Tuning of band-gap of phononic crystals with initial confining pressure*

Feng Rong-Xin(冯荣欣)[a] and Liu Kai-Xin(刘凯欣)[a)b)†]

[a] *LTCS and Department of Mechanics & Aerospace Engineering, College of Engineering, Peking University, Beijing* 100871, *China*

[b] *Center for Applied Physics and Technology, Peking University, Beijing* 100871, *China*

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The mechanism of the shift of the band-gap in phononic crystal (PC) with different initial confining pressures is studied experimentally and numerically. The experimental results and numerical analysis simultaneously indicate that the confining pressure can efficiently tune the location in and the width of the band-gap. The present work provides a basis for tuning the band-gap of phononic crystal in engineering applications.

Keywords: phononic crystals, band-gap, initial confining pressure, modified split-Hopkinson pressure bar apparatus

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1. Introduction

One or two kinds of materials embedded periodically in a matrix constitute phononic crystal (PC). PC possesses an important property in that it forbids the propagation of elastic wave with a special frequency,[1,2] and because of this it can be used in many engineering fields such as filters, transducers, and vibration isolation technology.[3] Normally, the materials of the components in PC with a special band-gap can be designed by selecting proper dimensions of periodic structures and suitable contrast of the elastic constants or the mass densities of the components. However, if the PC only has a constant distribution of band-gaps in the frequency domain, its application in most of the engineering fields will not be efficient enough. Therefore, the study on tuning and controlling the band-gap of PC by applying an electric field in smart material or loading an initial stress in common material has attracted much attention.[4−15] Although these kinds of smart materials each have a speedy response, the voltage used to control the band-gap is very high.[11−14] Therefore, the harsh conditions restrict the application of PC made from smart materials.

It is easier to tune the phononic band-gap with initial stress than the smart material. On the one hand, the pattern of nano/micro-scale periodic structure in PC is changed by initial stress, so that the phononic band-gap can be tuned by applying different stresses.[4−6] On the other hand, the speeds of P-wave and S-wave in an elastic solid can be affected by the initial stress.[16,17] This means that the initial stress can change the wavelength, thereby leading to the change of the location of band-gap. Actually, the effect of initial stress on the phononic band-gap has been theoretically studied in a rod-piezoelectric PC.[7] Experimental research on one-dimensional rod-PC made from common materials has been done.[18] Moreover, it is known that the band-gap of PC is also affected by the contrast of the Poisson's ratio between the embedded material and the matrix material.[19,20] When initial stress is exerted on the material, it restricts the strain in one direction and it may change the condition of the elastic wave through the PC. This effect is similar to the elastic wave propagating through materials with different elastic parameters.[21,22] Hence, the confining pressure changes the strain in the confining direction and therefore tunes the band-gap of the PC. However, up to now, this research work has not been carried out.

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†Corresponding author. E-mail: kliu@pku.edu.cn

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In this work, we first present an experimental research on the shift of the band-gap in steel/epoxy PC with an initial confining pressure by using a modified split-Hopkinson pressure bar apparatus (SHPB). Then, the energy transmission of the PC is simulated by the finite element method (FEM). Based on the above experimental results and the numerical simulation, the shift mechanism of the band-gap under confining pressure is clarified.

2. Experimental method and results

In our experiment, four unit cells constitute the specimens of the PC, and each unit includes two sub-cells as shown in Fig. 1 (steel and epoxy marked as 1 and 2, respectively). The diameters of all the sub-cells are 14.5 mm. The height of all sub-cells is 10 mm and the filling fraction of PC is 50%. The material constants are listed in Table 1.

The equipment with a confining pressure is a modified SHPB experiment system. The components of the system are an air-gun, a striker, an incident bar, a transmitted bar, and an oil vessel. The incident bar and the transmitted bar are made of steel with a Young's modulus of 210 GPa. The diameter of each bar is 14.5 mm and the lengths of the incident bar and the transmitted bar are 550 mm and 400 mm, respectively. The striker bar is a 5-mm long steel disc. A special oil vessel device is added to the conventional SHPB to exert confining pressure on the specimen. The values of the confining pressure can be obtained from the pressure meter.

Table 1. Material constants.

