The quantum Hall's effect: A quantum electrodynamic phenomenon

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We have applied Maxwell's equations to study the physics of quantum Hall's effect. The electromagnetic properties of this system are obtained. The Hall's voltage, $V_{\rm H} = 2\pi\hbar^2 n_s/e\,m$, where n_s is the electron number density, for a 2-dimensional system, and $h = 2\pi\hbar$ is the Planck's constant, is found to coincide with the voltage drop across the quantum capacitor. Consideration of the cyclotronic motion of electrons is found to give rise to Hall's resistance. Ohmic resistances in the horizontal and vertical directions have been found to exist before equilibrium state is reached. At a fundamental level, the Hall's effect is found to be equivalent to a resonant LCR circuit with $L_{\rm H} = 2\pi\, m/e^2 n_s$ and $C_{\rm H} = me^2/2\pi\hbar^2 n_s$ satisfying the resonance condition with resonant frequency equal to the inverse of the scattering (relaxation) time, τ_s. The Hall's resistance is found to be $R_{\rm H} = \sqrt{L_{\rm H}/C_{\rm H}}$. The Hall's resistance may be connected with the impedance that the electron wave experiences when it propagates in the 2-dimensional gas.

Keywords: quantum Hall effect, Maxwell equations, electromagnetism

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1. Introduction

The quantum Hall effect is one of the most fascinating phenomena in physics that dates back to Hall who discovered it in 1879.[1] It is shown experimentally to occur in integer and fractional aspects.[2−4] Despite this, no single theory exists at present that renders it fully comprehensible. We have recently proposed a complex quantum harmonic model to describe the fractional quantum Hall effect.[5,6] Owing to this model the Hall's electric field is found to be[6]

$$E_{\rm H} = \frac{1}{\tau_s}\sqrt{\frac{\hbar B}{e}}\,\frac{\Gamma(n+3/2)}{\Gamma(n+1)}, \quad n = 0, 1, 2, \ldots, \quad (1)$$

where

$$\tau_s = \frac{m}{\hbar\, n_s} = \frac{e}{2\pi I_0}. \quad (2)$$

The quantum number n designates the principal quantum number of the harmonic oscillator. An alternating current (ac) quantum Hall effect impedance was investigated by Cage and Jeffery.[7] They associated the imaginary part of the impedance with the capacitance and impedance of the Hall device two-dimensional electron gas (2DEG). Hence, this impedance is related to intrinsic properties of the device. The capacitance and impedance of a 4.6-mm long GaAs/AlGaAs device are found to be 0.0029 pF and 0.07 μH, respectively, and for a device length of 2.2 mm, they are found to be 0.0014 pF and 0.04 μH, respectively.[7] The Hall's effect can be thought of as equivalent to an LCR circuit. At resonance, the impedance of the circuit is equal to Hall's resistance. The resonance time is equal to the relaxation (collision) time of electrons in Hall's effect. The fine structure constant which determines the strength of electromagnetic interaction is related to the ratio between the vacuum impedance of the electromagnetic wave to the Hall's resistance. Moreover, the ratio of the magnetic charge to the electric charge is equal to Hall's resistance too. In this sense one can relate the Hall's resistance to the impendence of the electron wave in the two-dimensional electron gas (2DEG).

2. A model of electromagnetism

We trust that we should apply the Maxwell's equations to make a full description of Hall's effect.[1] The electromagnetic effects are prominent, and therefore, an electromagnetic treatment should be considered besides its quantum aspect. We believe that a transient electromagnetic wave is produced during a short period of time, τ_s. Because of this the Hall effect exhibits additional properties that were not dealt

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with before. One may define the time τ_s in Eq. (2) as a characteristic time of electrons. It is related to the relaxation (scattering) time of the two-dimensional material, or it may be related to the time during which the electron responds to the external fields. For a typical two-dimensional material, $n_s \sim 10^{15}$ m^{-2}, so that $\tau_s \sim 10^{-12}$ s. This value agrees with a recent estimate we have obtained for a two-dimensional Drude's conductance.[8] We will consider here the implications of applying Maxwell's equations to explore the quantum Hall effect.

