Approximate solution of the spin-one Duffin–Kemmer–Petiau (DKP) equation under a non-minimal vector Yukawa potential in (1+1)-dimensions

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We solve the Duffin–Kemmer–Petiau (DKP) equation with a non-minimal vector Yukawa potential in (1+1)-dimensional space-time for spin-1 particles. The Nikiforov–Uvarov method is used in the calculations, and the eigenfunctions as well as the energy eigenvalues are obtained in a proper Pekeris-type approximation.

Keywords: DKP equation, non-minimal vector Yukawa potential, Nikiforov–Uvarov method

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1. Introduction

Contrary to relativistic spin-zero and spin-1/2 particles, which have been extensively investigated in the literature, spin-one bosons have been studied by a considerably less number of physicists and mathematicians. To deal with such particles in wave equation formalism, we need to solve the Proca or Duffin–Kemmer–Petiau (DKP) equation. The latter is capable of investigating both spin-zero and spin-one particles in a unified basis,[1−7] and its equivalence with the Klein–Gordon (KG) and Proca equations is subject to serious doubt at the moment.[8,9] It is notable that investigating a spin-one particle can be performed within a gauge theory, namely the so-called compensator field[10] which is not the approach used by our study. The success of the DKP equation in describing the relativistic interactions of hadrons with nuclei goes beyond doubt.[11] Until now, the equation has been investigated with an oscillator term,[12−15] in the casual approach,[16,17] scattering of the K nucleus,[18] and within the five-dimensional Galilean invariance.[19] It is also studied for the Woods–Saxon potential,[20] vector and scalar Hulthen-type potential,[21] linear plus Coulomb-like potential,[22] vector Hulthén-type potential,[23] scalar Coulomb potential,[24] and smooth position-dependent mass potential.[25] In the present article we will consider the DKP equation in the presence of a non-minimal vector Yukawa potential in (1+1) dimensions for spin-one particles.

2. Review of the DKP equation

The DKP equation for a free boson is (we choose $\hbar = c = 1$)[3]

$$(\mathrm{i}\beta^{\mu}\partial_{\mu} - m)\Psi = 0, \tag{1}$$

where the matrices β^{μ} satisfy the algebra

$$\beta^{\mu}\beta^{\nu}\beta^{\lambda} + \beta^{\lambda}\beta^{\nu}\beta^{\mu} = g^{\mu\nu}\beta^{\lambda} + g^{\lambda\nu}\beta^{\mu}, \tag{2}$$

with μ, ν, and λ being 0, 1, 2, 3. The metric tensor is $g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ and for the five-component representation the 5×5 β^{μ} matrices are

$$\beta^0 = \begin{pmatrix} \Theta & \bar{0} \\ \bar{0}^{\mathrm{T}} & 0 \end{pmatrix},$$
$$\beta^i = \begin{pmatrix} \tilde{0} & \rho^i \\ -(\rho^i)^{\mathrm{T}} & 0 \end{pmatrix}, \quad i = 1, 2, 3, \tag{3}$$

where

$$\Theta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \rho^1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\rho^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \rho^3 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \tag{4}$$

$\bar{0}$, $\tilde{0}$ and 0 are 2×3, 2×2, and 3×3 zero matrices, respectively.

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Within the ten-component representation, the β^μ matrices are naturally 10×10 matrices which are given by

$$\beta^0=\begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^{\mathrm{T}} & 0 & I & 0 \\ \bar{0}^{\mathrm{T}} & I & 0 & 0 \\ \bar{0}^{\mathrm{T}} & 0 & 0 & 0 \end{pmatrix},\ \beta^i=\begin{pmatrix} 0 & \bar{0} & e_{\mathrm{i}} & \bar{0} \\ \bar{0}^{\mathrm{T}} & 0 & 0 & -\mathrm{i}S_{\mathrm{i}} \\ -e_{\mathrm{i}}^{\mathrm{T}} & 0 & 0 & 0 \\ \bar{0}^{\mathrm{T}} & -\mathrm{i}S_{\mathrm{i}} & 0 & 0 \end{pmatrix}, \quad (5)$$

with S matrices being 3×3 ones, $(S_{\mathrm{i}})_{jk}=-\mathrm{i}\varepsilon_{ijk}$ where ε_{ijk} takes the values 1, -1, 0 for an even permutation, an odd permutation, and repeated indices, respectively, and e_{i} matrices are 1×3, $(e_{\mathrm{i}})_{1j}=\delta_{ij}$, i.e. $e_1=(1,0,0)$, $e_2=(0,1,0)$, and $e_3=(0,0,1)$, I and 0 respectively represent the unit and null 3×3 matrices and $\bar{0}$s are the 1×3 ones.[4]

