Multi-symplectic wavelet splitting method for the strongly coupled Schrödinger system*

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(Received 24 May 2012; revised manuscript received 19 June 2012)

We propose a multi-symplectic wavelet splitting method to solve the strongly coupled nonlinear Schrödinger equations. Based on its multi-symplectic formulation, the strongly coupled nonlinear Schrödinger equations can be split into one linear multi-symplectic subsystem and one nonlinear infinite-dimensional Hamiltonian subsystem. For the linear subsystem, the multi-symplectic wavelet collocation method and the symplectic Euler method are employed in spatial and temporal discretization, respectively. For the nonlinear subsystem, the mid-point symplectic scheme is used. Numerical simulations show the effectiveness of the proposed method during long-time numerical calculation.

Keywords: multi-symplectic wavelet splitting method, symplectic Euler method, strongly coupled nonlinear Schrödinger equations

PACS: 02.30.Jr, 02.60.Cb, 02.70.Jn **DOI:** 10.1088/1674-1056/21/12/120202

1. Introduction

In this paper, we develop a multi-symplectic wavelet splitting (MSWS) method to solve the strongly coupled nonlinear Schrödinger (SCNLS) system

$$\begin{cases} \mathrm{i}\, u_t + \beta u_{xx} + [\alpha_1 |u|^2 \\ +(\alpha_1 + 2\alpha_2)|v|^2]u + \gamma u + \Gamma v = 0, \\ \mathrm{i}\, v_t + \beta v_{xx} + [\alpha_1 |v|^2 \\ +(\alpha_1 + 2\alpha_2)|u|^2]v + \gamma v + \Gamma u = 0, \end{cases} \tag{1}$$

where u and v are complex amplitudes or envelopes of two wave packets, β is the group velocity dispersion, and α_1 describes the self-focusing of a signal for the pulses in birefringent media. $(\alpha_1 + 2\alpha_2)$ is the cross-phase modulation and defines the integrability of SCNLS equations (1). The linear coupling parameter Γ accounts for the effects that arise from twisting of the fiber and elliptic deformation of the fiber. It is also referred to as linear birefringence or the relative propagation constant. Finally, γ appears as constant ambient potential called normalized birefringence. The strongly coupled nonlinear Schrödinger system has extensive applications in many areas of physics, including nonlinear optics and plasma physics.[1,2] In Refs. [3]–[6], a few numerical methods were proposed for solving the SCNLS equations and showed some excellent results.

The idea of the MSWS method we investigate in this paper combines the multi-symplectic wavelet collocation method in Refs. [7] and [8] and the multi-symplectic splitting method in Refs. [9]–[11]. Multi-symplectic methods, which can preserve the multi-symplectic conservation law under appropriate discretization, have attracted much attention in recent years.[12,13] Bridges and Reich in Ref. [14] proposed a multi-symplectic Preissman scheme. Reich in Ref. [15] proposed a multi-symplectic Runge–Kutta collocation method. In Ref. [16], Bridges and Reich introduced the idea of a multi-symplectic Fourier transform for multi-symplectic partial differential equations (PDEs). Then, Chen and Qin proposed a multi-symplectic Fourier pseudospectral method.[17] In addition, most of the multi-symplectic schemes are im-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10971226, 91130013, and 11001270), the National Basic Research Program of China (Grant No. 2009CB723802), the Research Innovation Fund of Hunan Province, China (Grant No. CX2011B011), and the Innovation Fund of National University of Defense Technology, China (Grant No. B120205).