2.1. Faraday's law

Equation (1) can be associated with Faraday's law

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}. \tag{3}$$

In Hall's effect, one has $\boldsymbol{B} = B\hat{k}$ and $\boldsymbol{E} = E_x\hat{i} + E_y\hat{j}$. The z component of the Faraday's equation yields,

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B}{\partial t}. \tag{4}$$

Let us consider two cases below.

For the first case we assume that $\partial E_x/\partial y = 0$, so that for a transient change of $\boldsymbol{E}$ and $\boldsymbol{B}$, one can write

$$\frac{E_y}{L} = -\frac{B}{\tau_s}, \tag{5}$$

where L is the length of the sample. This can be written as

$$E_y = -\frac{\phi_B}{\tau_s W}, \quad \phi_B = BA, \tag{6}$$

where W is the width of the sample and $A = WL$ is its area. Therefore, E_y represents magnetic flux per unit time per unit width of the sample. Thus, the term $\sqrt{\hbar B/e}$ in Eq. (1) can be associated with a magnetic flux per unit length. The Hall's voltage now becomes

$$V_{\rm H} = -\phi_B/\tau_s. \tag{7}$$

This represents the induced electromotive force (emf) during a time τ_s after the application of the magnetic field. If the flux is quantized, then for a single electron one has $\phi_B = h/e$.[9] Therefore, equations (2) and (7) yield

$$V_{\rm H} = -\frac{h}{e\,\tau_s} = -\frac{2\pi\hbar^2 n_s}{e\,m}. \tag{8}$$

This in fact represents the Hall's voltage in the absence of a magnetic field. It is thus a quantum property of the system concerned. It can be associated with Fermi energy ($\epsilon_{\rm F}$) for a two-dimensional system, which is given by[10]

$$\epsilon_{\rm F} = \frac{2\pi\hbar^2 n_s}{m}. \tag{9}$$

Hence,

$$\epsilon_{\rm F} = -eV_{\rm H}. \tag{10}$$

This is an interesting equation which clearly shows that the Hall voltage is a measure of Fermi energy. Therefore, equation (10) can be used to calculate the number density of the two-dimensional system. For a typical two-dimensional system, one has $n_s \sim 10^{15}$ m^{-2}, so that $\tau_s \sim 10^{-12}$ s, and hence $V_{\rm H} \sim 1$ mV where we have used the bare mass of the electron.[1)] Equation (8), $eV_{\rm H}\tau_s = h$, can be seen as expressing the Heisenberg's uncertainty relation, $\Delta E \Delta t \geq h$, where $\Delta E = eV_{\rm H}$ and $\Delta t = \tau_s$. In this case, we may relate τ_s to the lifetime of the Hall state. This rather long time may account for the plateaus observed in the quantum Hall effect.[2] It is also apparent that $eV_{\rm H} = \epsilon_{\rm F}$ represents an energy connected with the decay width of the quantum Hall transition. The electric potential energy associated with the Hall system is $U_{\rm H} = \frac{1}{2}\,C_{\rm H}V_{\rm H}^2$. Employing Eqs. (8) and (14) yields

$$U_{\rm H} = \frac{\hbar^2}{2m}\,(2\pi\, n_s).$$

This is equal to the kinetic energy of electrons with wave vector, $k_{\rm H} = \sqrt{2\pi\, n_s}$.

