Novel delay-distribution-dependent stability analysis for continuous-time recurrent neural networks with stochastic delay*

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In this paper, the problem of delay-distribution-dependent stability is investigated for continuous-time recurrent neural networks (CRNNs) with stochastic delay. Different from the common assumptions on time delays, it is assumed that the probability distribution of the delay taking values in some intervals is known a priori. By making full use of the information concerning the probability distribution of the delay and by using a tighter bounding technique (the reciprocally convex combination method), less conservative asymptotic mean-square stable sufficient conditions are derived in terms of linear matrix inequalities (LMIs). Two numerical examples show that our results are better than the existing ones.

Keywords: recurrent neural networks, stochastic delay, mean-square stability, linear matrix inequality

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1. Introduction

Over the past decades, recurrent neural networks (RNNs) have been well studied owing to their applications in image processing, signal processing, pattern recognition, and optimization problems.[1,2] In practice, due to the finite speed of information processing, time delays frequently occur and are often a source of instability and oscillations in RNNs. Considerable effort has been devoted to stability analysis of RNNs with delays, and a lot of results on this topic have been reported in the literature. To mention a few, robust stability was investigated in Refs. [2]–[18]. As for exponential stability, we refer readers to Refs. [19]–[21], and the references therein.

On the other hand, it can be seen from the existing references that the stability criteria were derived based only on the information of variation range of the time delay. Actually, the time delay in some RNNs is often existent in a stochastic fashion.[18,19] That is to say, the observation of communication delay of the data also includes the probability distribution of the delay values. For a given CRNNs, it can be measured that there exists a small number ε such that $\mathrm{Prob}\{\tau(t) > d\} < \varepsilon$, where d is a constant. In other words, the possible value of the delay may be very large, but the probability of the delay taking such a large value is very small. It is well known that, the more delay information is considered in the analysis and synthesis, the less conservative the sufficient condition will be. To the best of the authors' knowledge, up to now, little attention has been paid to the study of stability of CRNNs with stochastic delay when considering the information of both the variation range and the probability distribution of the time delay.

Recently, various approaches have been proposed to obtain stability criteria for time-delay neural networks such as the free-weighting matrix method[3–7,18–20] and the delay decomposition approach.[8–13] In those previous works, the expression $-\tau_{01}\int_{t-\tau_1}^{t-\tau(t)} \dot{\boldsymbol{z}}^{\mathrm{T}}(\xi)\boldsymbol{R}_2\dot{\boldsymbol{z}}(\xi)\mathrm{d}\xi$ was always enlarged as $-(\tau_1-\tau(t))\int_{t-\tau_1}^{t-\tau(t)} \dot{\boldsymbol{z}}^{\mathrm{T}}(\xi)\boldsymbol{R}_2\dot{\boldsymbol{z}}(\xi)\mathrm{d}\xi$. Unlike the previous works, the reciprocally convex combination technique was introduced in this paper. The term $-(\tau(t)-\tau_0)\int_{t-\tau_1}^{t-\tau(t)} \dot{\boldsymbol{z}}^{\mathrm{T}}(\xi)\boldsymbol{R}_2\dot{\boldsymbol{z}}(\xi)\mathrm{d}\xi$ is also taken into account. What is more, we can directly handle the

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inversely weighted convex combination of quadratic terms of integral quantities by utilizing the reciprocally convex combination technique.[22]

Motivated by the above statements, this paper deals with the problem of delay-distribution-dependent stability analysis for CRNNs with stochastic delay. We will present sufficient conditions in terms of LMIs such that the CRNNs are asymptotically stable in the mean-square. Since probabilistic interval delay and a reciprocally convex combination technique are adopted, the merit of the obtained results is less conservatism, which will be shown via detailed examples.

Notation Throughout the paper, the notation "$*$" is used as an ellipsis for terms induced by symmetry. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\boldsymbol{X}^{\mathrm{T}}$ denotes the transpose of matrix $\boldsymbol{X}$. Ones(m,n) is an m by n matrix of ones. $\boldsymbol{X} \geq \boldsymbol{Y}$ (respectively, $\boldsymbol{X} > \boldsymbol{Y}$), where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are symmetric matrices, means that $\boldsymbol{X}$–$\boldsymbol{Y}$ is positive-semidefinite (respectively, positive-definite). $E\{x\}$ stands for the expectation of the stochastic variable x. Prob$\{\cdot\}$ means the occurrence probability of the event "$\cdot$".

2. System description and preliminaries

Consider the following CRNNs with time-varying interval delay

$$\begin{aligned}\dot{\boldsymbol{x}}(t) =& -\boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{A}\boldsymbol{g}(\boldsymbol{x}(t)) \\ &+ \boldsymbol{B}\boldsymbol{g}(\boldsymbol{x}(t-\tau(t))) + \boldsymbol{u}, \end{aligned} \tag{1}$$

where $\boldsymbol{x}(\cdot) = [x_1(\cdot), x_2(\cdot), \ldots, x_n(\cdot)]^{\mathrm{T}} \in \mathbb{R}^n$ is the neuron state vector, $\boldsymbol{u} = [u_1, u_2, \ldots, u_n]^{\mathrm{T}} \in \mathbb{R}^n$ is a constant external input,

$$\boldsymbol{g}(\boldsymbol{x}(\cdot)) = [g_1(x_1(\cdot), g_2(x_2(\cdot), \ldots, g_n(x_n(\cdot)]^{\mathrm{T}} \in \mathbb{R}^n$$

denotes the neuron activation function,

$$\boldsymbol{C} = \mathrm{diag}\{c_1, c_2, \ldots, c_n\} > 0$$

is a positive diagonal matrix, and $\boldsymbol{A}$ and $\boldsymbol{B}$ are the connection weight matrix and the delayed connection weight matrix, respectively.