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plicit, and we need to solve nonlinear algebraic equations. In Ref. [18], Moore and Reich proposed an explicit Euler box scheme. In this paper, we investigate the multi-symplectic wavelet splitting method for the SCNLS equations. The multi-symplectic system is split into one linear subsystem and one nonlinear subsystem. For the nonlinear subsystem, the mid-point symplectic scheme is used. For the linear subsystem, the multi-symplectic wavelet collocation method and the symplectic Euler method are employed in spatial and temporal discretization, respectively. The wavelet collocation method makes the corresponding spatial differentiation matrix sparse and demands less computations. The advantage of the symplectic Euler scheme lies in the fact that the generated full-discrete scheme for the linear subsystem is a semi-explicit multi-symplectic scheme, which does not need to solve all of the nonlinear algebraic equations at every time step. Finally, we can compose the linear and nonlinear solutions to gain the MSWS method, which is convenient to be calculated and efficient during long-time numerical calculations.

This paper is organized as follows. In Section 2, the multi-symplectic formulation of the SCNLS equations and some of the corresponding conservation laws are presented. In Section 3, the MSWS method for the SCNLS equations is constructed. Numerical simulations of elastic and inelastic collisions are presented in Section 4, where the effectiveness of the proposed method is demonstrated. Finally, conclusions are made in Section 5.

2. Multi-symplectic formulation of the SCNLS equations

By letting $u = q_1 + \mathrm{i}q_2$, $v = q_3 + \mathrm{i}q_4$, the system (1) can be written as a system of real-value equations as follows:

$$\begin{cases} \dfrac{\partial}{\partial t}q_1 + \beta\dfrac{\partial^2}{\partial x^2}q_2 + [\alpha_1(q_1^2+q_2^2) + (\alpha_1+2\alpha_2)(q_3^2+q_4^2)]q_2 + \gamma q_2 + \Gamma q_4 = 0, \\ \dfrac{\partial}{\partial t}q_2 - \beta\dfrac{\partial^2}{\partial x^2}q_1 - [\alpha_1(q_1^2+q_2^2) + (\alpha_1+2\alpha_2)(q_3^2+q_4^2)]q_1 - \gamma q_1 - \Gamma q_3 = 0, \\ \dfrac{\partial}{\partial t}q_3 + \beta\dfrac{\partial^2}{\partial x^2}q_4 + [\alpha_1(q_3^2+q_4^2) + (\alpha_1+2\alpha_2)(q_1^2+q_2^2)]q_4 + \gamma q_4 + \Gamma q_2 = 0, \\ \dfrac{\partial}{\partial t}q_4 - \beta\dfrac{\partial^2}{\partial x^2}q_3 - [\alpha_1(q_3^2+q_4^2) + (\alpha_1+2\alpha_2)(q_1^2+q_2^2)]q_2 - \gamma q_3 - \Gamma q_1 = 0. \end{cases} \tag{2}$$

Introducing new variables p_1, p_2, p_3, p_4, which satisfy $p_i = \dfrac{\partial}{\partial x}q_i$ $(i = 1, 2, 3, 4)$, we can rewrite Eq. (2) as a first-order system

$$\begin{cases} \dfrac{\partial}{\partial t}q_1 + \beta\dfrac{\partial}{\partial x}p_2 + [\alpha_1(q_1^2+q_2^2) + (\alpha_1+2\alpha_2)(q_3^2+q_4^2)]q_2 + \gamma q_2 + \Gamma q_4 = 0, \\ \dfrac{\partial}{\partial t}q_2 - \beta\dfrac{\partial}{\partial x}p_1 - [\alpha_1(q_1^2+q_2^2) + (\alpha_1+2\alpha_2)(q_3^2+q_4^2)]q_1 - \gamma q_1 - \Gamma q_3 = 0, \\ \dfrac{\partial}{\partial t}q_3 + \beta\dfrac{\partial}{\partial x}p_4 + [\alpha_1(q_3^2+q_4^2) + (\alpha_1+2\alpha_2)(q_1^2+q_2^2)]q_4 + \gamma q_4 + \Gamma q_2 = 0, \\ \dfrac{\partial}{\partial t}q_4 - \beta\dfrac{\partial}{\partial x}p_3 - [\alpha_1(q_3^2+q_4^2) + (\alpha_1+2\alpha_2)(q_1^2+q_2^2)]q_2 - \gamma q_3 - \Gamma q_1 = 0, \\ \dfrac{\partial}{\partial x}q_1 = p_1, \quad \dfrac{\partial}{\partial x}q_2 = p_2, \quad \dfrac{\partial}{\partial x}q_3 = p_3, \quad \dfrac{\partial}{\partial x}q_4 = p_4. \end{cases} \tag{3}$$

