Adaptive fuzzy nonlinear inversion-based control for uncertain chaotic systems*

Liu Heng(刘 恒)[a)†], Yu Hai-Jun(余海军)[b)], and Xiang Wei(向 伟)[a)]

[a)] School of Mathematics and Computer Science, Huainan Normal University, Huainan 232038, China
[b)] School of Physics and Electronic Information, Huainan Normal University, Huainan 232038, China

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This paper presents a robust output feedback control method for uncertain chaotic systems, which comprises a nonlinear inversion-based controller with a fuzzy robust compensator. The proposed controller eliminates the unknown nonlinear function by using a fuzzy system, whose inputs are not the state variables but feedback error signals. The underlying stability analysis as well as parameter update law design are carried out by using the Lyapunov-based technique. The proposed method indicates that the nonlinear inversion-based control approach can also be applied to uncertain chaotic systems. Theoretical results are illustrated through two simulation examples.

Keywords: chaotic systems, nonlinear inversion-based control, adaptive fuzzy control, variable structure control

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1. Introduction

In the past two decades, chaotic systems have been widely used in system engineering because of their desirable features; namely a broadband spectrum, a noise-like wave form, and extreme sensitivity to initial conditions, unpredictable behavior, and parameter variations.[1,2] Chaos has been found in many engineering systems. Chaos theory and chaotic properties have many useful applications in engineering areas such as digital communication, secure communication, power quality and power electronic devices, chemical reaction analysis and design, biological systems, and information processing.[3−5] The problem of chaos control was firstly studied by Ott, Grebogi, and Yorke.[6] Since then, several control techniques have been successfully applied for the control of chaotic systems, including: PID control, observer-based control, adaptive feedback control, sliding mode control, adaptive fuzzy control, adaptive backstepping control, and so on.[7−13] However, when chaotic systems are uncertain, their control problem becomes more challenging. In this area, there are very few constructive procedures for designing satisfactory controllers .

Variable structure control (VSC) is regarded as a good method for controlling uncertain nonlinear systems.[14,15] The VSC provides a robust control scheme which is insensitive to modeling errors, parameter uncertainties, and external disturbances. Meanwhile, there is considerable motivation to use adaptive fuzzy technologies in the control of nonlinear systems. Fuzzy control has a special impact in the control community because the fuzzy controllers have an efficient framework for incorporating linguistic fuzzy information from human experts. Based on the universal approximation theorem,[16,17] some adaptive fuzzy control methods have been proposed for unknown chaotic systems.[2,9,13,18,19] The stability of the underlying closed-loop system is analyzed based on the Lyapunov stability theorem. To cope with the fuzzy approximation error, these adaptive controllers are commonly incorporated with a robust control term that can be a supervisory control,[16,17,20] variable structure control,[13,14] and sliding mode control.

Although the above control schemes work well, the main weak point is that the term including the nominal model of chaotic systems is modeled by neural networks or fuzzy logic systems. In other words, the chaotic system nominal model is usually not included in the controller design. Considering this weak point, the motivation of this paper aims at proposing a hybrid control approach that incorporates the chaotic

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†Corresponding author. E-mail: liuheng122@gmail.com

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system nominal model in the design of the adaptive fuzzy controller. The proposed approach combines a nonlinear inversion-based controller with a fuzzy compensator. The nominal system is controlled by the nonlinear inversion-based controller. The fuzzy logic systems are employed to approximate the unknown nonlinear functions and the VSC is used to eliminating the effects of approximation error. The proposed hybrid controller can guarantee the stability of closed-loop systems. Control simulations are conducted on a multi-scroll chaotic system and a gyro chaotic system to show the effectiveness of the proposed method.