3. DKP equation in the presence of a non-minimal vector potential

The DKP equation in a (1+1)-dimensional space is written as

$$(\mathrm{i}\beta^0\partial_0+\mathrm{i}\beta^1\partial_1-m-U)\Psi(x,t)=0, \quad (6)$$

where $\Psi(x,t)=\exp(-\mathrm{i}E_nt)\psi_n(x)$. Thus, after the removal of the time component, we consider one quantum number and write the 10-component stationary spinor as

$$\begin{aligned}\psi_n^{\mathrm{T}}(x)=&(\varphi_{n,1}(x),\varphi_{n,2}(x),\varphi_{n,3}(x),\varphi_{n,4}(x),\varphi_{n,5}(x),\\ &\varphi_{n,6}(x),\varphi_{n,7}(x),\varphi_{n,8}(x),\\ &\varphi_{n,9}(x),\varphi_{n,10}(x))^{\mathrm{T}}. \end{aligned}\quad (7)$$

We intend to investigate a matrix interaction of the general form $U=\beta^\mu A^\mu+\mathrm{i}[P,\beta^\mu]H_\mu$, where $[P,\beta^\mu]=P\beta^\mu-\beta^\mu P$ conserves the four-current,[13] $\beta^\mu A^\mu$ is minimally coupled but $\mathrm{i}[P,\beta^\mu]H_\mu$ is the non-minimal vector term. Hence in this paper we will consider the term $\mathrm{i}[P,\beta^\mu]H_\mu$. Therefore, study the $U=\mathrm{i}[P,\beta^1]H_1$ and $H_1(x)=(V_0\,\mathrm{e}^{-\alpha x})/x$, where α and V_0 are real parameters and $P=(\beta^\mu\beta_\mu-2)=\mathrm{diag}(1,\,1,\,1,\,1,\,0,\,0,\,0,\,0,\,0,\,0)$ is the projection operator. Substitution of the latter into Eq. (6) gives the following coupled equations

$$\begin{aligned}&\mathrm{i}\frac{\partial\varphi_{n,5}(x)}{\partial x}-m\varphi_{n,1}(x)-\mathrm{i}H_1\varphi_{n,5}(x)=0,\\ &E_n\varphi_{n,3}(x)-m\varphi_{n,6}(x)=0, \end{aligned}\quad (8)$$

$$E_n\varphi_{n,5}(x)-m\varphi_{n,2}(x)=0,$$

$$E_n\varphi_{n,4}(x)-m\varphi_{n,7}(x)=0, \quad (9)$$

$$E_n\varphi_{n,6}(x)-\mathrm{i}\frac{\partial\varphi_{n,10}(x)}{\partial x}-m\varphi_{n,3}(x)+\mathrm{i}H_1\varphi_{n,10}(x)=0, \qquad m\varphi_{n,8}(x)=0, \quad (10)$$

$$E_n\varphi_{n,7}(x)+\mathrm{i}\frac{\partial\varphi_{n,9}(x)}{\partial x}-m\varphi_{n,4}(x)-\mathrm{i}H_1\varphi_{n,9}(x)=0, \qquad \mathrm{i}\frac{\partial\varphi_{n,4}(x)}{\partial x}+m\varphi_{n,9}(x)+\mathrm{i}H_1\varphi_{n,4}(x)=0, \quad (11)$$

$$E_n\varphi_{n,2}(x)-\mathrm{i}\frac{\partial\varphi_{n,1}(x)}{\partial x}-m\varphi_{n,5}(x)-\mathrm{i}H_1\varphi_{n,1}(x)=0, \qquad \mathrm{i}\frac{\partial\varphi_{n,3}(x)}{\partial x}-m\varphi_{n,10}(x)+\mathrm{i}H_1\varphi_{n,3}(x)=0. \quad (12)$$