As reported in Refs. [18] and [19], network tomography, the technique that is concerned with inference of delay distribution of networked systems, makes it possible that the probability distribution of time delay $\tau(t)$ in practical systems is known in advance. Therefore, to fully use the delay information, a delay-distribution-dependent CRNN model is adopted here.

For the time-varying delay $\tau(t)$, we define a random variable $\alpha(t)$

$$\begin{aligned}\tau_1(t) =& \alpha(t)\tau(t), \\ & \text{and } \alpha(t) = 1, \text{ if } \tau(t) \in [\tau_0, \tau_1), \\ \tau_2(t) =& (1-\alpha(t))\tau(t), \\ & \text{and } \alpha(t) = 0, \text{ if } \tau(t) \in [\tau_1, \tau_2), \end{aligned} \tag{2}$$

where τ_1 is the medium value of the delay $\tau(t)$ and is assumed to be known, $\tau_1(t)$ means that the distribution of $\tau(t)$ is within a lower end range of $[\tau_0, \tau_1)$, while $\tau_2(t)$ means that the distribution of $\tau(t)$ is within a higher end range of $[\tau_1, \tau_2)$. It is assumed that $\dot{\tau}_1(t) \leq \mu_1$ and $\dot{\tau}_2(t) \leq \mu_2$. It is also assumed that $\alpha(t)$ in Eq. (2) is a Bernoulli-distributed sequence with

$$\begin{aligned}&\mathrm{Prob}\{\alpha(t) = 1\} = E\{\alpha(t)\} = \bar{\alpha}, \\ &\mathrm{Prob}\{\alpha(t) = 0\} = 1 - E\{\alpha(t)\} = 1 - \bar{\alpha}, \end{aligned} \tag{3}$$

where $0 \leq \bar{\alpha} \leq 1$ is a constant. It can be shown that

$$\begin{aligned}&E\{\alpha(t) - \bar{\alpha}\} = 0, \\ &E\{(\alpha(t) - \bar{\alpha})^2\} = \bar{\alpha}(1-\bar{\alpha}). \end{aligned} \tag{4}$$

In terms of the definition of $\alpha(t)$, the system (1) can be rewritten as

$$\begin{aligned}\dot{\boldsymbol{x}}(t) =& -\boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{A}\boldsymbol{g}(\boldsymbol{x}(t)) + \alpha(t)\boldsymbol{B}\boldsymbol{g}(\boldsymbol{x}(t-\tau_1(t))) \\ &+ (1-\alpha(t))\boldsymbol{B}\boldsymbol{g}(\boldsymbol{x}(t-\tau_2(t))) + \boldsymbol{u}. \end{aligned} \tag{5}$$

It is assumed that each neuron activation function in system (1), $g_j(\cdot), j = 1, 2, \ldots, n$, satisfies the following condition

$$\begin{aligned}&0 \leq \frac{g_j(x) - g_j(y)}{x - y} \leq \sigma_j, \\ &\forall x, y \in \mathbb{R}, x \neq y, j = 1, 2, \ldots, n, \end{aligned} \tag{6}$$

where σ_j $(j = 1, 2, \ldots, n)$ are positive constants.

Just like Ref. [18], we can prove that there exists an equilibrium point of system (5) according to Brouwer's fixed point theorem. Now, we denote $\boldsymbol{x}^* = [x_1^*, x_2^*, \ldots, x_n^*]^{\mathrm{T}}$ as the equilibrium point of system (5). Then, in order to simplify the equation, we make the following transformation by the change of variables $\boldsymbol{z}(\cdot) = \boldsymbol{x}(\cdot) - \boldsymbol{x}^*$. Under this transformation, system (5) can be rearranged as

$$\begin{aligned}\dot{\boldsymbol{z}}(t) =& -\boldsymbol{C}\boldsymbol{z}(t) + \boldsymbol{A}\boldsymbol{f}(\boldsymbol{z}(t)) + \alpha(t)\boldsymbol{B}\boldsymbol{f}(\boldsymbol{z}(t-\tau_1(t))) \\ &+ (1-\alpha(t))\boldsymbol{B}\boldsymbol{f}(\boldsymbol{z}(t-\tau_2(t))), \end{aligned} \tag{7}$$

where

$$f_j(z_j(\cdot)) = g_j(z_j(\cdot) + x_j^*) - g_j(x_j^*). \tag{8}$$

Note that the functions $f_j(\cdot)$ $(j = 1, 2, \ldots, n)$ satisfy the following condition

$$0 \leq \frac{f_j(z_j)}{z_j} \leq \sigma_j,$$
$$f_j(0) = 0, \forall z_j \neq 0, j = 1, 2, \ldots, n, \tag{9}$$

which is equivalent to

$$f_j(z_j)[f_j(z_j) - \sigma_j z_j] \leq 0,$$
$$f_j(0) = 0, j = 1, 2, \ldots, n. \tag{10}$$