2. Problem formulation and preliminaries

Consider an MIMO chaotic dynamic system which can be represented by the following form

$$\begin{aligned} y_1^{(r_1)} &= f_1(x) + \sum_{j=1}^{p} g_{1j}u_j, \\ &\vdots \\ y_p^{(r_p)} &= f_p(x) + \sum_{j=1}^{p} g_{pj}u_j, \end{aligned} \tag{1}$$

where $x = [y_1, \dot{y}_1, \ldots, y_1^{(r_1-1)}, \ldots, y_p, \dot{y}_p, \ldots, y_p^{(r_p-1)}]^{\mathrm{T}} \in R^r, r = r_1 + r_2 + \ldots + r_p$ is the system state vector which is assumed to be available for measurement. $u = [u_1, \ldots, u_p]^{\mathrm{T}} \in R^p$ and $y = [y_1, \ldots, y_p]^{\mathrm{T}} \in R^p$ are control input and output vector, respectively. $f_i(x), i = 1, \ldots, p$ are unknown nonlinear functions and $g_{ij}, i, j = 1, \ldots, p$ are unknown constant control gains.

If we denote

$$\begin{aligned} y^{(r)} &= [y_1^{(r_1)}, \ldots, y_p^{(r_p)}]^{\mathrm{T}}, \\ F(x) &= [f_1(x), \ldots, f_p(x)]^{\mathrm{T}}, \\ G &= \begin{bmatrix} g_{11} & \cdots & g_{1p} \\ \vdots & \ddots & \vdots \\ g_{p1} & \cdots & g_{pp} \end{bmatrix}, \end{aligned}$$

then, equation (1) can be rewritten as

$$y^{(r)} = F(x) + Gu. \tag{2}$$

Remark 1 In fact system (2) is really large and lots of chaotic systems can be described as Eq. (2), namely: unified chaotic systems,[4,5,12] multi-scroll chaotic systems,[21−24] Chua's circuit,[25] Arneodo's chaotic system,[26] Genesio chaotic systems, gyro systems, Duffing and Holmes' system, Rössler's system, and many others.

The objective of this paper is to construct an adaptive fuzzy controller such that the system output y tracks the reference signal $y_d = [y_{d1}, \ldots, y_{dp}]^{\mathrm{T}}$ and all the signals in the closed-loop system remain bounded. To meet the objective, we make the following assumptions.

Assumption 1 The desired trajectory signal $x_d = [x_{\mathrm{d1}}^{\mathrm{T}}, \ldots, x_{\mathrm{dp}}^{\mathrm{T}}, y_{d1}^{(r_1)}, \ldots, y_{dp}^{(r_p)}]$ with $x_{\mathrm{di}} = [y_{di}, \dot{y}_{di}, \ldots, y_{di}^{(r_i-1)}]^{\mathrm{T}}, i = 1, \ldots, p$ is continuous, bounded, and available for measurement. Therefore we have $x_d \in \Omega \subset R^{r+p}$, where Ω is a known compact set.

Assumption 2 The control gain matrix G is positive-definite.

Let us define the tracking error as

$$e_i = y_{di} - y_i, \quad i = 1, \ldots, p \tag{3}$$

and the filtered tracking error as

$$s_i = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda_i\right)^{r_i-1} e_i(t), \quad i = 1, \ldots, p, \tag{4}$$

equation (4) can be rewritten as

$$\begin{aligned} s_i &= e_i^{(r_i-1)} + (r_i - 1)\lambda_i e_i^{(r_i-2)} + \ldots \\ &\quad + (r_i - 1)\lambda_i^{r_i-2}\dot{e}_i + \lambda_i^{r_i-1} e_i. \end{aligned} \tag{5}$$

Notice if we select $\lambda_i > 0$, then the roots of the polynomial $H_i(s) = s^{r_1-1} + (r_i - 1)\lambda_i s^{r_i-2} + \cdots + (r_i - 1)\lambda_i^{r_i-2}\dot{s} + \lambda_i^{r_i-1} = 0$ are all in the left-half complex plane. In other words, the objective of this work becomes the design of a controller to force the filtered tracking error $s_i(t) \to 0$. Differentiating s_i with respect to time we have

$$\dot{s}_i = v_i - f_i(x) - \sum_{j=1}^{p} g_{ij}u_j \tag{6}$$

with

$$\begin{aligned} &v_i = y_{d1}^{(r_i)} + \gamma_{1,r_i-1} e_1^{(r_i-1)} + \cdots + \gamma_{1,1}\dot{e}_i, \\ &\gamma_{i,j} = \frac{(r_i - 1)!}{(r_i - j)!(j - 1)!}\lambda_i^{r_i-j}, \\ &i = 1, \ldots, p, j = 1, \ldots, r_i - 1. \end{aligned}$$

Denote $s = [s_1, \ldots, s_p]^{\mathrm{T}}, v = [v_1, \ldots, v_p]^{\mathrm{T}}$, then equation (4) can be rewritten in the following compact form

$$\dot{s} = v - F(x) - Gu. \tag{7}$$

Thereafter, equation (7) will be used in the construction of the controller as well as the stability analysis.