2.2. Quantum (Hall) capacitance

The concept of quantum capacitance has been introduced by Luryi where he showed that the two-dimensional electron gas (2DEG) manifests itself as a capacitor in series, whose capacitance per unit area equals[11]

$$c_{\rm Q} = \frac{me^2}{\pi\,\hbar^2}. \tag{11}$$

He pointed out that it is a property of a 2DEG, and corresponds to the field penetrating through a highly conducting two-dimensional metal. In the present model, we consider the capacitance-time constant relation

$$\tau_s = R_{\rm H}\,C_{\rm H}, \tag{12}$$

[1)] We are taking m as a bare mass of the electron. Effective mass of the electron should be used instead for actual values.

where τ_s represents the time constant of the capacitor, $C_{\rm H}$ is the Hall's capacitance, and $R_{\rm H}$ is the fractional Hall's resistance given by[5]

$$R_{\rm H} = \frac{2n \pm 1}{n} \frac{h}{e^2}, \tag{13}$$

where n is the principal quantum number of the oscillator. Substituting Eqs. (2) and (12) into Eq. (13) yields[2)]

$$C_{\rm H} = \frac{n}{2n \pm 1} \frac{me^2}{2\pi\hbar^2 n_s}, \tag{14}$$

where each energy level is composed of two states with $\pm$. It is interesting to remark that the Hall's voltage, as defined in Eq. (10), coincides with the voltage drop across the quantum capacitor, as shown by Luryi.[11]

Using Eq. (9), equation (14) can be written as

$$C_{\rm H} = \frac{n}{2n \pm 1} \frac{e^2}{\epsilon_{\rm F}}. \tag{15}$$

Now for a single electron, when the spin is considered, the Hall's capacitance per unit area will be equal to the quantum capacitance, viz., $C_{\rm H}/A = c_{\rm Q}$, for $n = 1$ in the (–) state. It is, therefore, very interesting that for a single electron Hall's phenomenon occurs without the application of a magnetic field. Hence, the quantum capacitance represents the smallest possible capacitance that can be measured in a two-dimensional system. With the above estimates, one finds for a typical two-dimensional system, $C_{\rm H} \sim 10^{-16}$ F (with electron bare mass). This agrees with the one obtained in Ref. [7]. It is thus evident that the Hall effect is a macroscopic quantum electrodynamic effect.

2.3. Ampere's law

Ampere's law can be written as

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu \boldsymbol{J} + \frac{1}{c^2} \frac{\partial \boldsymbol{E}}{\partial t}, \tag{16}$$

where μ is the permeability of the sample under consideration.

Let us apply this law in Hall's effect, where $\boldsymbol{B} = B\hat{k}$ and $\boldsymbol{E} = E_x\hat{i} + E_y\hat{j}$. Now, considering the y component of Eq. (16), we can obtain

$$-\frac{\partial B}{\partial x} = \frac{1}{c^2} \frac{\partial E_y}{\partial t}. \tag{17}$$

For a transient time, one has

$$-\frac{B}{L} = \frac{1}{c^2} \frac{E_y}{\tau_s}. \tag{18}$$

Therefore, one can define the Hall's voltage by

$$V_{\rm H} = -\frac{\phi_B\, c^2 \tau_s}{L^2}. \tag{19}$$

For a single electron, one has $\phi_B = h/e$, so that equations (19) and (2) give

$$V_{\rm H} = -\frac{2\pi mc^2}{e\, n_s L^2}. \tag{20}$$

Equating Eqs. (8) and (20) yields

$$n_s = \frac{mc}{\hbar L}. \tag{21}$$

Now consider the x component of Eq. (16), we have

$$\frac{B}{W} = \mu J_x + \frac{1}{c^2} \frac{E_x}{\tau_s}. \tag{22}$$

This can be written as, using Eq. (2),

$$V_x + \frac{J_x L\, m}{\varepsilon\, \hbar\, n_s} = \frac{2\pi mc^2}{W^2 n_s e}. \tag{23}$$

Using Eq. (20), equation (23) can be expressed as

$$V_x + \frac{J_x L\, m}{\varepsilon\, \hbar\, n_s} = -V_{\rm H} \left(\frac{L}{W}\right)^2. \tag{24}$$