Material	Mass density ρ/(kg/m^3)	Young modulus E/GPa	Poisson's ratio
Steel	7780	210.6	0.3
Epoxy	31180	4.35	0.367

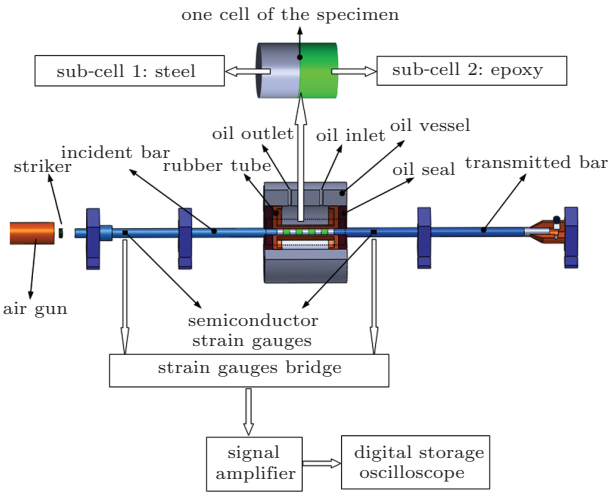


Fig. 1. (colour online) Schematic diagram of the experimental system.

Figure 1 shows the sketch of the experimental equipment. In the first step of the experiment, the confining stress is applied to the specimen by the oil vessel. Pressure up to 30 MPa is obtained from a fluid pump with machine oil as the confining medium. In a second step, the striker is accelerated by an air gun. A pulse signal is generated by the striker through impacting the incident bar. The semiconductor strain gauges on the incident bar and the transmitted bar capture the input and output signals, which are amplified by a signal amplifier and stored by a digital storage oscilloscope (as shown in Fig. 1). The stored signals are shown in Fig. 2 when different confining stresses are applied.

A fast-Fourier transform (FFT) method is used to transform these signals into those in the frequency domain. By computing the ratio of the output signal amplitude to the input signal amplitude, the frequency response function (FRF) can be used to describe the band-gap of the PCs. The FRF is a function of frequency and is given by

$$H(f) = 20 \log_{10} \frac{X(f)}{F(f)}. \tag{1}$$

Here, $X(f)$ is the amplitude of the transmitted wave signal through the FFT analysis, $F(f)$ is the amplitude of the incident wave signal, and $H(f)$ is the decibels (dB) of the attenuated transmission of the signal.

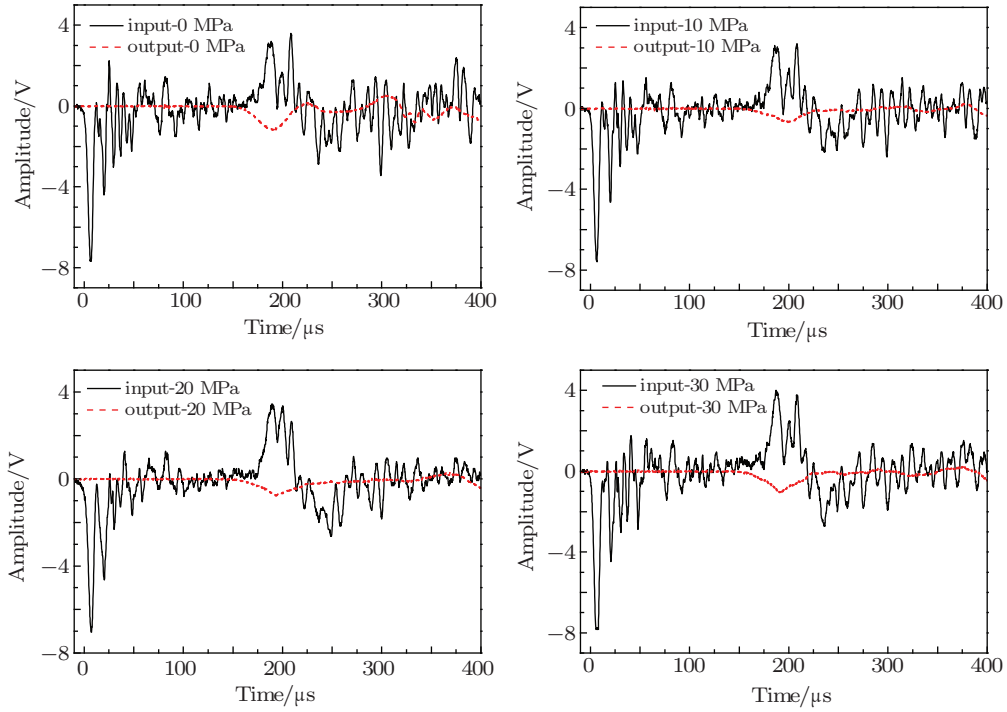


Fig. 2. (colour online) Signals from incident/transmitted bar with different confining stresses.