2.1. Description of the fuzzy logic system

The fuzzy logic system that employs singleton fuzzification, sum-product inference, and center-of-sets defuzzification is modeled by

$$\alpha(x) = \frac{\sum_{j=1}^{N} \theta_j \prod_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{N} \left[\prod_{i=1}^{n} \mu_{F_i^j}(x_i)\right]}, \tag{8}$$

where $\alpha(x)$ is the output of the fuzzy system, x is the input vector, $\mu_{F_i^j}(x_i)$ is x_i's membership of the j-th rule and θ_j is the centroid of the j-th consequent set. Equation (4) can be rewritten as the following equation

$$\alpha(x) = \theta^{\mathrm{T}} \psi(x) \tag{9}$$

with $\theta = [\theta_1, \ldots, \theta_N]^{\mathrm{T}}$, $\psi(x) = [p_1(x), p_2(x), \ldots, p_N(x)]^{\mathrm{T}}$, and the fuzzy basis function can be written as

$$p_j(x) = \frac{\prod_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{N} \left[\prod_{i=1}^{n} \mu_{F_i^j}(x_i)\right]}.$$

Suppose there are N rules of the fuzzy system used to approximate the unknown function $\alpha(x)$.

Rule i: if x_1 is F_1^i and $\cdots$ and x_n is F_n^i then $\alpha(x)$ is B^i, $i = 1, 2, \ldots, N$.

2.2. Adaptive fuzzy output feedback controller design

If the exact model of the chaotic system dynamics is precisely known, the following input–output nonlinear inversion-based controller (NIC) can be obtained

$$u = G^{-1}(-F(x) + v + Ks), \tag{10}$$

where $K = \mathrm{diag}[k_1, k_2, \ldots, k_p]$ with $k_i > 0$.

However, in most practical situations, precise knowledge of $F(x)$ and G is difficult to obtain because of time-varying uncertainties and measurement errors. In this paper, we assume that $F(x)$ and G can be separated as nominal parts, denoted by $F_1(x)$ and G_1, and uncertain parts, prescribed by $\triangle F(x)$ and $\triangle G$, respectively. As in Assumption 2, we assume that G_1 is positive-definite. Then we have

$$F(x) = F_1(x) + \triangle F(x),\ G = G_1 + \triangle G. \tag{11}$$

Therefore, our objective is to design an adaptive fuzzy output feedback controller which comprises of an NIC controller and a fuzzy robust compensator for unknown chaotic systems. The controller with two components can be described as

$$u(t) = u_0(t) + u_{\mathrm{c}}(t), \tag{12}$$

where $u_0(t)$ is the NIC and can be constructed as

$$u_0(t) = G_1^{-1}(v - F_1(x) + Ks). \tag{13}$$

Introducing Eqs. (11)–(13) to Eq. (7) gives

$$\dot{s} = -Ks - \alpha(x, u) - G_1 u_{\mathrm{c}}, \tag{14}$$

where $\alpha(x, u) = \triangle F(x) - \triangle G u = [\alpha_1(x, u), \ldots, \alpha_p(x, u)]^{\mathrm{T}} \in R^p$ is the modeling uncertainty. To achieve a favorable tracking performance, the compensating controller $u_{\mathrm{c}}(t)$ in Eq. (12) is given by

$$u_{\mathrm{c}}(t) = G_1^{-1}[-u_n(t) - u_r(t)], \tag{15}$$

where $u_n(t) = [u_{n1}, \ldots, u_{np}]^{\mathrm{T}} \in R^p$ is the fuzzy compensator to approximate the online unknown function $\alpha(x, u)$.