Actually, system (3) can be written as the Hamiltonian PDEs

$$Mz_t + Kz_x = \nabla_z S(z), \tag{4}$$

where

$$M=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$K=\beta\begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$z=[q_1,q_2,p_1,p_2,q_3,q_4,p_3,p_4]^{\mathrm{T}}$ and the Hamiltonian function is

$$\begin{aligned} S(z)=&\frac{1}{2}[\beta(p_1^2+p_2^2+p_3^2+p_4^2)\\ &+\frac{1}{2}\alpha_1(q_1^2+q_2^2)^2+\frac{1}{2}\alpha_1(q_3^2+q_4^2)^2]\\ &+\frac{1}{2}(\alpha_1+2\alpha_2)(q_1^2+q_2^2)(q_3^2+q_4^2)\\ &+\frac{1}{2}\gamma(q_1^2+q_2^2+q_3^2+q_4^2)\\ &+\Gamma(q_1q_3+q_2q_4). \end{aligned}$$

We can prove that equation (4) has multi-symplectic conservation law[14]

$$\omega_t+\kappa_x=0 \tag{5}$$

with $\omega=\frac{1}{2}\mathrm{d}z\wedge M\mathrm{d}z$, $\kappa=\frac{1}{2}\mathrm{d}z\wedge K\mathrm{d}z$.

In addition, when $S(z)$ is independent of x and t, the system (4) has local energy conservation law

$$E_t+F_x=0 \tag{6}$$

with $E(z)=S(z)-\frac{1}{2}z^{\mathrm{T}}Kz_x$, $F(z)=\frac{1}{2}z^{\mathrm{T}}Kz_t$, and momentum conservation law

$$I_t+G_x=0 \tag{7}$$

with $G(z)=S(z)-\frac{1}{2}z^{\mathrm{T}}Mz_t$, $I(z)=\frac{1}{2}z^{\mathrm{T}}Mz_x$.

3. Multi-symplectic wavelet splitting method for the SCNLS equations

3.1. Wavelet collocation method for space discretization

In Refs. [7] and [8], based on the autocorrelation function, a wavelet collocation method was used for space discretization for the multi-symplectic PDEs (4) by Zhu *et al.* and many important properties were obtained.

Consider the multi-symplectic PDEs (4) with periodic boundary condition $z(0,t)=z(L,t)$, where L is an integer. For a fixed scale $J=$ constant, we set $z=[u_1,u_2,\ldots,u_d]^{\mathrm{T}}$ and approximate function $u_i(x,t)$ $(i=1,2,\ldots,d)$ by an interpolation operator $I_Ju_i(x,t)$, which interpolates $u_i(x,t)$ at collocation points $x_l=l/2^J$, for $l=0,1,\ldots,N-1$, $N=L\cdot 2^J$. The interpolation operator $I_Ju_i(x,t)$ has the form of

$$I_Ju_i(x,t)=\sum_{l=0}^{N-1}u_{i_l}\theta(2^Jx-l), \tag{8}$$

where $u_{i_l}=u_i(x_l,t)$, and $\theta(x)$ is the autocorrelation function.[7] According to the interpolation property, we have $I_Ju_i(x_l,t)=u_i(x_l,t)$.

Applying the wavelet collocation method for spatial discretization of multi-symplectic system (4), we obtain

$$M(z_l)_t+K(I_Jz)_x|_{x=x_l}=\nabla_zS(z_l). \tag{9}$$

where

$$I_Jz=[I_Ju_1,I_Ju_2,\ldots,I_Ju_d]^{\mathrm{T}}$$

and $z_l=[u_{1_l},u_{2_l},\ldots,u_{d_l}]^{\mathrm{T}}$.