Remark 1 In this paper, we assume a certain probability distribution of the time delay. It is worth mentioning that when $\alpha(t) = 0$ or 1, which means $\tau(t) = \tau_1(t) = \tau_2(t)$, equation (7) reduces to $\dot{\boldsymbol{z}}(t) = -\boldsymbol{C}\boldsymbol{z}(t) + \boldsymbol{A}\boldsymbol{f}(\boldsymbol{z}(t)) + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{z}(t - \tau(t)))$, which is exactly the system treated in Refs. [3]–[14]. In this sense, the model considered in this paper is more general than those in the literature. Therefore, it is of significance to investigate the stability for the CRNNs described by Eqs. (7)–(10).

Remark 2 Note that system (7) makes full use of the probability distribution information of the delay. Therefore, less conservative results can be obtained than those without considering the nonuniform nature of the delay distribution, such as those in Refs. [5] and [8]. This will be demonstrated by numerical examples.

Towards the end of this section, we introduce the following lemma that is useful in deriving the main results of this paper.

Lemma 1[22] Let $f_1, f_2, \ldots, f_N : \mathbb{R}^m \to \mathbb{R}$ have positive values in an open subset $\boldsymbol{D}$ of $\mathbb{R}^m$. Then, the reciprocally convex combination of f_i over $\boldsymbol{D}$ satisfies

$$\min_{\{\delta_i | \delta_i > 0, \sum_i \delta_i = 1\}} \sum_i \frac{1}{\delta_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t)$$

subject to:

$$\left\{ g_{i,j} : \mathbb{R}^m \to \mathbb{R}, g_{j,i}(t) \triangleq g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \geq 0 \right\}.$$

3. Main results

To facilitate further development, equation (7) can be rewritten as

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{\varphi}(t) + (\alpha(t) - \bar{\alpha})\boldsymbol{\omega}(t), \tag{11}$$

where

$$\begin{aligned} \boldsymbol{\varphi}(t) &= -\boldsymbol{C}\boldsymbol{z}(t) + \boldsymbol{A}\boldsymbol{f}(\boldsymbol{z}(t)) + \bar{\alpha}\boldsymbol{B}\boldsymbol{f}(\boldsymbol{z}(t - \tau_1(t))) \\ &\quad + (1 - \bar{\alpha})\boldsymbol{B}\boldsymbol{f}(\boldsymbol{z}(t - \tau_2(t))), \\ \boldsymbol{\omega}(t) &= \boldsymbol{B}[\boldsymbol{f}(\boldsymbol{z}(t - \tau_1(t))) - \boldsymbol{f}(\boldsymbol{z}(t - \tau_2(t)))]. \end{aligned}$$

Theorem 1 For some given scalars $\tau_0 \leq \tau_1 \leq \tau_2, \tau_{01}, \tau_{12}, \mu_1, \mu_2, \bar{\alpha}$, the system in Eq. (11) with Eq. (10) is asymptotically stable in the mean square, if there exist positive-definite matrices $\boldsymbol{P}$, $\boldsymbol{Q}_i$ $(i = 1, \ldots, 7)$, $\boldsymbol{R}_j$, $\boldsymbol{S}_j = \mathrm{diag}\{s_{j1}, s_{j2}, \ldots, s_{jn}\}$ $(j = 1, 2, 3)$, $\boldsymbol{\lambda} = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$, $\boldsymbol{\sigma} = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_n\}$, and $\boldsymbol{M}_1$, $\boldsymbol{M}_2$ of appropriate dimensions such that the following linear matrix inequalities (LMIs) conditions hold

$$\begin{bmatrix} \boldsymbol{R}_2 & \boldsymbol{M}_1 \\ * & \boldsymbol{R}_2 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \boldsymbol{R}_3 & \boldsymbol{M}_2 \\ * & \boldsymbol{R}_3 \end{bmatrix} \geq 0,$$
$$\boldsymbol{\Xi} = \begin{bmatrix} \boldsymbol{\Xi}_{11} & * & * \\ \boldsymbol{\Xi}_{21} & \boldsymbol{\Xi}_{22} & * \\ \boldsymbol{\Xi}_{31} & 0 & \boldsymbol{\Xi}_{33} \end{bmatrix} < 0, \tag{12}$$