Combination of the above equations yields $\varphi_{n,3}=\varphi_{n,4}$,

$$\frac{\mathrm{d}^2\varphi_{n,3}(x)}{\mathrm{d}x^2}+(E_n^2-m^2-H_1^2)\varphi_{n,3}(x)+\frac{\partial H_1}{\partial x}\varphi_{n,3}(x)=0. \quad (13)$$

Substituting $H_1=V_0\,\mathrm{e}^{-\alpha x}/x$ into Eq. (13) we have,

$$\frac{\mathrm{d}^2\varphi_{n,3}(x)}{\mathrm{d}x^2}+\Big[E_n^2-m^2-\Big(\frac{V_0\,\mathrm{e}^{-\alpha x}}{x}\Big)^2\Big]\varphi_{n,3}(x)-V_0\Big(\frac{\alpha x+1}{x^2}\Big)\mathrm{e}^{-\alpha x}\varphi_{n,3}(x)=0. \quad (14)$$

Now, we introduce the Pekeris-type approximations[26]

$$\frac{1}{x^2}\approx\frac{4\alpha^2\,\mathrm{e}^{-\alpha x}}{(1-\,\mathrm{e}^{-2\alpha x})^2}, \quad (15a)$$

$$\frac{1}{x^2}\approx\frac{4\alpha^2\,\mathrm{e}^{-2\alpha x}}{(1-\,\mathrm{e}^{-2\alpha x})^2}, \quad (15b)$$

and use the change of variable $z=\mathrm{e}^{-2\alpha x}$ to bring Eq. (14) into the following form:

$$\begin{aligned}&\frac{\mathrm{d}^2\hat{\varphi}_{n,3}(z)}{\mathrm{d}z^2}+\frac{(1-z)}{z(1-z)}\frac{\mathrm{d}\hat{\varphi}_{n,3}(z)}{\mathrm{d}z}\\ &+\frac{1}{z^2(1-z)^2}\Big[\Big(\frac{E_n^2-m^2}{4\alpha^2}-V_0^2+\frac{V_0}{2}\Big)z^2\\ &+\Big(-V_0-\frac{V_0}{2}+\frac{m^2-E_n^2}{2\alpha^2}\Big)z+\Big(\frac{E_n^2-m^2}{4\alpha^2}\Big)\Big]\hat{\varphi}_{n,3}(z)\\ &=0, \end{aligned}\quad (16)$$

where $\hat{\varphi}_{n,3}(z)=\varphi_{n,3}(x(z))$.

4. Nikiforov–Uvarov method

This powerful mathematical tool enables us to solve second order differential equations. Consider the

following differential equation:[27,28]

$$\Big\{\frac{\mathrm{d}^2}{\mathrm{d}s^2}+\frac{\alpha_1-\alpha_2 s}{s(1-\alpha_3 s)}\frac{\mathrm{d}}{\mathrm{d}s}+\frac{1}{[s(1-\alpha_3 s)]^2}\times\big[-\xi_1 s^2+\xi_2 s-\xi_3\big]\Big\}\psi_n(s)=0. \quad (17)$$

According to the Nikiforov–Uvarov method, the eigenfunctions are[28]

$$\psi_n(s) = N_n s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-\alpha_{13}/\alpha_3} \times \mathrm{P}_n^{(\alpha_{10}-1,\alpha_{11}/\alpha_3-\alpha_{10}-1)}(1-2\alpha_3 s), \quad (18)$$

where P_n is an orthogonal Jacobi-polynomial.