To obtain the equations for z_l, the crucial step is to express the one order spatial partial derivatives at collocation points x_l in terms of the values z_l. Making one-time differential with Eq. (8) and evaluating the resulting expressions at collocation points, we obtain

$$\left.\frac{\partial I_Ju_i(x,t)}{\partial x}\right|_{x_l}=\sum_{m=0}^{N-1}u_{i_l}\left.\frac{\mathrm{d}\theta(2^Jx-m)}{\mathrm{d}x}\right|_{x_l}=(B_1U_i)_l, \tag{10}$$

where B_1 is an $N\times N$ matrix with elements

$$(B_1)_{lm}=\left.\frac{\mathrm{d}\theta(2^Jx-m)}{\mathrm{d}x}\right|_{x_l}=2^J\theta'(l-m),$$

and $U_i=[u_{i_0},u_{i_1},\ldots,u_{i_{N-1}}]^{\mathrm{T}}$. For more details, see Refs. [7] and [8] and references therein.

3.2. Multi-symplectic splitting method for SCNLS

The multi-symplectic form (4) can be split into sub-multi-symplectic systems

$$Mz_t + K_i z_x = \nabla_z S^i(z), \quad i = 1, 2, \ldots, N, \tag{11}$$

where

$$K = \sum_{i=1}^{N} K_i, \ S(z) = \sum_{i=1}^{N} S^i(z).$$

The multi-symplectic conservation laws of subsystems (11) have been proved in Ref. [9]

$$\omega_t + \kappa_{ix} = 0 \tag{12}$$

with

$$\omega = \frac{1}{2}\mathrm{d}z \wedge M\,\mathrm{d}z, \quad \kappa_i = \frac{1}{2}\mathrm{d}z \wedge K_i\,\mathrm{d}z,$$

$i = 1, 2, \ldots, N$.

For the SCNLS equations (1), it can be split into a linear subproblem

$$\mathscr{L}: \ \begin{cases} \mathrm{i}u_t + \beta u_{xx} + \gamma u + \Gamma v = 0, \\ \mathrm{i}v_t + \beta v_{xx} + \gamma v + \Gamma u = 0, \end{cases} \tag{13}$$

and a nonlinear subproblem

$$\mathscr{N}: \ \begin{cases} \mathrm{i}u_t + [\alpha_1|u|^2 + (\alpha_1 + 2\alpha_2)|v|^2]u = 0, \\ \mathrm{i}v_t + [\alpha_1|v|^2 + (\alpha_1 + 2\alpha_2)|u|^2]v = 0. \end{cases} \tag{14}$$

Both the linear subproblem (13) and the nonlinear subproblem (14) can be written into multi-symplectic systems (11) with

$$\begin{aligned} \mathscr{L}: \ K_1 &= K, \\ S^1(z) &= \frac{1}{2}\beta(p_1^2 + p_2^2 + p_3^2 + p_4^2) \\ &\quad + \frac{1}{2}\gamma(q_1^2 + q_2^2 + q_3^2 + q_4^2) \\ &\quad + \Gamma(q_1 q_3 + q_2 q_4), \\ \mathscr{N}: \ K_2 &= \mathbf{0}, \\ S^2(z) &= \frac{1}{4}[\alpha_1(q_1^2 + q_2^2)^2 + \alpha_1(q_3^2 + q_4^2)^2] \\ &\quad + \frac{1}{2}(\alpha_1 + 2\alpha_2)(q_1^2 + q_2^2)(q_3^2 + q_4^2). \end{aligned}$$

