Generalized projective synchronization of fractional-order complex networks with nonidentical nodes

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This paper investigates the synchronization problem of fractional-order complex networks with nonidentical nodes. The generalized projective synchronization criterion of fractional-order complex networks with order $0 < q < 1$ is obtained based on the stability theory of the fractional-order system. The control method which combines active control with pinning control is then suggested to obtain the controllers. Furthermore, the adaptive strategy is applied to tune the control gains and coupling strength. Corresponding numerical simulations are performed to verify and illustrate the theoretical results.

Keywords: fractional-order complex networks, generalized projective synchronization, pinning control, adaptive coupling law

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1. Introduction

Complex networks exist everywhere in various fields in the real world, such as the Internet, the World Wide Web, biological neural networks, social communities, electrical power grids, etc. Complex networks have attracted tremendous attention from various fields, including physical, economical, social and biological sciences since the discovery of the small-world effect[1] and the scale-free feature[2] of complex networks. Particularly, the control and synchronization problem of complex networks has been a focus for many researchers in recent years.[3−11]

On the other hand, the application of fractional calculus rapidly grows because it can describe and model an object more accurately than classical integer methods. Some fractional-order dynamical systems can also behave chaotically, such as the fractional-order Lorenz system, the fractional-order Chen system, the fractional-order Lü system, the fractional-order Duffing system, and so on. Therefore, synchronization of complex networks with fractional-order chaotic nodes has been studied due to its practical application. In Ref. [12], synchronization of fractional-order chaotic systems with a ring connection has been investigated. Network synchronization of star-coupled fractional nonlinear oscillators has been studied in Ref. [13]. Outer synchronization between two different fractional-order general complex dynamical networks has been discussed in Ref. [14]. In Ref. [15] the authors discussed the projective synchronization of a complex networks with fractional-order chaotic nodes, etc.

As is known to all, real-world complex networks usually consist of a large number of nodes. However, it is impractical to assume that all network nodes are identical since some real-world complex networks may consist of different types of nodes. Moreover, it is impractical to add controllers to all nodes in the networks to achieve synchronization. Therefore, it is necessary to investigate synchronization of complex networks with nonidentical nodes. Pinning control has attracted the attention of researchers because the idea of pinning control is to synchronize whole networks by controlling a fraction of network nodes.[16−18]

To the best of our knowledge, generalized projective synchronization (GPS) of fractional-order complex networks with nonidentical nodes has not been considered yet to date. Motivated by the above discussion, we combine active control with pinning control to synchronize the whole networks in this paper.

The rest of this paper is organized as follows. In Section 2, the preliminaries and problem description of fractional-order complex networks are presented. In Section 3, the controllers are designed and some GPS

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criteria are obtained. In Section 4, a numerical example is given to illustrate the effectiveness of our method. Finally, some concluding remarks are presented in Section 5.

2. Preliminaries and problem description

Till now, there are three different definitions for fractional derivatives, that is, Riermann–Liouville, Grünwald-Letnikov, and Caputo definitions.

Definition 1 (See Ref. [19]) The Caputo fractional derivative of order q of a continuous function $f : R^+ \to R$ is defined as follows:

$$\mathrm{D}^q f(t) = J^{m-q} f^{(m)}(t), \quad q > 0, \tag{1}$$

where m is the first integer which is not less than q. J^β is the β order Riemann–Liouville integral operator which can be described with expression

$$J^\beta g(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-\tau)^{\beta-1} g(\tau)\,\mathrm{d}\tau, \tag{2}$$

where Γ stands for the gamma function,

$$\Gamma(z) = \int_0^\infty t^{\mathrm{z}-1}\,\mathrm{e}^{-\mathrm{t}}\mathrm{d}t. \tag{3}$$

Consider a fractional-order complex dynamical networks consisting of N nonidentical nodes, which is characterized by

$$\frac{\mathrm{d}^q \boldsymbol{x}_i(t)}{\mathrm{d}t^q} = f_i(\boldsymbol{x}_i(t)) + c\sum_{j=1}^{N} G_{ij}\boldsymbol{A}\boldsymbol{x}_j, \quad i = 1, 2, \ldots, N, \tag{4}$$

where $0 < q < 1$ is the fractional order; $\boldsymbol{x}_i \in R^n$ is the state vector of the i-th node; $f_i \in R^n$ is a smooth nonlinear function describing the local dynamics of the nodes; $c > 0$ is the coupling strength; $\boldsymbol{A} \in R^{n\times n}$ is an inner-coupling matrix satisfying $\boldsymbol{A} > 0$; $\boldsymbol{G} = (G_{ij})_{N\times N}$ is a diffusive matrix satisfying $\sum_{j=1}^{N} G_{ij} = 0$.