The energy eigenvalues can be obtained from Refs. [27] and [28] as

$$\alpha_2 n-(2n+1)\alpha_5+(2n+1)(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}) + n(n-1)\alpha_3+\alpha_7+2\alpha_3\alpha_8+2\sqrt{\alpha_8\alpha_9}=0, \quad (19)$$

where

$$\alpha_4=\frac{1}{2}(1-\alpha_1),\quad \alpha_5=\frac{1}{2}(\alpha_2-2\alpha_3),\quad \alpha_6=\alpha_5^2+\xi_1,$$
$$\alpha_7=2\alpha_4\alpha_5-\xi_2,\quad \alpha_8=\alpha_4^2+\xi_3,$$
$$\alpha_9=\alpha_3\alpha_7+\alpha_3^2\alpha_8+\alpha_6,\quad \alpha_{10}=\alpha_1+2\alpha_4+2\sqrt{\alpha_8},$$
$$\alpha_{11}=\alpha_2-2\alpha_5+2(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}),$$
$$\alpha_{12}=\alpha_4+\sqrt{\alpha_8},\quad \alpha_{13}=\alpha_5-(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}),$$
$$\alpha_{10}^*=\alpha_1+2\alpha_4-2\sqrt{\alpha_8},$$
$$\alpha_{11}^*=\alpha_2-2\alpha_5+2(\sqrt{\alpha_9}-\alpha_3\sqrt{\alpha_8}),$$
$$\alpha_{12}^*=\alpha_4-\sqrt{\alpha_8},\quad \alpha_{13}^*=\alpha_5-(\sqrt{\alpha_9}-\alpha_3\sqrt{\alpha_8}). \quad (20)$$

Therefore, we can use the following eigenfunctions in some case

$$\psi_n(s) = N_n\, s^{\alpha_{12}^*}(1-\alpha_3 s)^{-\alpha_{12}^*-\alpha_{13}^*/\alpha_3} \times \mathrm{P}_n^{(\alpha_{10}^*-1,\alpha_{11}^*/\alpha_3-\alpha_{10}^*-1)}(1-2\alpha_3 s). \quad (21)$$

5. Exact bound-state solutions

A comparison of Eq. (16) with Eq. (17) indicates the correspondence

$$\alpha_1=1,\quad \alpha_2=1,\quad \alpha_3=1,$$
$$\xi_1=\frac{m^2-E_n^2}{4\alpha^2}+V_0^2-\frac{V_0}{2},$$
$$\xi_2=\frac{m^2-E_n^2}{2\alpha^2}-V_0-\frac{V_0}{2},$$
$$\xi_3=\frac{m^2-E_n^2}{4\alpha^2}. \quad (22)$$

Equation (19) determines the rest of coefficients as

$$\alpha_4=0,\ \alpha_5=-\frac{1}{2},\ \alpha_6=\frac{1}{4}+V_0^2-\frac{V_0}{2}+\frac{m^2-E_n^2}{4\alpha^2},$$
$$\alpha_7=\frac{V_0}{2}+V_0+\frac{E_n^2-m^2}{2\alpha^2},$$
$$\alpha_8=\frac{m^2-E_n^2}{4\alpha^2},\quad \alpha_9=\frac{1}{4}+V_0^2+V_0,$$
$$\alpha_{10}=1+\frac{\sqrt{m^2-E_n^2}}{\alpha},$$
$$\alpha_{11}=2+2\Big(\sqrt{\frac{1}{4}+V_0^2+V_0}+\frac{\sqrt{m^2-E_n^2}}{2\alpha}\Big),$$
$$\alpha_{12}=\frac{\sqrt{m^2-E_n^2}}{2\alpha},$$
$$\alpha_{13}=-\frac{1}{2}-\Big(\sqrt{\frac{1}{4}+V_0^2+V_0}+\frac{\sqrt{m^2-E_n^2}}{2\alpha}\Big). \quad (23)$$