Furthermore, the nonlinear subproblem (14) can also be written as an infinite Hamiltonian system

$$z_t = J^{-1}\frac{\delta}{\delta z}H(z), \quad z = [q_1,\ q_2,\ q_3,\ q_4]^{\mathrm{T}}, \tag{15}$$

with a symplectic conservation law

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega = 0, \tag{16}$$

where

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix},$$

$\omega = \frac{1}{2}\mathrm{d}z \wedge M\,\mathrm{d}z$ and the Hamiltonian function is

$$\begin{aligned} H(z) = \int \Big(&\frac{1}{4}[\alpha_1(q_1^2 + q_2^2)^2 + \alpha_1(q_3^2 + q_4^2)^2] \\ &+ \frac{1}{2}(\alpha_1 + 2\alpha_2)(q_1^2 + q_2^2)(q_3^2 + q_4^2)\Big)\,\mathrm{d}x. \end{aligned} \tag{17}$$

Discreting Eq. (13) by using the wavelet collocation method in space, we can obtain a semi-discrete system

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{q}_1 + \beta B_1^2 \boldsymbol{q}_2 + \gamma \boldsymbol{q}_2 + \Gamma \boldsymbol{q}_4 = 0, \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{q}_2 - \beta B_1^2 \boldsymbol{q}_1 - \gamma \boldsymbol{q}_1 - \Gamma \boldsymbol{q}_3 = 0, \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{q}_3 + \beta B_1^2 \boldsymbol{q}_4 + \gamma \boldsymbol{q}_4 + \Gamma \boldsymbol{q}_2 = 0, \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{q}_4 - \beta B_1^2 \boldsymbol{q}_3 - \gamma \boldsymbol{q}_3 - \Gamma \boldsymbol{q}_1 = 0, \end{cases} \tag{18}$$

where

$$\boldsymbol{q}_i = [q_{i_0}, q_{i_1}, \ldots, q_{i_{N-1}}]^{\mathrm{T}}, \quad i = 1, 2, 3, 4.$$

We apply the symplectic Euler scheme[19] for semi-discrete multi-symplectic form of Eq. (18) and obtain the equations

$$\begin{cases} \boldsymbol{q}_1^{n+1} = \boldsymbol{q}_1^n - \Delta t(\beta B_1^2 \boldsymbol{q}_2^{n+1} + \gamma \boldsymbol{q}_2^{n+1} + \Gamma \boldsymbol{q}_4^{n+1}), \\ \boldsymbol{q}_2^{n+1} = \boldsymbol{q}_2^n + \Delta t(\beta B_1^2 \boldsymbol{q}_1^n + \gamma \boldsymbol{q}_1^n + \Gamma \boldsymbol{q}_3^n), \\ \boldsymbol{q}_3^{n+1} = \boldsymbol{q}_3^n - \Delta t(\beta B_1^2 \boldsymbol{q}_4^{n+1} + \gamma \boldsymbol{q}_4^{n+1} + \Gamma \boldsymbol{q}_2^{n+1}), \\ \boldsymbol{q}_4^{n+1} = \boldsymbol{q}_4^n + \Delta t(\beta B_1^2 \boldsymbol{q}_3^n + \gamma \boldsymbol{q}_3^n + \Gamma \boldsymbol{q}_1^n), \end{cases} \tag{19}$$

where $\boldsymbol{q}_i^n = [q_{i_0}^n, q_{i_1}^n, \ldots, q_{i_{N-1}}^n]^{\mathrm{T}}$, $q_{i_j}^n$ is the approximation to $q_i(x_j, t_n)$, $i = 1, 2, 3, 4$, $j = 0, 1, \ldots, N-1$.

It is obvious that the semi-explicit scheme (19) for the linear subproblem (13) does not need to solve all of the nonlinear algebraic equations every time step. Actually, the second and fourth lines are entirely explicit, but the first and third lines need to solve the nonlinear algebraic equation at every spatial grid.