2.4. Gauss's law

Gauss's law can be written as

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon}, \tag{25}$$

where ρ is the charge per unit volume. For Hall's effect one has, $\boldsymbol{B} = B\hat{k}$ and $\boldsymbol{E} = E_x\hat{i} + E_y\hat{j}$. Hence,

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = \frac{\rho}{\varepsilon}. \tag{26}$$

In a two-dimensional system, one can write

$$\rho = \frac{n_s\, e}{d}, \tag{27}$$

where d is the thickness of the sample. Applying Eq. (27) in Eq. (26), one can write

$$\frac{E_x}{L} + \frac{E_y}{W} = \frac{n_s\, e}{d\, \varepsilon}, \tag{28}$$

where ε is the permittivity of the sample material. This can be cast in the form

$$V_{\rm H} = \frac{n_s W^2 e}{d\, \varepsilon} - \left(\frac{W}{L}\right)^2 V_x. \tag{29}$$

One can define the capacitance

$$c_g = \frac{d\, \varepsilon}{W^2}, \tag{30}$$

[2)] We have ignored here the spin of the electron.

as a geometric capacitance per unit area (n_s) of the system. This can be calculated if we plot V_x versus $V_{\rm H}$ for a given two-dimensional system, and from which ε can be obtained.

Now comparing Eq. (24) with Eq. (29), and using Eqs. (2) and (21) yield the horizontal velocity of the electron, where, $J_x = n_s e\, v_x/d$, as

$$v_x = \frac{L}{\tau_s} = c. \tag{31}$$

This result is surprising but not unexpected, since in the relativistic Dirac's theory the electron is found to travel at the speed of light.[12] This implies that the electric and magnetic influences are carried by an electromagnetic wave.

For the second case we assume now that $\partial E_x/\partial y \neq 0$, so that equation (4) reads

$$\frac{E_y}{L} - \frac{E_x}{W} = -\frac{B}{\tau_s}. \tag{32}$$

This can be written as

$$V_{\rm H} - V_x = -\frac{BA}{\tau_s}, \tag{33}$$

where $A = WL$ is the sample area, and BA is the flux through the area. For a single electron $BA = h/e$, so that equations (29), (33), and (2) read

$$V_{\rm H} = \left[1 + \frac{L^2}{W^2}\right]^{-1} \left(\frac{n_s e L^2}{d\,\varepsilon} - \frac{2\pi\hbar^2 n_s e}{me^2}\right). \tag{34}$$

Now using Eq. (20), Eq. (33) together with Eq. (2) yields

$$V_x = \left(1 - \left(\frac{n_s \hbar L}{mc}\right)^2\right) V_{\rm H}.$$

Therefore, $n_s \leq mc/\hbar L$. Now equation (34) can be written as

$$\frac{1}{C_{\rm H}} = \frac{1}{C_1} + \frac{1}{-C_2}, \tag{35}$$

where

$$C_1 = \frac{d\,\varepsilon}{L^2\beta}, \quad C_2 = \frac{me^2}{2\pi\hbar^2\beta}, \quad \beta = \frac{n_s}{1 + L^2/W^2}, \tag{36}$$

where β has a dimension of $\rm m^{-2}$. We can call C_1 the geometric capacitance per unit area (depends on dimensions), and C_2 the quantum capacitance per unit area (depends on fundamental constants, e.g., m, e, $\hbar$). It seems that C_1 is the equivalent capacitance of $C_{\rm H}$ and C_2 connected in series. C_2 can be compared with that obtained in Ref. [7].