In the following, GPS of the networks (4) is investigated. In order to obtain GPS, some controllers will be added to the networks. Thus, the controlled networks can be described by

$$\frac{\mathrm{d}^q \boldsymbol{x}_i(t)}{\mathrm{d}t^q} = f_i(\boldsymbol{x}_i(t)) + c\sum_{j=1}^{N} G_{ij}\boldsymbol{A}\boldsymbol{x}_j + u_i(t), \quad i = 1, 2, \ldots, N, \tag{5}$$

where $u_i \in R^n$, $i = 1, 2, \ldots, N$ are the controllers to be designed.

Here, let $s(t)$ be a solution of an isolated node satisfying

$$\frac{\mathrm{d}^q \boldsymbol{s}(t)}{\mathrm{d}t^q} = g(\boldsymbol{s}(t)). \tag{6}$$

Define error vectors as $\boldsymbol{e}_i(t) = \boldsymbol{x}_i(t) - \boldsymbol{\Lambda}\boldsymbol{s}(t), i = 1, 2, \ldots, N$, according to the controlled network (5), the error dynamical networks is described by

$$\begin{aligned}\frac{\mathrm{d}^q \boldsymbol{e}_i(t)}{\mathrm{d}t^q} &= f_i(\boldsymbol{x}_i(t)) - \boldsymbol{\Lambda} g(\boldsymbol{s}(t)) \\ &\quad + c\sum_{j=1}^{N} G_{ij}\boldsymbol{A}\boldsymbol{e}_j(t) + u_i(t), \\ &\quad i = 1, 2, \ldots, N,\end{aligned} \tag{7}$$

where $\boldsymbol{\Lambda}$ is a scaling matrix.

Our goal is to design appropriate controllers such that the controlled networks (5) can achieve GPS, that is, $\lim\limits_{t\to\infty} \|\boldsymbol{e}_i(t)\| = \lim\limits_{t\to\infty} \|\boldsymbol{x}_i(t) - \boldsymbol{\Lambda}\boldsymbol{s}(t)\| = 0$, $i = 1, 2, \ldots, N$.

To achieve the above objective, we need the following lemmas and assumption.

Lemma 1 (See Ref. [20]) For a given autonomous fractional-order linear system

$$\frac{\mathrm{d}^q \boldsymbol{x}(t)}{\mathrm{d}t^q} = \boldsymbol{W}\boldsymbol{x} \tag{8}$$

with $\boldsymbol{x}(0) = \boldsymbol{x}_0$, where $\boldsymbol{x} \in R^n$ is the state vector.

(i) Asymptotically stable if and only if $|\arg(\lambda_i)| > q\pi/2, i = 1, 2, \ldots, n$, where $\arg(\lambda_i)$ denotes the argument of the eigenvalue λ_i of $\boldsymbol{W}$. In this case, the component of the state decays towards 0 like $t^{-\mathrm{q}}$;

(ii) Stable if and only if either it is asymptotically stable or those critical eigenvalues which satisfy $|\arg(\lambda_i)| = q\pi/2$, have a geometric multiplicity one.

Due to the above Lemma 1, Ref. [21] gained the following result.

Lemma 2 For a given autonomous fractional-order nonlinear system

$$\frac{\mathrm{d}^q \boldsymbol{x}(t)}{\mathrm{d}t^q} = f(\boldsymbol{x}(t)) \tag{9}$$

with $\boldsymbol{x}(0) = \boldsymbol{x}_0$, where $\boldsymbol{x} \in R^n$ is the state vector, $f(\boldsymbol{x}(t))$ is a nonlinear function vector, if for $0 < q < 1$, there exists $\boldsymbol{P} > 0$, such that

$$J = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\frac{\mathrm{d}^q \boldsymbol{x}}{\mathrm{d}t^{\mathrm{q}}} \le 0, \tag{10}$$

then system (9) is asymptotically stable.