For the nonlinear subproblem (14), we choose the mid-point implicit scheme in time direction. Then, we obtain a symplectic scheme for the nonlinear subproblem

$$\begin{cases} q_{1_j}^{n+1} = q_{1_j}^n - \Delta t\{[\alpha_1((q_{1_j}^{1/2})^2 + (q_{2_j}^{1/2})^2) + (\alpha_1 + 2\alpha_2)((q_{3_j}^{1/2})^2 + (q_{4_j}^{1/2})^2)]q_{2_j}^{1/2}\}, \\ q_{2_j}^{n+1} = q_{2_j}^n + \Delta t\{[\alpha_1((q_{1_j}^{1/2})^2 + (q_{2_j}^{1/2})^2) + (\alpha_1 + 2\alpha_2)((q_{3_j}^{1/2})^2 + (q_{4_j}^{1/2})^2)]q_{1_j}^{1/2}\}, \\ q_{3_j}^{n+1} = q_{3_j}^n - \Delta t\{[\alpha_1((q_{3_j}^{1/2})^2 + (q_{4_j}^{1/2})^2) + (\alpha_1 + 2\alpha_2)((q_{1_j}^{1/2})^2 + (q_{2_j}^{1/2})^2)]q_{4_j}^{1/2}\}, \\ q_{4_j}^{n+1} = q_{4_j}^n + \Delta t\{[\alpha_1((q_{3_j}^{1/2})^2 + (q_{4_j}^{1/2})^2) + (\alpha_1 + 2\alpha_2)((q_{1_j}^{1/2})^2 + (q_{2_j}^{1/2})^2)]q_{3_j}^{1/2}\}, \end{cases} \tag{20}$$

where $q_{i_j}^{1/2} = \frac{1}{2}(q_{i_j}^n + q_{i_j}^{n+1})$, $i = 1, 2, 3, 4$, $j = 0, 1, \ldots, N-1$.

In this paper, the solution may be advanced from one time level to the next by means of the following second-order formula

$$u_i(x, t + \Delta t) = \exp\left(\frac{\Delta t}{2}\mathscr{N}\right)\exp(\Delta t\mathscr{L}) \times \exp\left(\frac{\Delta t}{2}\mathscr{N}\right)u_i(x, t). \tag{21}$$

4. Numerical simulations

In order to show the advantage of the proposed method, we conduct two numerical examples for the MSWS method and the multi-symplectic Fourier pseudospectral (MSFP) method in Ref. [20]. All of the solutions are computed with time-step $\Delta t = 0.005$ and periodic boundary conditions in $[-30, 30]$. We choose grid number $N = 300$ for all examples. The autocorrelation function AD30[8] is used for the wavelet collocation method in the following examples. In addition, we solve the implicit parts of the scheme by using the fixed-point iteration method. Since the shapes of waves obtained by the MSWS method and the MSFP method are similar in the two examples, the shapes obtained by the MSFP method are not shown in this paper.

The global energy error (GE) is defined by

$$\mathrm{GE} = \sum_{k=1}^{N-1}(E_k^n - E_k^0)\Delta x, \tag{22}$$

where E^0 is the initial energy and

$$E_k^n = -\frac{\beta}{2}\left[\left(\frac{u_k^n - u_{k-1}^n}{\Delta x}\right)^2 + \left(\frac{v_k^n - v_{k-1}^n}{\Delta x}\right)^2\right] + \frac{\alpha_1}{4}(|u_k^n|^4 + |v_k^n|^4) + \frac{\alpha_1 + 2\alpha_2}{2}(|u_k^n|^2|v_k^n|^2) + \frac{\gamma}{2}(|u_k^n|^2 + |v_k^n|^2) + \Gamma \cdot \mathrm{Re}\{\overline{u}_k^n v_k^n\}.$$

Example 1: Elastic collisions

Elastic collisions are those from which the newly formed shapes reemerge under deformation. We consider the elastic collisions of two solitons by choosing the parameters $\beta = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = -1/6$, $\gamma = 1.0$, $\Gamma = 1.0$ with initial condition

$$\begin{cases} u(x, 0) = \sqrt{2}\,\mathrm{sech}\left(x + \frac{1}{2}D_0\right)\exp(\mathrm{i}V_0 x/4), \\ v(x, 0) = \sqrt{2}\,\mathrm{sech}\left(x - \frac{1}{2}D_0\right)\exp(-\mathrm{i}V_0 x/4), \end{cases} \tag{23}$$

where $D_0 = 25$ and $V_0 = 1.0$.