In this paper, we propose an adaptive controller using a fuzzy system $\hat{\alpha}(E, \theta)$ to approximate $\alpha(x, u)$ whose inputs are the feedback error signals instead of the state variables. Here $E = [e_1, \dot{e}_1, \ldots, e_1^{r_1 - 1}, \ldots, e_p, \dot{e}_p, \ldots e_p^{r_p - 1}]$. In practice, there is no guarantee that parameter vector θ converge to their ideal values because the persistent excitation (PE) condition is hard to satisfy. Furthermore, the modeling error of the system can only be reduced to within some bound because of structural errors. Since the main objective of a fuzzy controller is to track the reference signals, we employ $\hat{\alpha}(E, \theta)$ to approximate the unknown nonlinear function.

Then, the nonlinear function $\alpha_i(x, u)$ can be approximated, over the compact set Ω, by the fuzzy logic systems (9) as

$$u_{ni}(t) = \hat{\alpha}_i(E, \theta_i) = \theta_i^{\mathrm{T}} \psi(E), \quad i = 1, 2, \ldots, p. \tag{16}$$

Let us define the ideal parameters of θ_i as:

$$\theta_i^* = \arg \min_{\theta_i} \left[\sup_{x \in \Omega} |\alpha_i(x, u) - \hat{\alpha}_i(E, \theta_i)|\right]. \tag{17}$$

Note that the ideal parameter θ_i^* is only for the purposes of analysis, and the value of θ_i^* is not needed when implementing the controller. Define the parameter estimation errors and the fuzzy approximation errors as follows:

$$\tilde{\theta}_i = \theta_i - \theta_i^*, \tag{18}$$

$$\varepsilon_i(E) = \alpha_i(x, u) - \hat{\alpha}_i(E, \theta_i^*) \tag{19}$$

with $\hat{\alpha}_i(E,\theta_i^*) = \theta_i^*\psi(E)$. As in literature,[2,9,13,16,17] we can assume that the fuzzy approximation error is bounded for all $x \in \Omega$, i.e., $|\varepsilon_i(E)| < \bar{\varepsilon}_i, \bar{\varepsilon}_i$ is an unknown constant. Let $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_n(x)]^{\mathrm{T}}$, $\bar{\varepsilon} = [\bar{\varepsilon}_1, \ldots, \bar{\varepsilon}_n]^{\mathrm{T}}$, then we can get $\varepsilon(E) \leq \bar{\varepsilon}$.

From the above analyses, we have

$$\begin{aligned} &\hat{\alpha}(E,\theta) - \alpha(x,u) \\ &= \hat{\alpha}(E,\theta) - \hat{\alpha}(E,\theta^*) + \hat{\alpha}(E,\theta^*) - \alpha(x,u) \\ &= \tilde{\theta}^{\mathrm{T}}\psi(E) - \varepsilon(E), \end{aligned} \tag{20}$$

where $\hat{\alpha}(E,\theta) = [\hat{\alpha}_1(E,\theta_1), \ldots, \hat{\alpha}_n(E,\theta_p)]^{\mathrm{T}}$.

Substituting Eqs. (15), (16), and (20) into Eq. (14) we have

$$\dot{s} = -Ks + \tilde{\theta}^{\mathrm{T}}\psi(E) - \varepsilon(E) + u_r(t). \tag{21}$$

Multiplying s^{T} to Eq. (21) gives

$$\begin{aligned} s^{\mathrm{T}}\dot{s} = &-s^{\mathrm{T}}Ks + \sum_{j=1}^{p} s_j\tilde{\theta}_j^{\mathrm{T}}\psi_j(E) - \sum_{j=1}^{p} s_j\varepsilon_j(E) \\ &+ \sum_{j=1}^{p} s_j u_{rj}(t). \end{aligned} \tag{22}$$

The VSC $u_{ri}(t)$ can be chosen as

$$u_{ri}(t) = -\hat{\bar{\varepsilon}}_i \,\mathrm{sign}(s_i), \quad i = 1, \ldots, p, \tag{23}$$

where $\hat{\bar{\varepsilon}}_i$ is the estimation of the unknown constant $\bar{\varepsilon}_i$.