where

$$\boldsymbol{\Xi}_{11} = \begin{pmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12} & \bar{\alpha}\boldsymbol{P}\boldsymbol{B} & (1-\bar{\alpha})\boldsymbol{P}\boldsymbol{B} & \boldsymbol{R}_1 & 0 & 0 & 0 & 0 \\ * & \boldsymbol{\Lambda}_{22} & \bar{\alpha}\boldsymbol{\lambda}\boldsymbol{B} & (1-\bar{\alpha})\boldsymbol{\lambda}\boldsymbol{B} & 0 & 0 & 0 & 0 & 0 \\ * & * & \boldsymbol{\Lambda}_{33} & 0 & 0 & \boldsymbol{S}_2\boldsymbol{\sigma} & 0 & 0 & 0 \\ * & * & * & \boldsymbol{\Lambda}_{44} & 0 & 0 & 0 & \boldsymbol{S}_3\boldsymbol{\sigma} & 0 \\ * & * & * & * & \boldsymbol{\Lambda}_{55} & \boldsymbol{R}_2 - \boldsymbol{M}_1 & \boldsymbol{M}_1 & 0 & 0 \\ * & * & * & * & * & \boldsymbol{\Lambda}_{66} & \boldsymbol{R}_2 - \boldsymbol{M}_1 & 0 & 0 \\ * & * & * & * & * & * & \boldsymbol{\Lambda}_{77} & \boldsymbol{R}_3 - \boldsymbol{M}_2 & \boldsymbol{M}_2 \\ * & * & * & * & * & * & * & \boldsymbol{\Lambda}_{88} & \boldsymbol{R}_3 - \boldsymbol{M}_2 \\ * & * & * & * & * & * & * & * & -\boldsymbol{Q}_7 - \boldsymbol{R}_3 \end{pmatrix},$$

$\boldsymbol{\Xi}_{22} = \boldsymbol{\Xi}_{33} = -\mathrm{diag}\{\boldsymbol{R}_1, \boldsymbol{R}_2, \boldsymbol{R}_3\}$,
$\boldsymbol{\Xi}_{21} = \boldsymbol{\Pi} \times \mathrm{Ones}(3,9) \times \mathrm{diag}\{-\boldsymbol{C}, \boldsymbol{A}, \bar{\alpha}\boldsymbol{B}, (1-\bar{\alpha})\boldsymbol{B}, 0, 0, 0, 0, 0\}$,
$\boldsymbol{\Xi}_{31} = \boldsymbol{\Pi} \times \mathrm{Ones}(3,9) \times \mathrm{diag}\left\{0, 0, \sqrt{\bar{\alpha}(1-\bar{\alpha})}\boldsymbol{B}, -\sqrt{\bar{\alpha}(1-\bar{\alpha})}\boldsymbol{B}, 0, 0, 0, 0, 0\right\}$

with
$\boldsymbol{\Lambda}_{11} = -\boldsymbol{P}\boldsymbol{C} - \boldsymbol{C}\boldsymbol{P} + \boldsymbol{Q}_1 + \boldsymbol{Q}_3 + \boldsymbol{Q}_5 - \boldsymbol{R}_1$, $\boldsymbol{\Lambda}_{12} = \boldsymbol{P}\boldsymbol{A} - \boldsymbol{C}\boldsymbol{\lambda} + \boldsymbol{\sigma}\boldsymbol{S}_1$, $\boldsymbol{\Lambda}_{22} = \boldsymbol{\lambda}\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{\lambda} + \boldsymbol{Q}_2 + \boldsymbol{Q}_4 - 2\boldsymbol{S}_1$,
$\boldsymbol{\Lambda}_{33} = (\mu_1 - 1)\boldsymbol{Q}_2 - 2\boldsymbol{S}_2$, $\boldsymbol{\Lambda}_{44} = (\mu_2 - 1)\boldsymbol{Q}_4 - 2\boldsymbol{S}_3$, $\boldsymbol{\Lambda}_{55} = -\boldsymbol{Q}_5 + \boldsymbol{Q}_6 - \boldsymbol{R}_1 - \boldsymbol{R}_2$, $\boldsymbol{\Lambda}_{66} = (\mu_1 - 1)\boldsymbol{Q}_1 - 2\boldsymbol{R}_2 + \boldsymbol{M}_1 + \boldsymbol{M}_1^{\mathrm{T}}$,
$\boldsymbol{\Lambda}_{77} = -\boldsymbol{Q}_6 + \boldsymbol{Q}_7 - \boldsymbol{R}_2 - \boldsymbol{R}_3$, $\boldsymbol{\Lambda}_{88} = (\mu_2 - 1)\boldsymbol{Q}_3 - 2\boldsymbol{R}_3 + \boldsymbol{M}_2 + \boldsymbol{M}_2^{\mathrm{T}}$, $\tau_{01} = \tau_1 - \tau_0$, $\tau_{12} = \tau_2 - \tau_1$,
$\boldsymbol{\Pi} = \mathrm{diag}\{\tau_0\boldsymbol{R}_1, \tau_{01}\boldsymbol{R}_2, \tau_{12}\boldsymbol{R}_3\}$.