Assumption 1 There exists constant matrix $\boldsymbol{T}_i$, $i = 1, 2, \ldots, N$ such that

$$\begin{aligned}&(\boldsymbol{x}-\boldsymbol{y})^{\mathrm{T}}(f_i(\boldsymbol{x}) - f_i(\boldsymbol{y})) \le (\boldsymbol{x}-\boldsymbol{y})^{\mathrm{T}}\boldsymbol{T}_i\boldsymbol{A}(\boldsymbol{x}-\boldsymbol{y}), \\ &\forall \boldsymbol{x}, \boldsymbol{y} \in R^n.\end{aligned} \tag{11}$$

3. GPS criteria of complex networks

In this section, we present the main results for GPS criteria of the controlled networks (5). To realize GPS of the controlled networks (5), the method which combines active control with pinning control is employed to control the networks. Assume that l nodes are selected, where $l = \lfloor \delta N \rfloor$ represents the integer part of the real number δN. Without loss of generality, rearrange the order of the nodes based on the biggest degree scheme, and let the first l nodes be controlled.

Theorem 1 Suppose that Assumption (11) holds and $\boldsymbol{A}$ is a positive-definite matrix, then GPS of the controlled networks (5) is achieved under the following controllers:

$$\begin{aligned} u_i(t) &= \boldsymbol{\Lambda} g(\boldsymbol{s}(t)) - f_i(\boldsymbol{\Lambda}\boldsymbol{s}(t)) - k_i \boldsymbol{A}\boldsymbol{e}_i, \quad i = 1, \ldots, l, \\ u_i(t) &= \boldsymbol{\Lambda} g(\boldsymbol{s}(t)) - f_i(\boldsymbol{\Lambda}\boldsymbol{s}(t)), \quad i = l+1, \ldots, N, \end{aligned} \tag{12}$$

if the condition

$$\boldsymbol{\Theta} + c\boldsymbol{G} - \boldsymbol{K} \leq 0, \tag{13}$$

is satisfied, where $\boldsymbol{K} = \mathrm{diag}(k_1, k_2, \ldots, k_l, 0, 0, \ldots, 0)$, k_i are positive constants, and

$$\boldsymbol{\Theta} = \mathrm{diag}(\theta_1, \theta_2, \ldots, \theta_N),$$

$\theta_i = \|\boldsymbol{T}_i\|$ and $\boldsymbol{T}_i$ defined in expression (11).

Proof Consider the following function

$$J(t) = \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t) \frac{\mathrm{d}^q \boldsymbol{e}_i(t)}{\mathrm{d}t^q}. \tag{14}$$

Substituting Eqs. (7) and (12) into Eq. (14), we have

$$\begin{aligned} J(t) &= \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t) \Big[f_i(\boldsymbol{x}_i(t)) - \boldsymbol{\Lambda} g(\boldsymbol{s}(t)) \\ &\quad + c \sum_{j=1}^{N} G_{ij} \boldsymbol{A}\boldsymbol{e}_j(t) \Big] + \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t) u_i(t) \\ &= \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t) [f_i(\boldsymbol{x}_i(t)) - f_i(\boldsymbol{\Lambda}\boldsymbol{s}(t))] \\ &\quad + c \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t) \sum_{j=1}^{N} G_{ij} \boldsymbol{A}\boldsymbol{e}_j(t) \\ &\quad - \sum_{i=1}^{l} k_i \boldsymbol{e}_i^{\mathrm{T}}(t) \boldsymbol{A}\boldsymbol{e}_i(t) \\ &\leq \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t) \boldsymbol{T}_i \boldsymbol{A}\boldsymbol{e}_i(t) + c \sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t) G_{ij} \boldsymbol{A}\boldsymbol{e}_j(t) \\ &\quad - \sum_{i=1}^{l} k_i \boldsymbol{e}_i^{\mathrm{T}} \boldsymbol{A}\boldsymbol{e}_i(t) \\ &\leq \boldsymbol{e}^{\mathrm{T}}(t) [(\boldsymbol{\Theta} + c\boldsymbol{G} - \boldsymbol{K}) \otimes \boldsymbol{A}] \boldsymbol{e}(t), \end{aligned}$$