In order to meet the control objective, $\hat{\bar{\varepsilon}}_i$ and the fuzzy parameters θ_i are updated by

$$\dot{\theta}_i = -\sigma_i s_i \psi(E), \tag{24}$$

$$\dot{\hat{\bar{\varepsilon}}}_i = \rho_i |s_i|, \tag{25}$$

where $\rho_i, \sigma_i > 0$ are design parameters.

Now we are ready to give the following results.

Theorem 1 Considering system (1). Suppose that Assumptions 1 and 2 are satisfied. The control law defined by Eqs. (12), (13), (15), and (16) with the adaption law given by Eqs. (24) and (25) can guarantee the following properties: the tracking error and their derivatives converge asymptotically to zero, i.e., $e_i^j \to 0$ when $t \to \infty$ for $i = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, r_i - 1$, and all signals in the closed-loop systems remain bounded.

Proof Let us consider the following Lyapunov function candidate

$$V = \frac{1}{2}s^{\mathrm{T}}s + \frac{1}{2\rho_i}\sum_{j=1}^{p}\tilde{\bar{\varepsilon}}_j^2 + \frac{1}{2\sigma_j}\sum_{j=1}^{p}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j, \tag{26}$$

where $\tilde{\bar{\varepsilon}}_j = \hat{\bar{\varepsilon}}_j - \bar{\varepsilon}_j$ are the estimation errors of $\bar{\varepsilon}_j$. The time derivative of V is given by

$$\dot{V} = s^{\mathrm{T}}\dot{s} + \frac{1}{\rho_i}\sum_{j=1}^{p}\tilde{\bar{\varepsilon}}_j\dot{\tilde{\bar{\varepsilon}}}_j + \frac{1}{\sigma_j}\sum_{j=1}^{p}\tilde{\theta}_j^{\mathrm{T}}\dot{\theta}_j. \tag{27}$$

From Eq. (22), we have

$$\begin{aligned} s^{\mathrm{T}}\dot{s} \leq &-s^{\mathrm{T}}Ks + \sum_{j=1}^{p} s_j\tilde{\theta}_j\psi_j(E) + \sum_{j=1}^{p}|s_j|\bar{\varepsilon}_j \\ &+ \sum_{j=1}^{p} s_j u_{rj}(t). \end{aligned} \tag{28}$$

Using Eq. (23), inequality (28) becomes

$$s^{\mathrm{T}}\dot{s} \leq -s^{\mathrm{T}}Ks + \sum_{j=1}^{p} s_j\tilde{\theta}_j\psi_j(E) - \sum_{j=1}^{p}|s_j|\tilde{\bar{\varepsilon}}_j. \tag{29}$$

Substituting expressions (24), (25), and (29) into Eq. (27), and after some straight manipulations, yields

$$\dot{V} \leq -s^{\mathrm{T}}K\dot{s}. \tag{30}$$

Then V is always negative, which implies that s_i, $\tilde{\bar{\varepsilon}}$, and $\tilde{\theta} \in L_\infty$. Since s_i, $\tilde{\bar{\varepsilon}}$, and $\tilde{\theta} \in L_\infty$, it implies that x and u are also bounded. Because V is a non-increasing function of time and bounded from below, we know the $\lim_{t\to\infty} V(t) = V(\infty)$ exists. Let us integrate inequality (30) from 0 to ∞, we can obtain

$$\int_0^\infty \sum_{j=1}^{p} k_j s_j^2(t)\mathrm{d}t \leq V(0) - V(\infty), \tag{31}$$

which means $s_i \in L_2$. Then, from Eq. (21) we have $\dot{s} \in L_\infty$.

Finally, since $s_i \in L_2 \cap L_\infty$ and $\dot{s} \in L_\infty$, by using Barbalat's lemma,[27,28] we have $s_i(t) \to 0$ as $t \to \infty$. Therefore, $e_i^j \to 0$ when $t \to \infty$ for $i = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, r_i - 1$. This completes the prof.

Remark 2 To deal with the chattering effects, the discontinuous functions $\mathrm{sign}(s_i)$ can be replaced by any equivalent smooth function, such as arctan, tanh. However, if we use these smooth functions, the tracking errors will only converge to a neighborhood of the origin.

3. Simulation results

Two chaotic systems are utilized to show the effectiveness of the proposed hybrid control scheme.