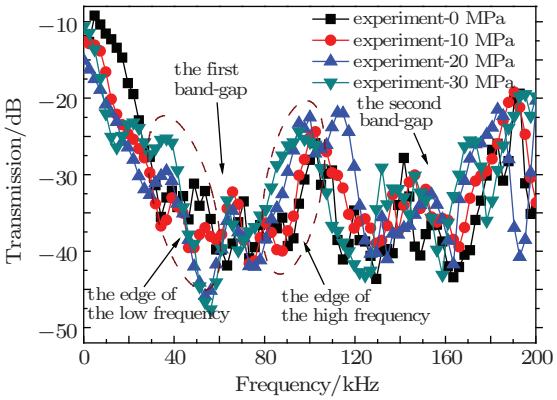


Fig. 3. (colour online) Band-gap location of steel–epoxy PCs with different initial confining pressures.

In this work, the different values of initial confining pressure σ_0 (0, 10, 20, and 30 MPa) are applied to the PC sequentially. Their effects on the transmission of the signal are shown in Fig. 3. The curves represent different transmissions when the specimens are loaded under different values of initial confining pressure. Two regions of large attenuation can be seen in each curve and are so-called band-gaps. Comparing the four curves of the steel/epoxy PC shown in Fig. 3, it can be found that the high frequency edge of the first band-gap gradually shifts toward lower frequencies when the confining pressure increases. A similar behaviour is found for the second band-gap. These results clearly show the band-gap relocation induced by initial confining pressures, thus, we may tune the band-gap of PC by properly changing this parameter.

3. Numerical simulation by the finite element method

In the experiment, it is shown that the band-gap is changed by the initial confining pressure. To further investigate its effect on tuning the band-gap of phononic crystals, we use FEM to simulate the specimen (the steel/epoxy PC). Software ANSYS 12.0 is used to build up the axisymmetric model, in which the dimensions of the model are comparable to those of the specimen. Since the coherence of the displacement and stress on the interface of steel and epoxy, the interface between these two materials is "glued" together in PC. The epoxy can be regarded as being embedded in the steel matrix. The maximum size of the element in the model is set to be 1.44 mm. In cylindrical coordinates, the location of the exciting surface is $Z = 0$ cm and the location of the responsive surface is $Z = 9$ cm. The location of the confining surface is $R = 7.2$ mm along with the specimen (shown in Fig. 4).

The harmonic response analysis module of ANSYS 12.0 is used to calculate the powerful transmis-

sion of the PC. We set the value of the 1 micro-strain on the nodes on the exciting surface. When different confining pressures (0, 10, 20, and 30 MPa) are exerted on the confining surface, the different response data can be obtained from a central node in the responsive surface. The style of data is "Amplitude/Phase". In order to compare them with the experimental results, we show the logarithm of the ratio of the output signal amplitude to the input signal amplitude (these amplitudes are obtained from the simulation).

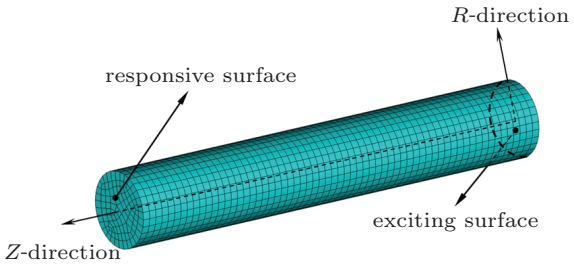


Fig. 4. (colour online) Meshes of the PC in ANSYS.

