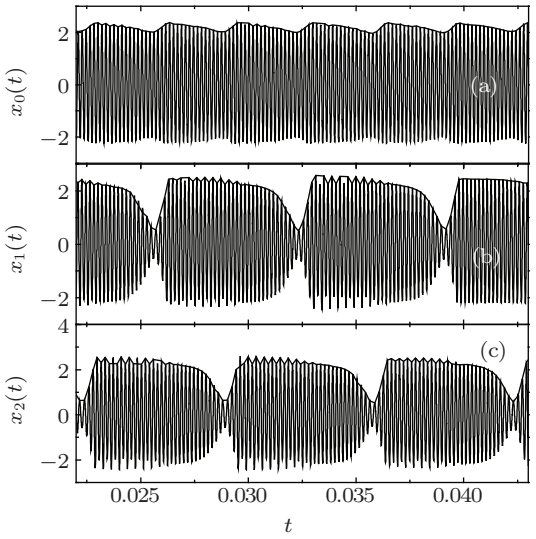


Fig. 3. (a)–(c) The time series of $x_i(t)$ ($i = 0, 1, 2$) of three coupled Rossler circuits for $R_{42} = 3$ kΩ, $R_5 = 16$ kΩ. The envelope (bold lines) of $x_1(t)$ is antiphase to that of $x_2(t)$ while that of $x_0(t)$ has double frequency to that of $x_1(t)$ and $x_2(t)$.

When R_5 of circuit 0 is around 12 kΩ, the system is in an incoherent state for $R_{42} < 4.5$ kΩ. However, circuits 1 and 2 are in antiphase synchronous states for $R_{42} = 4.5$ kΩ $\sim$ 5.5 kΩ, while circuit 0 oscillates at the same frequency (about 6.5 kHz) as itself before coupling as shown in Figs. 4(a)–4(c).

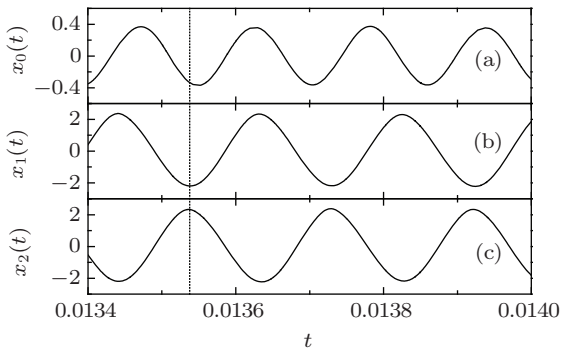


Fig. 4. (a)–(c) The time series of $x_i(t)$ ($i = 0, 1, 2$) of three coupled Rossler circuits for $R_{42} = 5$ kΩ, $R_5 = 12$ kΩ. $x_1(t)$ is antiphase to $x_2(t)$ while $x_0(t)$ has about a 4:3 frequency to $x_1(t)$.

The system transits to the multirhythmic pattern state for $R_{42} = 5.5$ kΩ $\sim$ 7.5 kΩ, where circuits 1 and 2 are in antiphase synchronous states while circuit 0 is in a half period oscillation pattern with a small amplitude as shown in Fig. 5. For even larger R_{42}, the whole coupled system becomes amplitude death (AD)[26−29] owing to the parameter mismatches where all circuits

stop oscillating and come to a steady state. The total phase diagram of R_5 and R_{42} are presented in Fig. 6.

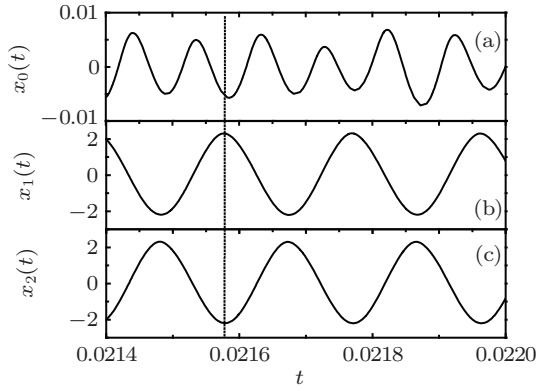


Fig. 5. (a)–(c) The time series of $x_i(t)$ $(i = 0, 1, 2)$ of three coupled Rossler circuits for $R_{42} = 6$ kΩ, $R_5 = 12$ kΩ. $x_1(t)$ is antiphase to $x_2(t)$ while $x_0(t)$ has a half period oscillation.