Example 1 A three-dimensional (3D) saturated multi-scroll chaotic system

A 3D saturated multi-scroll chaotic system can be described by[21−24,29,30]

$$\begin{aligned}\dot{x}_1 &= x_2 - d_2 s(x_2; k_2, h_2, p_2, q_2)/b + u_1 \\ &= f_1(x) + u_1, \\ \dot{x}_2 &= x_3 - d_3 s(x_3; k_3, h_3, p_3, q_3)/c + u_2 \\ &= f_2(x) + u_2, \\ \dot{x}_3 &= -ax_1 - bx_2 - cx_3 + \sum_{i=1}^{3} d_i s(x_i; k_i, h_i, p_i, q_i) + u_3 \\ &= f_3(x) + u_3, \end{aligned} \tag{32}$$

where the saturated function series $s(x; k, h, p, q)$ is described as

$$s(x; k, h, p, q) = \begin{cases} 2q + 1, & \text{if } x > qh + 1, \\ k(x - ih) + 2ik, & \text{if } |x - ih| \le 1, -p \le i \le q, \\ (2i + 1)k, & \text{if } ih + 1 < x < (i + 1)h - 1, -p \le i < q, \\ -(2p + 1)k, & \text{if } x < -ph - 1. \end{cases}$$

When $a = d_1 = 0.7$, $b = c = d_2 = d_3 = 0.8$, $k_1 = 100$, $h_1 = 200$, $k_2 = k_3 = 40$, $h_2 = h_3 = 80$, $p_i = q_i = 2$, the unforced system (32) has a 3D $6 \times 6 \times 6$-grid scroll chaotic attractor, as shown in Fig. 1.

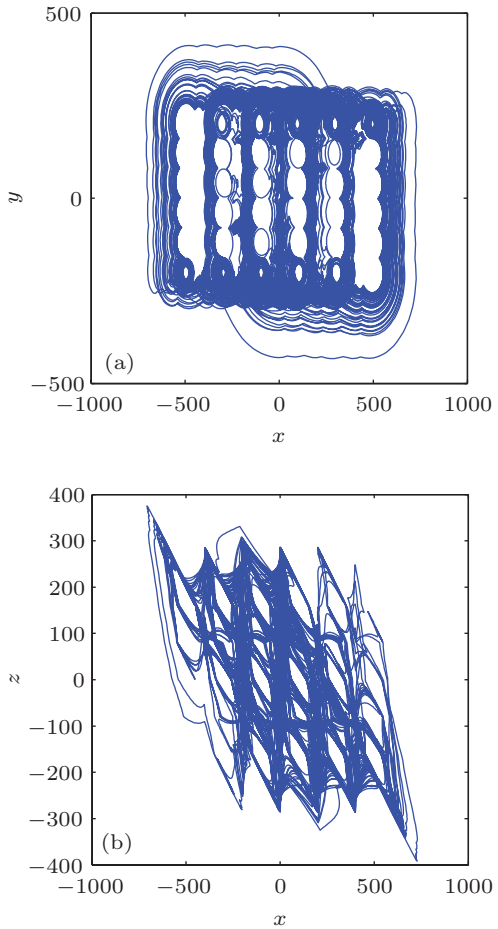


Fig. 1. (colour online) 3D multi-scroll attractor (a) x–y plane; (b) x–z plane.

System (32) can be written as the form (2). In simulation, let $F_1(x) = [f_1(x), f_2(x), f_3(x)]^{\mathrm{T}}$, $\triangle F(x) = F_1(x) \sin t/2$, $G_1 = \mathrm{diag}[1, 1, 1]$, $\triangle G = G_1/2$. Since $r_i = 1, i = 1, 2, 3$, we have $s_i = e_i$.

The initial values are chosen as $x_1(0) = 5$, $x_2(0) = -5$, $x_3(0) = -4$, $\theta_i(0) = \bar{0}$, $\hat{\bar{\varepsilon}}_i(0) = 0$, $i = 1, 2, 3$. The desired trajectories are $x_{\mathrm{d}1} = x_{\mathrm{d}2} = x_{\mathrm{d}3} = \sin t$.