where $\boldsymbol{e}(t) = [\boldsymbol{e}_1^{\mathrm{T}}(t), \boldsymbol{e}_2^{\mathrm{T}}(t), \ldots, \boldsymbol{e}_{\mathrm{N}}^{\mathrm{T}}(t)]^{\mathrm{T}}$.

Since $\boldsymbol{A} > 0$, it is easy to see that if inequality (13) is satisfied, we have $J(t) \leq 0$. From Lemma 2, the controlled networks (5) can achieve GPS. This completes the Proof.

Remark 1 It is easy to see that we choose control gains and coupling strength which are usually larger than the needed one in order to satisfy the conditions (13) in Theorem 1. Therefore, we employ an adaptive strategy to tune control gains $k_i, i = 1, 2, \ldots, l$ and coupling strength c adaptively.

Without loss of generality, the error dynamical networks (7) with the adaptive coupling law can be described by

$$\begin{aligned} \frac{\mathrm{d}^q \boldsymbol{e}_i(t)}{\mathrm{d}t^q} &= f_i(\boldsymbol{x}_i(t)) - \boldsymbol{\Lambda} g(\boldsymbol{s}(t)) \\ &\quad + c(t) \sum_{j=1}^{N} G_{ij} \boldsymbol{A}\boldsymbol{e}_j(t) + u_i(t), \\ &\quad i = 1, 2, \ldots, N, \end{aligned} \tag{15}$$

$$\frac{\mathrm{d}^q c(t)}{\mathrm{d}t^q} = \alpha \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t) \boldsymbol{A}\boldsymbol{e}_i(t), \tag{16}$$

where α is a small positive constant.

Theorem 2 Suppose that Assumption (11) holds and $\boldsymbol{A}$ is a positive-definite matrix, then GPS of the controlled networks (5) is achieved under the following controllers:

$$\begin{aligned} u_i(t) &= \boldsymbol{\Lambda} g(\boldsymbol{s}(t)) - f_i(\boldsymbol{\Lambda}\boldsymbol{s}(t)) - k_i(t) \boldsymbol{A} e_i(t), \\ &\quad i = 1, \ldots, l, \end{aligned} \tag{17}$$

$$u_i(t) = \boldsymbol{\Lambda} g(\boldsymbol{s}(t)) - f_i(\boldsymbol{\Lambda}\boldsymbol{s}(t)), i = l+1, \ldots, N, \tag{18}$$

$$\frac{\mathrm{d}^q k_i(t)}{\mathrm{d}t^q} = q_i \boldsymbol{e}_i^{\mathrm{T}}(t) \boldsymbol{A}\boldsymbol{e}_i(t), i = 1, 2, \ldots, l, \tag{19}$$

where q_i are positive constants.

Proof Consider the following function

$$\begin{aligned} J(t) &= \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t) \frac{\mathrm{d}^{\mathrm{q}} \boldsymbol{e}_i(t)}{\mathrm{d}t^q} + \sum_{i=1}^{l} \frac{k_i(t) - \tilde{k}_i}{q_i} \frac{\mathrm{d}^q k_i(t)}{\mathrm{d}t^{\mathrm{q}}} \\ &\quad + \frac{c(t) - \tilde{c}}{\alpha} \frac{\mathrm{d}^q c(t)}{\mathrm{d}t^{\mathrm{q}}}, \end{aligned} \tag{20}$$

where $\tilde{k}_i$ $(i = 1, 2, \ldots, l)$ and $\tilde{c}$ are positive constants to be determined below, and satisfying $k_i(t) \to \tilde{k}_i$ $(i = 1, 2, \ldots, l)$, $c(t) \to \tilde{c} (t \to \infty)$.