Proof Construct a new Lyapunov function as

$$V(\boldsymbol{z}(t)) = \sum_{i=1}^{3} V_i(\boldsymbol{z}(t)), \tag{13}$$

$$V_1(\boldsymbol{z}(t)) = \boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{z}(t) + 2\sum_{i=1}^{n}\lambda_i\int_0^{z_i(t)} f_i(s)\mathrm{d}s,$$

$$\begin{aligned} V_2(\boldsymbol{z}(t)) = {} & \int_{t-\tau_1(t)}^{t} [\boldsymbol{z}^{\mathrm{T}}(s)\boldsymbol{Q}_1\boldsymbol{z}(s) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(s))\boldsymbol{Q}_2\boldsymbol{f}(\boldsymbol{z}(s))]\mathrm{d}s \\ & + \int_{t-\tau_2(t)}^{t} [\boldsymbol{z}^{\mathrm{T}}(s)\boldsymbol{Q}_3\boldsymbol{z}(s) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(s))\boldsymbol{Q}_4\boldsymbol{f}(\boldsymbol{z}(s))]\mathrm{d}s + \int_{t-\tau_0}^{t} \boldsymbol{z}^{\mathrm{T}}(s)\boldsymbol{Q}_5\boldsymbol{z}(s)\mathrm{d}s \\ & + \int_{t-\tau_1}^{t-\tau_0} \boldsymbol{z}^{\mathrm{T}}(s)\boldsymbol{Q}_6\boldsymbol{z}(s)\mathrm{d}s + \int_{t-\tau_2}^{t-\tau_1} \boldsymbol{z}^{\mathrm{T}}(s)\boldsymbol{Q}_7\boldsymbol{z}(s)\mathrm{d}s, \end{aligned}$$

$$\begin{aligned} V_3(\boldsymbol{z}(t)) = {} & \tau_0\int_{-\tau_0}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{z}}(s)\mathrm{d}s\mathrm{d}\theta + \tau_{01}\int_{-\tau_1}^{-\tau_0}\int_{t+\theta}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{z}}(s)\mathrm{d}s\mathrm{d}\theta \\ & + \tau_{12}\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_3\dot{\boldsymbol{z}}(s)\mathrm{d}s\mathrm{d}\theta. \end{aligned}$$

Let $\mathscr{L}$ be the weak infinitesimal generator of the random process. Then, the expectation of $\mathscr{L}V(\boldsymbol{z}(t))$ for the evolution of $V(\boldsymbol{z}(t))$ along the solution of system (11) is given by

$$E\{\mathscr{L}V_1(\boldsymbol{z}(t))\} = 2E\{\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{z}}(t) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t))\boldsymbol{\lambda}\dot{\boldsymbol{z}}(t)\} = 2[\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{P} + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t))\boldsymbol{\lambda}]\boldsymbol{\varphi}(t), \tag{14}$$

$$\begin{aligned} E\{\mathscr{L}V_2(\boldsymbol{z}(t))\} \leq {} & \boldsymbol{z}^{\mathrm{T}}(t)(\boldsymbol{Q}_1 + \boldsymbol{Q}_3 + \boldsymbol{Q}_5)\boldsymbol{z}(t) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t))(\boldsymbol{Q}_2 + \boldsymbol{Q}_4)\boldsymbol{f}(\boldsymbol{z}(t)) \\ & - (1-\mu_1)[\boldsymbol{z}^{\mathrm{T}}(t-\tau_1(t))\boldsymbol{Q}_1\boldsymbol{z}(t-\tau_1(t)) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t-\tau_1(t)))\boldsymbol{Q}_2\boldsymbol{f}(\boldsymbol{z}(t-\tau_1(t)))] \\ & - (1-\mu_2)[\boldsymbol{z}^{\mathrm{T}}(t-\tau_2(t))\boldsymbol{Q}_3\boldsymbol{z}(t-\tau_2(t)) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t-\tau_2(t)))\boldsymbol{Q}_4\boldsymbol{f}(\boldsymbol{z}(t-\tau_2(t)))] \\ & + \boldsymbol{z}^{\mathrm{T}}(t-\tau_0)(\boldsymbol{Q}_6 - \boldsymbol{Q}_5)\boldsymbol{z}(t-\tau_0) + \boldsymbol{z}^{\mathrm{T}}(t-\tau_1)(\boldsymbol{Q}_7 - \boldsymbol{Q}_6)\boldsymbol{z}(t-\tau_1) \\ & - \boldsymbol{z}^{\mathrm{T}}(t-\tau_2)\boldsymbol{Q}_7\boldsymbol{z}(t-\tau_2), \end{aligned} \tag{15}$$

$$\begin{aligned} E\{\mathscr{L}V_3(\boldsymbol{z}(t))\} = {} & E\Big\{\dot{\boldsymbol{z}}^{\mathrm{T}}(t)[\tau_0^2\boldsymbol{R}_1 + \tau_{01}^2\boldsymbol{R}_2 + \tau_{12}^2\boldsymbol{R}_3]\dot{\boldsymbol{z}}(t) - \tau_0\int_{t-\tau_0}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{z}}(s)\mathrm{d}s \\ & - \tau_{01}\int_{t-\tau_1}^{t-\tau_0} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{z}}(s)\mathrm{d}s - \tau_{12}\int_{t-\tau_2}^{t-\tau_1} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_3\dot{\boldsymbol{z}}(s)\mathrm{d}s\Big\}. \end{aligned} \tag{16}$$

According to (11), we obtain

$$E\{\dot{\boldsymbol{z}}^{\mathrm{T}}(t)\boldsymbol{\Theta}\dot{\boldsymbol{z}}(t)\} = \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\Theta}\boldsymbol{\varphi}(t) + \bar{\alpha}(1-\bar{\alpha})\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\Theta}\boldsymbol{\omega}(t), \tag{17}$$

where

$$\boldsymbol{\Theta} = \tau_0^2\boldsymbol{R}_1 + \tau_{01}^2\boldsymbol{R}_2 + \tau_{12}^2\boldsymbol{R}_3.$$