The fuzzy systems have e_1, e_2, and e_3 as inputs. For each variable of inputs, we define six Gaussian membership functions uniformly distributed on the interval $[-5, 5]$.

The design parameters are selected as follows: $k_i = 0.5$, $\rho_i = 20$, $\sigma_i = 10$, $i = 1, 2, 3$. Note that k_i are chosen small in order to see the contribution of the fuzzy systems. The discontinuous function $\mathrm{sign}(e(t))$ has been replaced by smooth function $\arctan(20e(t))$.

Figure 2 shows the simulation results of the proposed scheme. From the simulation results, we can see that the tracking is significantly decreased by using the proposed method in comparison to the control scheme in Ref. [28]. The proposed controller can achieve a better performance in the presence of disturbances and system uncertainties. This is because the control scheme proposed in Ref. [28] does not include a nominal model, and the main controller is only from the fuzzy system which is usually sensitive to its structure and the number of fuzzy rules. However, in the proposed controller, the main controller is an inversion-based controller with the fuzzy system modeling the uncertainties at every step with compensation by the variable structure controller.

Example 2 Gyro chaotic system

The equation governing the motion of the symmetric gyro mounted on a vibrating base in terms of the rotation angle γ (i.e., the angle between the verti-

cal axis and the spin axis of the gyro), is given by[31]

$$\ddot{\gamma} + \vartheta^2 \frac{(1-\cos\gamma)^2}{\sin^3\gamma} - a\sin\gamma + b\dot{\gamma} + c\dot{\gamma}^3 = f\sin\omega t\sin\gamma, \tag{33}$$

where $f\sin\omega t$ is a parametric excitation, $c\dot{\gamma}^3$ and $b\dot{\gamma}$ are nonlinear and linear damping terms, respectively. $\vartheta^2(1-\cos\gamma)^2/\sin^3\gamma - a\sin\gamma$ is nonlinear resilience force. System (33) presents chaotic behavior with $f = 35$, $\vartheta = 10$, $a = 1$, $b = 0.5$, $c = 0.05$, $\omega = 2$, and states initial values of $x(0) = [1.6, 0]^{\mathrm{T}}$, is shown in Fig. 3.