Substituting Eqs. (15), (16) into Eq. (17), we have

$$J(t) = \sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t) \Big[f_i(\boldsymbol{x}_i(t)) - \boldsymbol{\Lambda} g(\boldsymbol{s}(t)) \Big.$$

$$
\begin{aligned}
&+c(t)\sum_{j=1}^{N}G_{ij}\boldsymbol{A}\boldsymbol{e}_j(t)\Big]+\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)u_i(t)\\
&+\sum_{i=1}^{l}\frac{k_i(t)-\tilde{k}_i}{q_i}q_i\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{A}\boldsymbol{e}_i(t)\\
&+\frac{c(t)-\tilde{c}}{\alpha}\alpha\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}e_i(t)\\
=&\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)[f_i(\boldsymbol{x}_i(t))-f_i(\boldsymbol{\Lambda}\boldsymbol{s}(t))\\
&+c(t)\sum_{j=1}^{N}G_{ij}\boldsymbol{A}\boldsymbol{e}_j(t)]-\sum_{i=1}^{l}k_i(t)\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}e_i(t)\\
&+\sum_{i=1}^{l}(k_i(t)-\tilde{k}_i)\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{A}e_i(t)\\
&+(c(t)-\tilde{c})\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}e_i(t)\\
\le&\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{T}_i\boldsymbol{A}\boldsymbol{e}_i(t)\\
&+c(t)\sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)G_{ij}\boldsymbol{A}\boldsymbol{e}_j(t)\\
&-\sum_{i=1}^{l}\tilde{k}_i\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{A}e_i(t)\\
&+(c(t)-\tilde{c})\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{A}e_i(t)\\
\le&\ \boldsymbol{e}^{\mathrm{T}}[(\theta\boldsymbol{I}_{\mathrm{N}}+c(t)\boldsymbol{G}-\tilde{\boldsymbol{K}}\\
&+(c(t)-\tilde{c})\boldsymbol{I}_{\mathrm{N}})\otimes\boldsymbol{A}]\boldsymbol{e}(t),
\end{aligned}
$$

where $\boldsymbol{e}(t)=[\boldsymbol{e}_1^{\mathrm{T}}(t),\boldsymbol{e}_2^{\mathrm{T}}(t),\ldots,\boldsymbol{e}_{\mathrm{N}}^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\theta=\max\|\boldsymbol{T}_i\|$, $\tilde{\boldsymbol{K}}=\mathrm{diag}(\tilde{k}_1,\ldots,\tilde{k}_l,0,\ldots,0)$. If we choose an initial condition $c(0)>0$ and let α be small enough, then $c(t)$ cannot increase quickly. In order to obtain GPS, we choose $k_i(0)>0$ $(i=1,2,\ldots,l)$ and $c(0)>0$ such that $\theta\boldsymbol{I}_{\mathrm{N}}+c(t)\boldsymbol{G}-\tilde{\boldsymbol{K}}+(c(t)-\tilde{c})\boldsymbol{I}_{\mathrm{N}}\le 0$ in the time evolution process. Therefore, according to Theorem 1, GPS of fractional-order complex networks can be achieved. After reaching synchronization, $\boldsymbol{e}_i(t)\to 0$, $i=1,2,\ldots,N$, and $\mathrm{d}^qk_i(t)/\mathrm{d}^qt\to 0$ $(i=1,2,\ldots,l)$, $\mathrm{d}^qc(t)/\mathrm{d}^qt\to 0$, so $k_i(t)\to\tilde{k}_i$ $(i=1,2,\ldots,l)$, $c(t)\to\tilde{c}(t\to\infty)$ are satisfied. This completes the Proof.

4. Numerical simulation

In this section, we take an NW small-world networks for example to verify the effectiveness of the proposed GPS criteria. We assume that the networks consists of N nonidentical nodes, which is described by

$$
\begin{aligned}
&\frac{\mathrm{d}^q\boldsymbol{x}_i(t)}{\mathrm{d}t^q}=f_i(\boldsymbol{x}_i(t))+c\sum_{i=1}^{N}G_{ij}\boldsymbol{A}x_j(t),\\
&\quad i=1,2,\ldots,N,
\end{aligned}
\tag{21}
$$

where $N=50$, $\boldsymbol{A}=\mathrm{diag}(2,1,1)$, and $\boldsymbol{G}=(G_{ij})_{\mathrm{N}\times\mathrm{N}}$ is a diffusive matrix.