According to the Jensen integral inequality approach, we have

$$-\tau_0 \int_{t-\tau_0}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{z}}(s)\mathrm{d}s \leq \begin{bmatrix} \boldsymbol{z}(t) \\ \boldsymbol{z}(t-\tau_0) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\boldsymbol{R}_1 & \boldsymbol{R}_1 \\ * & -\boldsymbol{R}_1 \end{bmatrix} \begin{bmatrix} \boldsymbol{z}(t) \\ \boldsymbol{z}(t-\tau_0) \end{bmatrix}. \tag{18}$$

On the other hand

$$\begin{aligned} -\tau_{01} \int_{t-\tau_1}^{t-\tau_0} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{z}}(s)\mathrm{d}s &= -\tau_{01} \int_{t-\tau_1}^{t-\tau_1(t)} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{z}}(s)\mathrm{d}s - \tau_{01} \int_{t-\tau_1(t)}^{t-\tau_0} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{z}}(s)\mathrm{d}s \\ &\leq -\delta_1[\boldsymbol{z}(t-\tau_1(t)) - \boldsymbol{z}(t-\tau_1)]^{\mathrm{T}}\boldsymbol{R}_2[\boldsymbol{z}(t-\tau_1(t)) - \boldsymbol{z}(t-\tau_1)] \\ &\quad - \delta_2[\boldsymbol{z}(t-\tau_0) - \boldsymbol{z}(t-\tau_1(t))]^{\mathrm{T}}\boldsymbol{R}_2[\boldsymbol{z}(t-\tau_0) - \boldsymbol{z}(t-\tau_1(t))], \end{aligned} \tag{19}$$

where

$$\delta_1 = \frac{\tau_{01}}{\tau_1 - \tau_1(t)}, \quad \delta_2 = \frac{\tau_{01}}{\tau_1(t) - \tau_0}.$$

According to Lemma 1, expression (19) can be turned into the following form

$$\leq -\begin{bmatrix} \boldsymbol{z}(t-\tau_0) - \boldsymbol{z}(t-\tau_1(t)) \\ \boldsymbol{z}(t-\tau_1(t)) - \boldsymbol{z}(t-\tau_1) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{R}_2 & \boldsymbol{M}_1 \\ * & \boldsymbol{R}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{z}(t-\tau_0) - \boldsymbol{z}(t-\tau_1(t)) \\ \boldsymbol{z}(t-\tau_1(t)) - \boldsymbol{z}(t-\tau_1) \end{bmatrix}. \tag{20}$$

Similarly, we can show that

$$-\tau_{12} \int_{t-\tau_2}^{t-\tau_1} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_3\dot{\boldsymbol{z}}(s)\mathrm{d}s \leq -\begin{bmatrix} \boldsymbol{z}(t-\tau_1) - \boldsymbol{z}(t-\tau_2(t)) \\ \boldsymbol{z}(t-\tau_2(t)) - \boldsymbol{z}(t-\tau_2) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{R}_3 & \boldsymbol{M}_2 \\ * & \boldsymbol{R}_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{z}(t-\tau_1) - \boldsymbol{z}(t-\tau_2(t)) \\ \boldsymbol{z}(t-\tau_2(t)) - \boldsymbol{z}(t-\tau_2) \end{bmatrix}, \tag{21}$$

in expressions (20) and (21), $\begin{bmatrix} \boldsymbol{R}_2 & \boldsymbol{M}_1 \\ * & \boldsymbol{R}_2 \end{bmatrix} \geq 0$, $\begin{bmatrix} \boldsymbol{R}_3 & \boldsymbol{M}_2 \\ * & \boldsymbol{R}_3 \end{bmatrix} \geq 0$, respectively.

From Eq. (10), let $\boldsymbol{S}_j = \mathrm{diag}\{s_{j1}, s_{j2}, \ldots, s_{jn}\}$ $(j = 1, 2, 3)$. Then we obtain

$$2[\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t))\boldsymbol{S}_1\boldsymbol{\sigma}\boldsymbol{z}(t) - \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t))\boldsymbol{S}_1\boldsymbol{f}(\boldsymbol{z}(t))] \geq 0, \tag{22}$$

$$2[\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t-\tau_1(t)))\boldsymbol{S}_2\boldsymbol{\sigma}\boldsymbol{z}(t-\tau_1(t)) - \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t-\tau_1(t)))\boldsymbol{S}_2\boldsymbol{f}(\boldsymbol{z}(t-\tau_1(t)))] \geq 0, \tag{23}$$

$$2[\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t-\tau_2(t)))\boldsymbol{S}_3\boldsymbol{\sigma}\boldsymbol{z}(t-\tau_2(t)) - \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t-\tau_2(t)))\boldsymbol{S}_3\boldsymbol{f}(\boldsymbol{z}(t-\tau_2(t)))] \geq 0. \tag{24}$$