4. Discussion

The energy transmission of the numerical simulation (solid line) and the corresponding experimental results (solid line with solid circles) with the same initial confining pressure are given in Fig. 5. In general, a similar trend can be found both in the numerical simulation and in the experimental results as shown in Fig. 3; the band-gap can be clearly seen. Taking the first band-gap for illustration, when the initial confining pressure rises from 0 MPa to 30 MPa, the first band-gap interval of steel–epoxy PC changes from [23.0507; 95.0334] kHz to [23.0259; 94.6604] kHz. As indicated by the dashed line in Fig. 5, the lower and higher edges of the first band-gap can be found in the simulation results. They shift with the initial confining pressure increasing. From the simulation results in Fig. 5, we can obtain the variations of the edges of the first band-gap (see Fig. 6). In Fig. 6, both of the lower (line with solid squares) and higher (line with solid circles) edges of the band-gap shift toward lower frequencies with the increase of initial confining pressure. Moreover, the width of the first band-gap (the difference between the lower edge and the higher edge) of the PC becomes smaller when the initial confining pressure turns larger (from 0 MPa to 30 MPa). This trend is also confirmed by our experimental results.

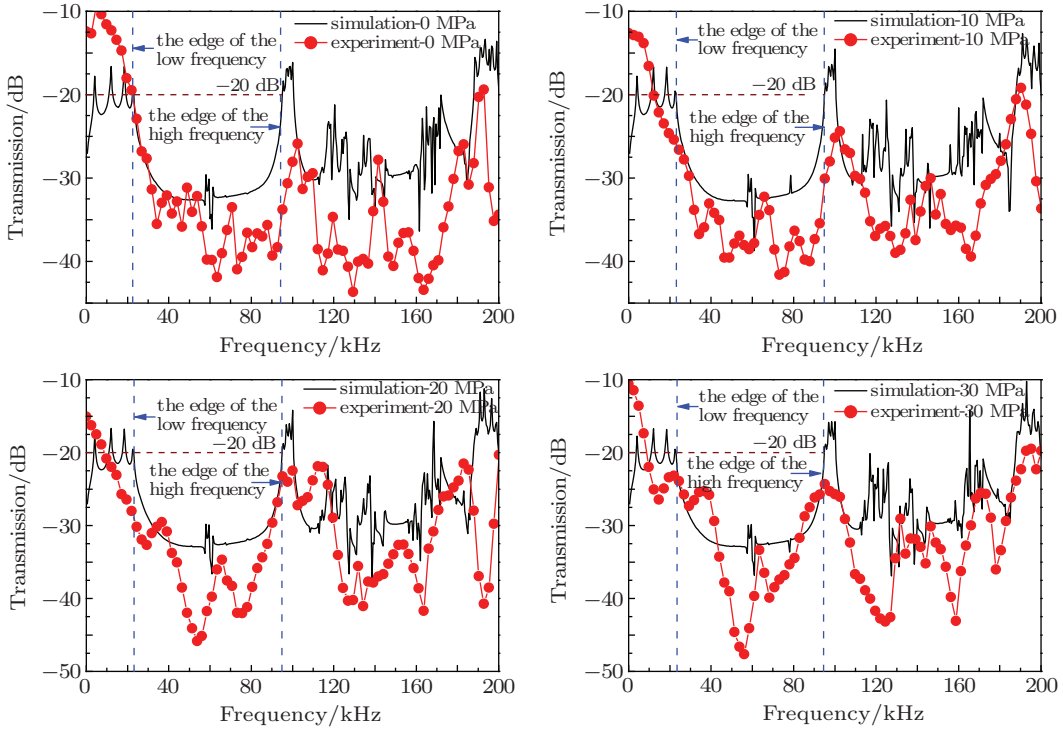


Fig. 5. (colour online) Transmissions of energy for steel–epoxy PC with four initial confining pressures (0, 10, 20, and 30 MPa).

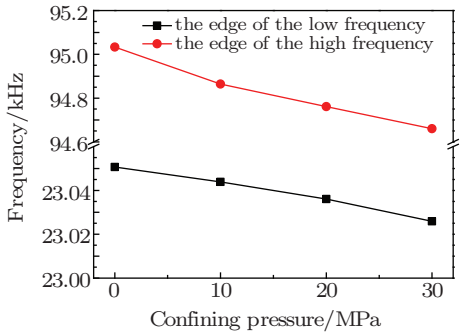


Fig. 6. (colour online) Edges of the first band-gap of the steel/epoxy PC with different initial confining pressures.