The isolated node dynamics is depicted by the fractional-order Liu system

$$
\begin{cases}
\dfrac{\mathrm{d}^qs_1}{\mathrm{d}t^q}=-s_1-s_2^2,\\
\dfrac{\mathrm{d}^qs_2}{\mathrm{d}t^q}=2.5s_2-4s_1s_3,\\
\dfrac{\mathrm{d}^qs_3}{\mathrm{d}t^q}=-5s_3+4s_1s_2.
\end{cases}
\tag{22}
$$

The fractional-order Liu system (19) exhibits chaotic behavior for order $q>0.916$.

The nodes dynamics of the networks are depicted by the fractional-order Lü system

$$
\begin{cases}
\dfrac{\mathrm{d}^qx_{i1}}{\mathrm{d}t^q}=36(x_{i2}-x_{i1}),\\
\dfrac{\mathrm{d}^qx_{i2}}{\mathrm{d}t^q}=20x_{i2}-x_{i1}x_{i3},\\
\dfrac{\mathrm{d}^qx_{i3}}{\mathrm{d}t^q}=-3x_{i3}+x_{i1}x_{i2},
\end{cases}
\tag{23}
$$

and the fractional-order Chen system

$$
\begin{cases}
\dfrac{\mathrm{d}^qx_{i1}}{\mathrm{d}t^q}=35(x_{i2}-x_{i1}),\\
\dfrac{\mathrm{d}^qx_{i2}}{\mathrm{d}t^q}=-7x_{i1}-x_{i1}x_{i3}+28x_{i2},\\
\dfrac{\mathrm{d}^qx_{i3}}{\mathrm{d}t^q}=-3x_{i3}+x_{i1}x_{i2}.
\end{cases}
\tag{24}
$$

The fractional-order Lü system (23) and Chen system (24) exhibit chaotic behavior for order $q>0.9156$ and $q>0.8244$, respectively. In order to obtain GPS of the networks (21), we choose the first 10 nodes to be pinned based on the biggest degree scheme. The initial values $\boldsymbol{x}_i(0)$ are chosen in $[-10,10]$ for $1\le i\le 50$ randomly, and $\boldsymbol{s}(0)=[4,5,6]^{\mathrm{T}}$. The simulation results are shown in Fig. 1. From the simulation results, one can conclude that GPS of the networks (21) is achieved by using the proposed method in this paper. Comparing with works in Refs. [13]–[15], the control method in this paper of only adding feedback controllers to a fraction of the network nodes can reduce the control cost, and it tunes control gains and coupling strength adaptively which avoids the difficulty

in choosing appropriate control gains and coupling strength.