Combining expressions (14)–(24) and using Schur complements, we obtain

$$E\{\mathscr{L}V(\boldsymbol{z}(t))\} \leq \boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{\Xi}\boldsymbol{\xi}(t), \tag{25}$$

where

$$\begin{aligned} \boldsymbol{\xi}^{\mathrm{T}}(t) = \big[&\boldsymbol{z}^{\mathrm{T}}(t),\ \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t)),\ \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t-\tau_1(t))),\ \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{z}(t-\tau_2(t))),\ \boldsymbol{z}^{\mathrm{T}}(t-\tau_0), \\ &\boldsymbol{z}^{\mathrm{T}}(t-\tau_1(t)),\ \boldsymbol{z}^{\mathrm{T}}(t-\tau_1),\ \boldsymbol{z}^{\mathrm{T}}(t-\tau_2(t)),\ \boldsymbol{z}^{\mathrm{T}}(t-\tau_2)\big]. \end{aligned} \tag{26}$$

According to Theorem 1, we have $E\{\mathscr{L}V(\boldsymbol{z}(t))\} < 0$. This means that $E\{\mathscr{L}V(\boldsymbol{z}(t))\} < -\rho\|\boldsymbol{z}(t)\|^2$ for a sufficiently small $\rho > 0$ ensures system (11) asymptotically stable in the mean square. This completes the proof.

As point out in Remark 1, when $\alpha(t) = 1$ or 0, that is, there is only one delay interval with $\tau(t) = \tau_1(t) = \tau_2(t)$, system (7) is equivalent to

$$\dot{\boldsymbol{z}}(t) = -\boldsymbol{C}\boldsymbol{z}(t) + \boldsymbol{A}\boldsymbol{f}(\boldsymbol{z}(t)) + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{z}(t-\tau(t))). \tag{27}$$

In this case, $\tau(t)$ takes values in the interval $[\tau_0, \tau_1]$ and satisfies $\dot{\tau}(t) \leq \mu$. Based on Theorem 1, the following corollary is obtained.

Corollary 1 For some given scalars $\tau_0 \leq \tau_1, \tau_{01}, \mu$, the system in Eq. (27) with Eq. (10) is asymptotically stable in the mean square, if there exist positive-definite matrices $\boldsymbol{P}$, $\boldsymbol{Q}_i$ $(i = 1, \ldots, 4)$, $\boldsymbol{R}_j$, $\boldsymbol{S}_j$ =

$\mathrm{diag}\{s_{j1}, s_{j2}, \ldots, s_{jn}\}$ $(j = 1, 2)$, $\boldsymbol{\lambda} = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$, $\boldsymbol{\sigma} = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_n\}$, and $\boldsymbol{M}_1$ of appropriate dimensions such that the following LMIs conditions hold

$$\begin{bmatrix} \boldsymbol{R}_2 & \boldsymbol{M}_1 \\ * & \boldsymbol{R}_2 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \boldsymbol{\Upsilon}_{11} & * \\ \boldsymbol{\Upsilon}_{21} & \boldsymbol{\Upsilon}_{22} \end{bmatrix} < 0, \tag{28}$$

where

$$\boldsymbol{\Upsilon}_{11} = \begin{pmatrix} \boldsymbol{\Gamma}_{11} & \boldsymbol{P}\boldsymbol{A} - \boldsymbol{C}\boldsymbol{\lambda} + \boldsymbol{\sigma}\boldsymbol{S}_1 & \boldsymbol{P}\boldsymbol{B} & \boldsymbol{R}_1 & 0 & 0 \\ * & \boldsymbol{\lambda}\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{\lambda} + \boldsymbol{Q}_2 - 2\boldsymbol{S}_1 & \boldsymbol{\lambda}\boldsymbol{B} & 0 & 0 & 0 \\ * & * & (\mu-1)\boldsymbol{Q}_2 - 2\boldsymbol{S}_2 & 0 & \boldsymbol{S}_2\boldsymbol{\sigma} & 0 \\ * & * & * & \boldsymbol{\Gamma}_{44} & \boldsymbol{R}_2 - \boldsymbol{M}_1 & \boldsymbol{M}_1 \\ * & * & * & * & \boldsymbol{\Gamma}_{55} & \boldsymbol{R}_2 - \boldsymbol{M}_1 \\ * & * & * & * & * & -\boldsymbol{Q}_4 - \boldsymbol{R}_2 \end{pmatrix},$$

$\boldsymbol{\Upsilon}_{22} = -\mathrm{diag}\{\boldsymbol{R}_1, \boldsymbol{R}_2\}$,

$\boldsymbol{\Upsilon}_{21} = \mathrm{diag}\{\tau_0\boldsymbol{R}_1, \tau_{01}\boldsymbol{R}_2\} \times \mathrm{Ones}(2, 6) \times \mathrm{diag}\{-\boldsymbol{C}, \boldsymbol{A}, \boldsymbol{B}, 0, 0, 0\}$ with

$$\boldsymbol{\Gamma}_{11} = -\boldsymbol{P}\boldsymbol{C} - \boldsymbol{C}\boldsymbol{P} + \boldsymbol{Q}_1 + \boldsymbol{Q}_3 - \boldsymbol{R}_1, \ \boldsymbol{\Gamma}_{44} = -\boldsymbol{Q}_3 + \boldsymbol{Q}_4 - \boldsymbol{R}_1 - \boldsymbol{R}_2, \ \boldsymbol{\Gamma}_{55} = (\mu-1)\boldsymbol{Q}_1 - 2\boldsymbol{R}_2 + \boldsymbol{M}_1 + \boldsymbol{M}_1^{\mathrm{T}}.$$

Remark 3 In Corollary 1, if setting $\boldsymbol{Q}_1 = \boldsymbol{Q}_2 = 0$, we can obtain a delay-derivative-independent stability criterion of CRNNs for unknown μ. The reason is that this criterion is not interrelated to the parameter μ when $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ are fixed to zero.