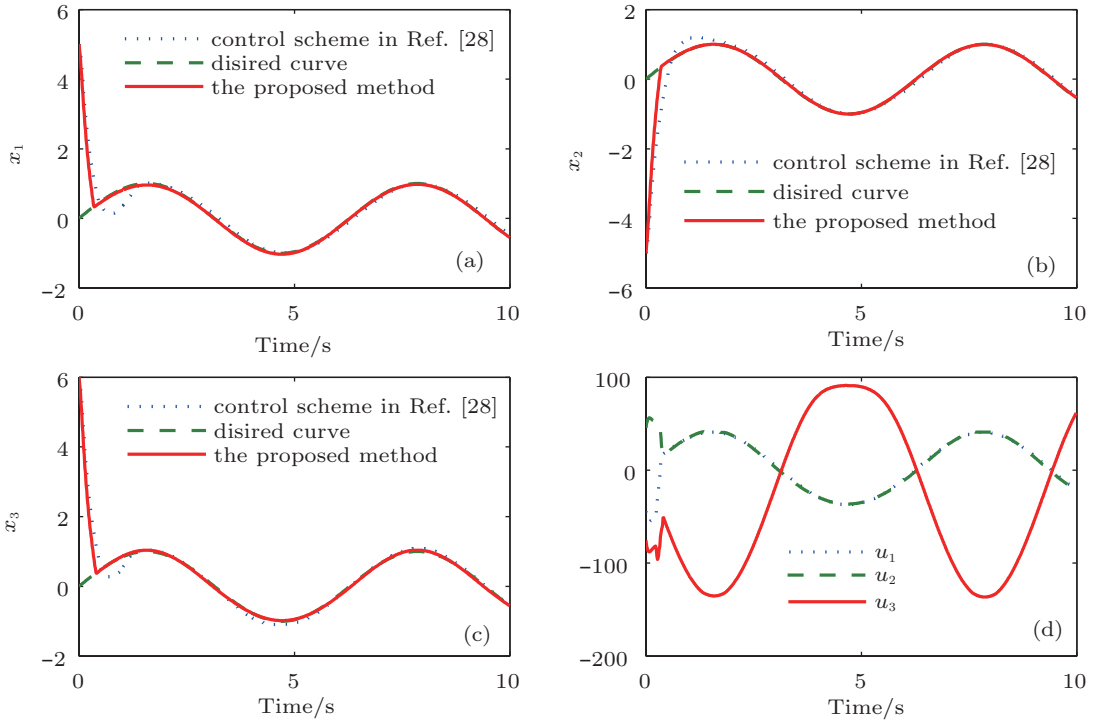


Fig. 2. (color online) Tracking results of Example 1. (a) Tracking of x_1. (b) Tracking of x_2. (c) Tracking of x_3. (d) The control input.

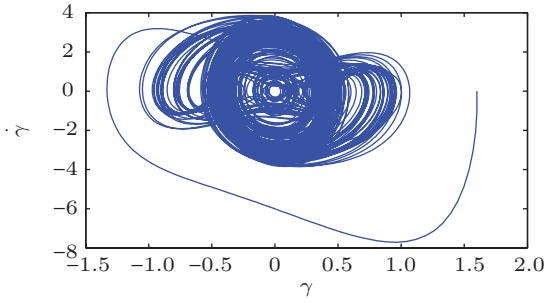


Fig. 3. (color online) Gyro state space trajectory.

The forced gyro system (33) can be rewritten as

$$\ddot{\gamma} = F(\Gamma) + u, \tag{34}$$

where $\Gamma = [\gamma, \dot{\gamma}]^{\mathrm{T}}$ is the state vector,

$$F(\Gamma) = f\sin\omega t\sin\gamma - \vartheta^2\frac{(1-\cos\gamma)^2}{\sin^3\gamma} + a\sin\gamma - b\dot{\gamma} - c\dot{\gamma}^3.$$

In the simulation, let

$$F_1(\Gamma) = F(\Gamma), \quad \triangle F(\Gamma) = F(\Gamma)\sin t/2,$$

$G_1 = 1$, $\triangle G = 1/2$. The initial values are chosen as $\theta(0) = \bar{0}$, $\hat{\bar{\varepsilon}}(0) = 0$.

Let the desired trajectory is

$$x_d(t) = 1.25 - 7.5\,\mathrm{e}^{-t} + 7/20\,\mathrm{e}^{-4t}.$$

The fuzzy system has $e, \dot{e}$ as inputs. For each variable of inputs, we define eleven Gaussian membership functions uniformly distributed on the interval $[-10, 10]$.

The design parameters are chosen as follows: $\lambda = 1$, $\rho = 20$, $\sigma = 10$, $k = 0.5$. The discontinuous function $\mathrm{sign}(s)$ has been replaced by smooth function $\arctan(20s)$.

Figure 4 shows the simulation results of the proposed scheme. These results show the good tracking performance of the proposed method.

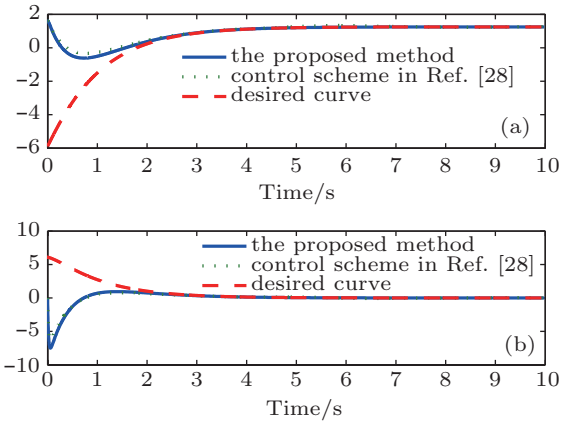


Fig. 4. (color online) Simulation results: (a) tracking of γ, (b) tracking of $\dot{\gamma}$.

4. Conclusion

This paper proposed a nonlinear inversion-based control scheme with fuzzy compensation for uncertain MIMO chaotic systems. The fuzzy logic systems were used to cancel system uncertainties. The variable structure controller was employed to eliminate the effects of the fuzzy approximation errors. The hybrid control method we proposed demonstrated effective and robust control performance.

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