It is also apparent from Eqs. (1) and (9) that one can define the Hall's capacitance as

$$C_{\rm H} = \frac{me^2}{2n_s\hbar^2 W}\sqrt{\frac{\hbar}{e\,B}}\,\frac{\Gamma(n+1)}{\Gamma(n+\frac{3}{2})},$$

$$C_{\rm H} = \frac{\pi\, e^2}{\epsilon_{\rm F} W}\sqrt{\frac{\hbar}{e\,B}}\,\frac{\Gamma(n+1)}{\Gamma(n+\frac{3}{2})}.$$

For $n = 0$ one has

$$C_{\rm H} = \frac{me^2}{\pi\sqrt{2}\, n_s^{3/2}\hbar^2 W}.$$

For $n_s = 10^{16}$ $\rm m^{-2}$, one finds $C_{\rm H} = 4.7/W \times 10^{-24}$ F, where W is measured in meters. The corresponding electric field will be

$$E_{\rm H} = \frac{\sqrt{2}\,\pi\hbar^2 n_s^{3/2}}{me}.$$

Consequently, one has $E_{\rm H} = 3.4 \times 10^5$ V/m. It is found by Casimir that an attractive force ($F_{\rm C}$) is developed when two conducting plates are brought very close to each other.[13] This force is deemed to result from quantum fluctuations of the vacuum. If we relate this force to a Casimir-like force, then the distance between the two plates is $a = (\pi\, c\, m/240\hbar\, n_s^{3/2})^{1/2}$. For a typical system this amounts to $a = 0.058$ μm. The electronic current is $I_{\rm H} = e\hbar\, n_s^{3/2} W/\sqrt{2}\, m$. This yields $I_{\rm H} = 13.1\, W$ (Amp.), where W is in meters.[6] The magnetic moment associated with this current is $\mu_{\rm H} = I_{\rm H}/n_s = \mu_{\rm B}\sqrt{2n_s}\, W$, where $\mu_{\rm B} = e\hbar/2m$. These quantities may emerge from quantum fluctuations of the two-dimensional systems. Thus, these fluctuations would set fundamental limits on the performance of 2DEG devices working at nonoscales.

2.5. General consideration of Hall's resistance

Now if we assume that the electrons execute cyclotronic motion with a multiple radius of $\hbar/mc$ (or, equivalently, quantization of orbital angular momentum of electrons in their cyclotronic motion), then the internal current produced by each electron, moving at the speed of light, is

$$I_c = \frac{e}{T} = \frac{e\, mc^2}{nh}, \quad n = 1, 2, 3, \ldots, \tag{37}$$

where T is the cyclotronic period. Now for a single electron, one has $\phi_B = h/e$, so that equation (37) can be written as

$$I_c = \frac{mc^2}{n\phi_B}. \tag{38}$$

Recall that the cyclotronic time can be related to the uncertainty time-energy relation, viz., $\Delta E\, \Delta t = h$,

via, $\Delta E = mc^2$, and $\Delta t = T$. The induced emf per cycle, as a result of applying a magnetic field B, is

$$\epsilon_c = \frac{\phi_B}{T} = \frac{\phi_B\, mc^2}{nh}, \tag{39}$$

so that the resulting resistance experienced is

$$R_c = \frac{\epsilon_c}{I_c} = \frac{h}{e^2}. \tag{40}$$

Interestingly, R_c is independent of n and is equal to the Hall's resistance.[2] Hence, equation (31) does not lead to physical contradictions. It is interesting that the speed of light drops out of the final equation. Moreover, equation (40) is independent of the assumption in Eq. (31), and can be obtained directly by dividing Eq. (39) by Eq. (37).

Alternatively, if we assume that the motion of each electron across the sample produces a voltage difference ΔV, when the magnetic field is applied, then

$$\Delta V = v_y B\, L. \tag{41}$$

The current developed by each electron is

$$I_y = \frac{e\, v_y}{W}. \tag{42}$$

The resistance encountered by each electron is

$$R_y = \frac{\Delta V_y}{I_y} = \frac{B\, W L}{e} = \frac{\phi_B}{e}. \tag{43}$$

Thus, for a single electron, one has $\phi_B = h/e$, so that equation (43) yields

$$R_y = \frac{h}{e^2}. \tag{44}$$

Once again this is the same as Eq. (40). Unlike Hall's resistance, which is defined by the vertical voltage divided by the horizontal current, the resistance Eq. (44) is the direct Ohmic resistance resulting from the motion of electrons across the sample.