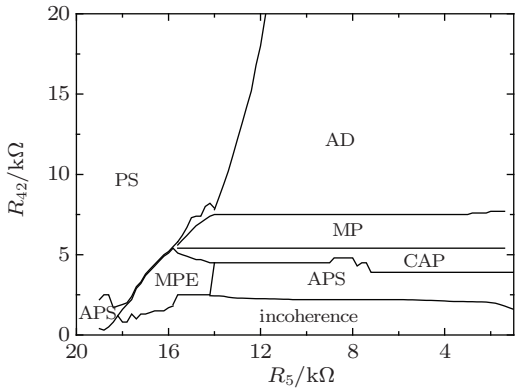


Fig. 6. The phase diagram of R_5 (parameter mismatch) versus R_{42} (coupling constant), where MP is a multirhythmic pattern, where circuit 0 is in half period oscillation while circuits 1 and 2 are in antiphase states. MPE is a multirhythmic pattern of the envelope of the time series. AD is the amplitude death state of all three oscillators. PS (APS) is an in-phase (antiphase) state between circuits 1 and 2.

To better understand the formation of the multirhythmic pattern in coupled chaotic oscillators, it is necessary to have a brief discussion. Based on the discussion in Ref. [19], it is obvious that the in-phase state and the antiphase state are two possible stable states since they have minimal coupling potential in chains or rings of coupled chaotic oscillator. What is more, it is easier to get the APS of circuits 1 and 2 when there are parameter mismatches between circuits 0 and 1. As the antiphase is established between circuits 1 and 2, the value of $x_1(t)+x_2(t)$ tends to become a small value $\xi(t)$. Then, circuit 0 can be modified to $\mathrm{d}U_0/\mathrm{d}t = F(U_0) + \varepsilon\xi(t)$, $U_0(x_0, y_0, z_0) \in R$, $F(U_0) = \{-\alpha(\delta x_0 + y_0 + \beta z_0 - 2\varepsilon x_0)$, $\alpha((R_6/R_5)x_0 + \gamma y_0)$, $\alpha(g(x_0) - z_0)\}$. For proper ε, $\mathrm{d}U_0/\mathrm{d}t = F(U_0)$ has a stable fixed point. Therefore, the dynamics of circuit 0 are determined by the small driving signal $\xi(t)$. Interestingly, $\xi(t)$ is a half period oscillation compared to $x_1(t)$ as shown in Fig. 7, which results in a half period oscillation of $x_0(t)$, then the multirhythmic pattern forms. However, the regimes of the multirhythmic patterns based on the envelope are still unclear and need further study.

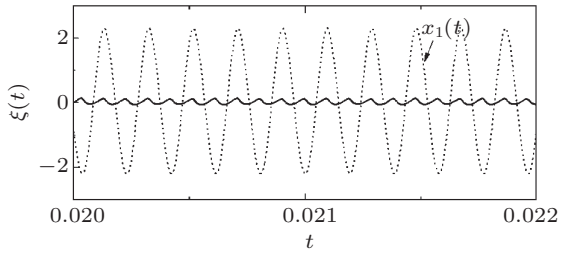


Fig. 7. The time series of $x_1(t)$ (dashed line) and $\xi(t)$ (solid line), where $\xi(t)$ is twice the oscillation to $x_1(t)$.

4. Conclusion

In summary, we have experimentally investigated the influence of parameter mismatches on multirhythmic pattern in a chain of coupled Rossler oscillators. A multirhythmic pattern which was reported in the coupled periodical biological oscillators is observed in coupled chaotic oscillators. Moreover, a new type of multirhythmic pattern based on the envelope of time series is firstly observed in the coupled Rossler oscillators with parameter mismatches. This multirhythmic pattern is not exclusive in chaotic oscillators with a system size $N = 3$. It can also be observed for a larger odd system size N, for example $N = 5$, 7 for proper parameters of R_5 and R_{42}. Since the multirhythmic pattern has a significant role on the control of animal locomotion,[30] for example, the anti-phase mode corresponds to the walking mode in bipedal animal and the in-phase mode corresponds to hopping mode. We hope the results in the present paper can be found useful for both theoretical study and experimental investigation in exploring the origins of various orderings in disordered chaotic systems[31] and control of locomotion.

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