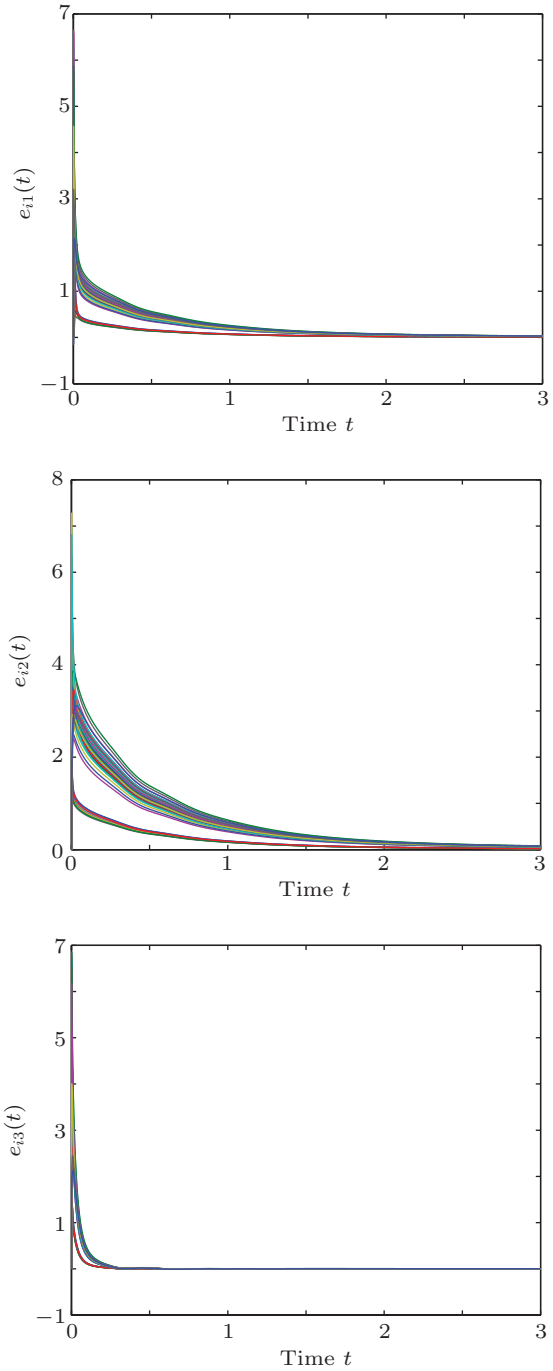


Fig. 1. Synchronization errors e_{i1}, e_{i2}, e_{i3} of the networks (21).

5. Conclusion

This paper investigates the GPS problem of fractional-order complex networks with nonidentical nodes. Based on the fractional-order system stability theory, a GPS criterion is proposed and the controllers are also designed. Furthermore, in order to avoid the difficulty in choosing appropriate control gains and coupling strength, an adaptive strategy is employed to tune control gains and coupling strength. Finally, numerical simulations are offered to show the effectiveness of the theoretical results. In the near future, we will study the synchronization of fractional-order complex networks with time delays and projective synchronization with delayed coupling in fractional-order complex networks simultaneously. Further, we will consider the application of GPS of fractional-order complex networks in secure communication.

References

[1] Watts D J and Strogatz S H 1998 *Nature* **393** 440
[2] Barabsi A L and Albert R 1999 *Science* **286** 509
[3] Wang X F and Chen G R 2002 *Int. J. Bifur. Chaos* **12** 187
[4] Lee T H and Park J H 2009 *Chin. Phys. Lett.* **26** 090507
[5] Wang X F and Chen G R 2002 *IEEE Trans. Circ. Syst.* I **49** 54
[6] Park J H 2009 *Mod. Phys. Lett.* B **23** 1889
[7] Liu T, Zhao J and Hill D J 2009 *Chaos, Solitons and Fractals* **40** 1506
[8] Ji D H, Lee D W, Koo J H, Won S C, Lee S M and Park J H 2011 *Nonlinear Dyn.* **65** 349
[9] Tu L L 2011 *Chin. Phys.* B **20** 030504
[10] Du R J, Dong G G, Tian L X, Zheng S and Sun M 2010 *Chin. Phys.* B **19** 070509
[11] Lee T H, Park J H, Jung H Y, Lee S M and Kwon O M 2012 *Nonlinear Dyn.* **69** 1081
[12] Delshad S S, Asheghan M M and Beheshti M H 2011 *Commun. Nonlinear Sci. Numer. Simul.* **16** 3815
[13] Wang J W and Zhang Y B 2010 *Phys. Lett.* A **374** 1464
[14] Wu X J and Lu H T 2010 *Chin. Phys.* B **19** 070511
[15] Wang M J, Wang X Y and Niu Y J 2011 *Chin. Phys.* B **20** 010508
[16] Liu Z B, Zhang H G and Sun Q Y 2010 *Chin. Phys.* B **19** 090506
[17] Yu W W, Chen G R and Lü J H 2009 *Automatica* **45** 429
[18] Zou Y L and Chen G R 2009 *Chin. Phys.* B **18** 3337
[19] Podlubny I 1999 *Fractional Differential Equations* (San Diego: Academic Press) p. 41
[20] Matignon D 1996 *Proceedings of the IEEE-SMC International Association for Mathematics and Computers in Simulation*, Lille, France, p. 963
[21] Hu J B, Han Y and Zhao L D 2009 *Acta Phys. Sin.* **58** 2235 (in Chinese)