2.6. Classical Hall effect

We would like here to derive Hall's resistance from Faraday's law ($\epsilon = -\partial\phi/\partial t$). The motion of horizontal and vertical electrons resembles the motion of electrons in an electric circuit. To this end, we suppose N_y electrons cross the width of the sample in a time W/v_y. Hence, the resistance experienced by the electrons will be

$$R_y = \frac{\epsilon_y}{I_y}, \quad \epsilon_y = B\, v_y L, \quad I_y = \frac{N_y\, e\, v_y}{W}. \tag{45}$$

Hence the vertical resistance will be

$$R_y = \frac{B}{n_y\, e}, \quad n_y = N_y/A, \tag{46}$$

where $A = WL$. The vertical motion of these electrons results from the action of Lorentz force.

Similarly, if N_x electrons cross the length of the sample, one has

$$R_x = \frac{\epsilon_x}{I_x}, \quad \epsilon_x = B\, v_x W, \quad I_x = \frac{N_x\, e\, v_x}{L}, \tag{47}$$

so that

$$R_x = \frac{B}{n_x\, e}, \quad n_x = N_x/A. \tag{48}$$

Thus, during a transient time there exist two types of Hall's resistances as a result of a change of magnetic flux during this time before electrons reach an equilibrium state. Since $n_y \ll n_x$, then $R_x \ll R_y$. It is therefore interesting that some Ohmic resistance, similar to the Hall's resistance, exits in the horizontal as well as vertical direction. However, these resistances are transient and when the equilibrium state is reach, they cease to exist.

3. Hall's inductance

One can now associate an inductance quantity with Hall's effect, i.e., $L_{\rm H}$. This is plausible since electrons inside the two-dimensional system are in circular motion so that they can be seen effectively as an inductor with N number of turns (electrons). Thus, during a transient time when the magnetic field is applied cyclotronic electrons react to the sudden change in the magnetic field producing an induced emf. Therefore, the Hall's circuit behaves as a resistor–inductor circuit. The emf developed (that may be equated to Hall voltage) in an inductor when a current changes across during a transient time can be defined as

$$V = -L\frac{{\rm d}I}{{\rm d}t}. \tag{49}$$

For a transient time, τ_s, one has, for a single electron,

$$L = -\frac{V\,\tau_s}{I}, \quad V = V_{\rm H}, \quad I = \frac{e}{\tau_s}. \tag{50}$$

Using Eqs. (2) and (20), we obtain, with $\phi_B = h/e$, the Hall's inductance

$$L_{\rm H} = -\frac{V\,\tau_s}{I} = \frac{\phi_B}{I} = \frac{h\tau_s}{e^2} = \frac{2\pi\, m}{e^2 n_s}. \tag{51}$$

We remark here that after application of the external magnetic field, the electrons oscillate so the circuit behaves as an ac, but very soon the electrons come to an

equilibrium, and thus the system acts as a dc. This behaviour is repeated each time the magnetic field is changed. In this way a sequence is generated. As the voltage stabilizes (saturates) after a few inductive times, this may help explain the observed plateaus in the quantum Hall effect. The inductance and capacitance studied here can be seen as due to quantum fluctuation imparted to electrons as the magnetic field is applied to the two-dimensional electron gas. These should find applications in nanodevices where their order of magnitudes are prominent. Devoret[14] has studied the quantum fluctuation in an electric circuit and showed that the quantum Hall's effect can be approximated with an LC circuit. In this case the impedance of the LC circuit coincides with Hall's resistance. Equation (51) inspires us to write $L_{\rm H}$ as, $L_{\rm H} = R_{\rm H}\tau_s = R_{\rm H}^2 C_{\rm H}$, using Eqs. (2), (12), and (13), where τ_s represents here the inductive time constant of the circuit. It is interesting that this inductance (intrinsic) depends only on the properties of the system under consideration. It is thus a characteristic quantity. For a system with $n_s \sim 10^{15}$ m^{-2} it amounts to a value of $L_{\rm H} \sim 2.23 \times 10^{-7}$ H. This result can be compared with that obtained in Ref. [7]. We can have some estimate of the Hall's mobility connected with the above quantities, viz., $\mu_{\rm H} = e\tau_s/m$. This can be written as, using Eq. (2), $\mu_{\rm H} = e/(n_s\hbar)$. It is independent of the electron mass. It amounts to $\mu_{\rm H} = 1.5\ {\rm V}^{-1}{\cdot}{\rm m}^2{\cdot}{\rm s}^{-1}$. This value agrees with the one found for graphene at room temperature.[15]