Remark 4 In this paper, we deal with the interval time-varying delay by using reciprocally convex combination technique.[22] To reduce the conservatism, $-\tau_{01}\int_{t-\tau_1}^{t-\tau(t)} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{z}}(s)\mathrm{d}s$ is not over bounded with $-(\tau_1-\tau(t))\int_{t-\tau_1}^{t-\tau(t)} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{z}}(s)\mathrm{d}s$, but rather $-(\tau(t)-\tau_0)\int_{t-\tau_1}^{t-\tau(t)} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{z}}(s)\mathrm{d}s$ is taken into account. What is more, we can directly handle the inversely weighted convex combination of quadratic terms of integral quantities.

4. Numerical examples

In this section, two examples are given to demonstrate the benefits of the proposed method.

Example 1 Consider the following CRNNs as in Refs. [5] and [8] with

$$\boldsymbol{C} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix}, \quad \sigma_1 = 0.4, \quad \sigma_2 = 0.8.$$

Now assuming that $\tau_0 = 0$ and $\mu = 0.8$, according to Corollary 1, we obtained that the maximum allowable delay bound is 2.3534. Also, if the variation probability of $\tau(t)$ is known, the corresponding upper bound for the delay can be solved by using Theorem 1. Here, we assume $\mu_1 = \mu_2 = \mu = 0.8$, $\tau_1 = 2.3534$, and $\bar{\alpha} = 0.1$ and we obtained the feasible variation intervals is $[0, 2.3534] \cup [2.3534, 4.6109]$. It is worth mentioning that τ_1 is not limited to the range of $[0, 2.3534]$. The corresponding upper bounds of τ_2 for various μ and $\bar{\alpha}$ derived by Theorem 1, and Corollary 1 and methods in Refs. [5] and [8] are listed in Table 1.

Table 1. Maximum allowable bound τ_2 for different μ and $\bar{\alpha}$.

Parameters	Methods	$\mu = 0.8$	$\mu = 0.9$	$\mu = 1.0$	$\mu = 1.1$
–	He[5]	2.3534	1.6050	1.5103	1.5103
–	Zhang[8]	2.5308	1.7457	1.5416	1.5139
–	Corollary 1	2.3534	1.6050	1.5103	1.5103
$\bar{\alpha} = 0.1$	Theorem 1	4.6109	3.0824	2.8632	2.8632
$\bar{\alpha} = 0.7$	Theorem 1	4.6141	3.0366	2.7799	2.7798

Example 2 Consider the CRNNs as those in Refs. [5] and [8] with

$$\boldsymbol{C} = \mathrm{diag}\{1.2769, 0.6231, 0.9230, 0.4480\},$$

$$\boldsymbol{\sigma} = \mathrm{diag}\{0.1137, 0.1279, 0.7994, 0.2368\},$$

$$\boldsymbol{A} = \begin{pmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{pmatrix},$$

$$\boldsymbol{B} = \begin{pmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{pmatrix}.$$

Now assuming that $\tau_0 = 0$ and $\mu = 0.9$, according to Corollary 1, we obtained that the maximum allowable delay bound is 1.7455. Similarly, we assume $\mu_1 = \mu_2 = \mu = 0.9$, $\tau_1 = 1.7$ and $\bar{\alpha}$=0.1 and we obtained the feasible variation intervals is $[0, 1.7] \cup [1.7, 2.1849]$ by using Theorem 1. The corresponding upper bounds of τ_2 for various $\bar{\alpha}$ derived by Theorem 1 and Corollary 1 and methods in Refs. [5] and [8] are listed in Table 2.

Table 2. Maximum allowable bound τ_2 with μ=0.9 and different $\bar{\alpha}$.

Parameters	Methods	τ_2
–	He[5]	2.0853
–	Zhang[8]	2.1306
–	Corollary 1	1.7455
$\bar{\alpha} = 0.1$	Theorem 1	2.1849
$\bar{\alpha} = 0.7$	Theorem 1	2.4433
$\bar{\alpha} = 0.9$	Theorem 1	2.8223

It is illustrated in Tables 1 and 2 that, when the variation probability of the delay is known, the maximum allowable delay bound may be increased. Obviously, our results are less conservativeness.

5. Conclusion

By defining a new Lyapunov function and by exploiting the probability interval of the time-varying delay, we have obtained new results for the stability analysis of CRNNs with stochastic delay. The merits of the proposed results in this paper are that we have adopted the probability distribution of the delay and a tighter bounding technique. Numerical examples have been given to demonstrate the effectiveness of the presented results and their improvement over existing results.

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