Thus, the eigenvalues and eigenfunctions are determined as follows:

$$(2n+1)\Big[\sqrt{\frac{1}{4}+V_0^2+V_0}+\sqrt{\frac{m^2-E_n^2}{4\alpha^2}}\Big] + n+n^2+\frac{1}{2}+V_0+\frac{V_0}{2} + \frac{1}{\alpha}\sqrt{\Big(\frac{1}{4}+V_0^2+V_0\Big)(m^2-E_n^2)}=0, \quad (24)$$

$$\hat{\varphi}_{n,3}(z) = N_n\, z^{-(m^2-E_n^2)^{1/2}/2\alpha}(1-z)^{1/2+\sqrt{1/4+V_0^2+V_0}} \times \mathrm{P}_n^{\left(-\sqrt{m^2-E_n^2}/\alpha,\,2\sqrt{1/4+V_0^2+V_0}\right)}(1-2z), \quad (25)$$

the Jacobi polynomial is

$$\mathrm{P}_n^{(c,d)}(z) = 2^{-n}\sum_{p=0}^{n}\binom{n+c}{P}\binom{n+d}{n-p}(1-z)^{n-p}(1+z)^p,$$

$$\mathrm{P}_n^{(c,d)}(z) = \frac{\Gamma(n+c+1)}{n!\Gamma(n+c+d+1)} \times\sum_{r=0}^{n}\binom{n}{r}\frac{\Gamma(n+c+d+r+1)}{\Gamma(r+c+1)}\Big(\frac{z-1}{2}\Big)^r, \quad (26)$$

where

$$\binom{n}{r}=\frac{n!}{r!\,(n-r)!}=\frac{\Gamma(n+1)}{\Gamma(r+1)\Gamma(n-r+1)}. \quad (27)$$

The curves of E_n versus V_0 for some values of n are plotted in Fig. 1. In addition, the energies for ($n=0$–8) are listed in Table 1. The wave functions for $n=$ (0–3) are plotted in Fig. 2.

6. Conclusion

We solve the (1+1)-dimensional DKP equation for spin-1 particles in the presence of a non-minimal vector Yukawa potential. The bound-state energies and the associated normalized wave functions are obtained by the Nikiforov–Uvarov method in the Pekeris-type approximation. Our results are useful for studying the relativistic spin-one particles and particularly spin-one mesons.

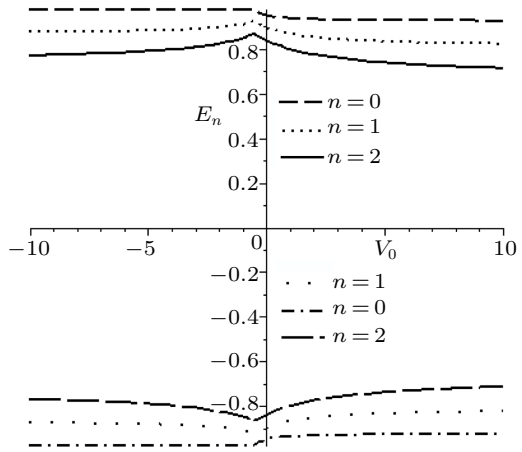


Fig. 1. Curves of E_n versus V_0 for some values of n ($m=1$, α=0.1).

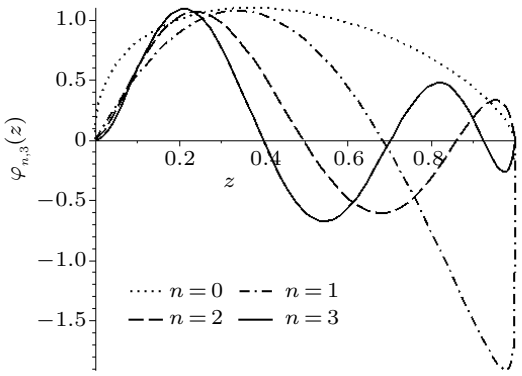


Fig. 2. Wavefunctions versus z for $n=0,1,2,3$.

Table 1. Energy eigenvalues for $V_0=-0.3$ and $m=1$.

n	E_n for $\alpha=-0.01$	E_n for $\alpha=-0.05$	E_n for $\alpha=-0.1$
0	−0.999965	−0.999132	−0.996522
1	−0.999792	−0.994792	−0.979003
2	−0.999530	−0.988186	−0.951864
3	−0.999169	−0.979015	−0.913171
4	−0.998708	−0.967186	−0.861275
5	−0.998147	−0.952594	−0.793561
6	−0.997485	−0.935107	−0.705480
7	−0.996723	−0.914559	−0.587941
8	−0.995860	−0.890738	−0.416717

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