Actually, the location of the band-gap can be influenced both by the geometrical dimensions of the cell and the elastic parameters of the material in the cell.[18–20] When we exert an initial confining pressure on the steel/epoxy PC, the strain in the radial direction of the PC, which is induced by the longitudinal waves, will be limited by the initial confining pressure. Namely, the initial confining pressure changes the elastic response of the cylinder to the longitudinal waves.[21,23] According to Hoger's theory of finite deformation in the right circular cylinder, the elastic coefficient of a body could be changed by the residual stress.[24] The corresponding displacement field is given as

$$u = ari_r + bzj_z, \tag{2}$$

where a and b are constants. The stress on the boundary is given by

$$\sigma = \begin{cases} 2b\mu' + (2a+b)\,\lambda' = 0, & \text{on } \partial B_z, \\ 2a\mu' + (2a+b)\,\lambda' = \sigma_0, & \text{on } \partial B_r, \end{cases} \tag{3}$$

where ∂B_r represents the confining surface; ∂B_z represents the top/bottom surface of the circular cylinders; σ_0 is the confining pressure. According to Eq. (3), we can obtain the incremental elastic lame constants to be

$$\mu' = \frac{\sigma_0}{2\,(a-b)}, \quad \lambda' = \frac{\sigma_0 b}{(2a+b)\,(b-a)}. \tag{4}$$

Hence, the Poission's ratio dependent on the confining pressure σ_0 is expressed as

$$\nu = \frac{\lambda_0 + \lambda'}{2\,[(\lambda_0 + \lambda') + (\mu_0 + \mu')]}, \tag{5}$$

where μ_0 and λ_0 are the lame constants of the circular cylinder in the initial state. A simulation through ANSYS is performed to calculate the length and the diameter of the deformed cylinder. The length and the diameter are $(1+b)\,Z_0$ and $2\,(1+a)\,r_0$, respectively and then a and b are obtained. As the initial confining pressure increases from 0 MPa to 30 MPa, the Poisson's ratio of the epoxy in PCs decreases from 0.367 to 0.339. Meanwhile the decrement of ratio of the steel in PCs is small and can be ignored.

On the other hand, the effect of the initial confining pressure on the longitudinal strain of the PC is very small compared with the radial one. This means that the confining pressure will not change the longitudinal expansion of the PC. Therefore, in this case, the location of the band-gap is not influenced by the change of the geometrical dimensions of the cell. It is affected only by the elastic parameter. In the following, we are going to investigate the variation of the band-gap with Poisson's ratio when a longitudinal wave is applied.

Three numerical examples in concordance with the above sample used in the experiment are calculated by the software ANSYS. However, the confining pressure is not applied in the simulation. Its effect is included in the Poisson's ratio of the epoxy in the PC. The Young's modulus and densities of steel/epoxy and the Poisson's ratio of steel are shown in Table 1. From Fig. 7, we can see that the high frequency edge of the first band-gap shifts toward lower frequencies with the decrement of the Poisson's ratio of epoxy. Meanwhile, the width of this band-gap becomes narrow, which is consistent with the findings in Fig. 6.

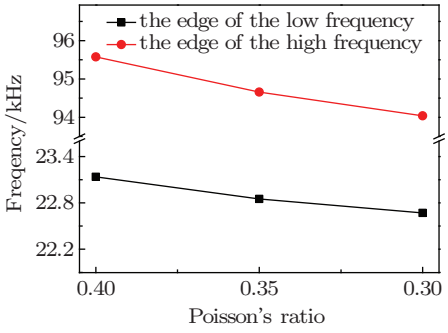


Fig. 7. (colour online) Variations of the band-gap with the equivalent Poisson's ratio of the epoxy.

5. Conclusions

The effect of initial confining pressure on the property of the PC is studied experimentally and numerically. By applying initial confining pressure, the location of the band-gap can be shifted toward the lower frequencies and the width of the band-gap can also be changed. The initial confining pressure affects the frequency of the band-gap due to the fact that it changes the Possion's ratio of the epoxy in PC. Thus,

we can obtain the desired frequency of the band-gap by choosing an appropriate confining pressure, which is beneficial to engineering applications in the future.

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