It is very remarkable that $C_{\rm H}$ and $L_{\rm H}$ in Eqs. (14) and (51) are related by the resonance condition of an LCR circuit, viz.,

$$\omega_s = \sqrt{\frac{1}{L_{\rm H}C_{\rm H}}}, \quad \omega_s = \frac{1}{\tau_s}. \tag{52}$$

The inductance in Eq. (51) can be related to a solenoid inductance of length ℓ, area A, and number of turns N, which is $L = \mu_0 N^2 A/\ell$. Equation (51) can be written as

$$L_{\rm H} = \frac{\mu_0}{(2r_e)\, n_s}, \tag{53}$$

where r_e is the classical electron radius. Hence, if we set in Eq. (53), $N = 1$, $A = n_s^{-1}$, and $\ell = 2r_e$, equation (53) coincides with a solenoid inductance mentioned above. Moreover, $L_{\rm H}$ and $C_{\rm H}$ are also related to $R_{\rm H}$ by

$$R_{\rm H} = \sqrt{\frac{L_{\rm H}}{C_{\rm H}}} = \frac{h}{e^2}. \tag{54}$$

Recall that the propagation of the electromagnetic wave in a vacuum has an impedance $Z_0 = \sqrt{\mu_0/\varepsilon_0} = 377\ \Omega$, where $L_{\rm H}/C_{\rm H}$ has the same dimension as μ_0/ε_0. Thus, the Hall's resistance, as defined in Eq. (54), may be connected with the impedance of propagation of matter waves in the two-dimensional electron gas (2DEG) devices. Or alternatively, it is the impedance resulting from the propagation of electromagnetic waves (transient oscillations) in a two-dimensional electron system. The impedance of the electromagnetic wave of frequency ω propagating in a conducting medium (σ) is given by $Z = \sqrt{\omega\mu_0/\sigma}$. In the quantum Hall effect the characteristic impedance (Hall's resistance) of 2DEG is the ratio of the magnetic charge (e_m) to electric charge (e). Hence, equation (54) implies that $e_m/e = h$. This is the familiar Dirac quantization for a magnetic monopole.[16,17] This result suggests that the impedance that an electron will experience, as it moves in metal, is the Hall's resistance (25.812 kΩ), while the impedance that the photon will experience, as it propagates in a vacuum, is $Z_0 = 377\ \Omega$. The two impedances can be related by the fine structure constant, $\alpha = Z_0/2R_{\rm H}$. This is commented on by Klitzing in his original note that led to the Nobel Prize in 1985.[18] The latter relation urges us to adopt a Hall resistance $R'_{\rm H} = 2h/e^2$ (as standard of resistance unit) so that α that defines the strength of electromagnetic force, viz., $\alpha = Z_0/R'_{\rm H}$ will have a simple expression. In the fractional quantum Hall effect this choice corresponds to a unique state with $n \to \infty$ (or $B \to 0$), see Eq. (13). We remark that the quantum Hall effect relates the interaction of photons to electrons. It is thus not only a quantum effect, but an electrodynamic phenomenon.

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