Figure 1 shows that the two waves emerge without any changes in their shapes, which indicates that the proposed method simulated the solitary wave well. Figure 2 represents the errors in global energy conservation. We notice that the MSWS method preserves the global energy conservation better than the MSFP method. The results obtained by using the MSWS method are also better than those obtained by the multi-symplectic partitioned Runge−Kutta method in Ref. [3]. In addition, the CPU time by the MSWS method is 427 s, which is about 60% less than that by the MSFP method.

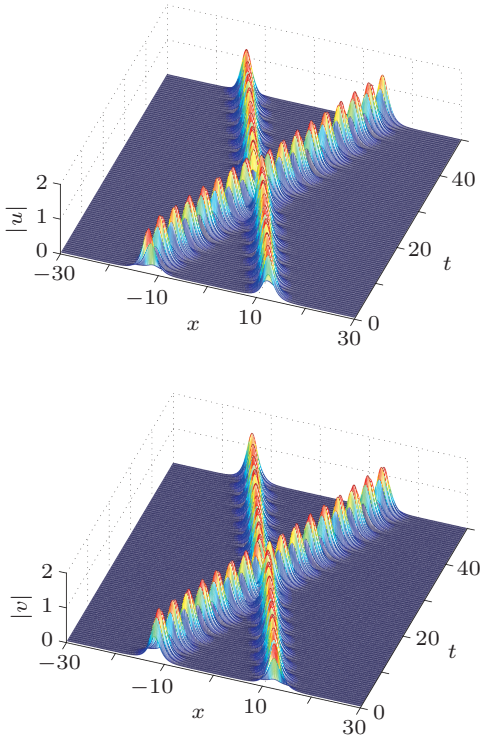


Fig. 1. (color online) Elastic collision of two solitons by the MSWS method with $\beta = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = -1/6$, $\gamma = 1.0$, $\Gamma = 1.0$.

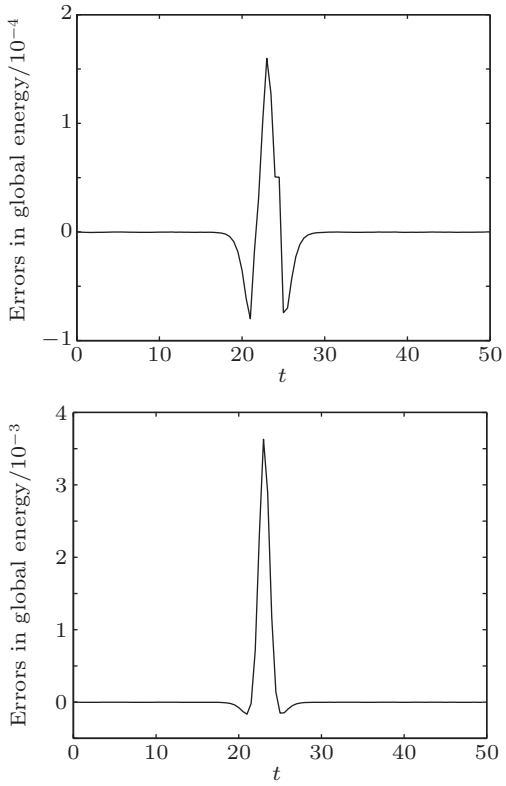


Fig. 2. The error of global energy for the elastic collision of two solitons. Up: MSWS method. Down: MSFP method.

Example 2: Inelastic collisions

In this example, we give collisions of two solitary waves of SCNLS equations (1) with initial condition (23). We choose the parameters $\beta = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1/2$, $\gamma = 0$, $\Gamma = 0$, $D_0 = 20$, $V_0 = 1.0$.

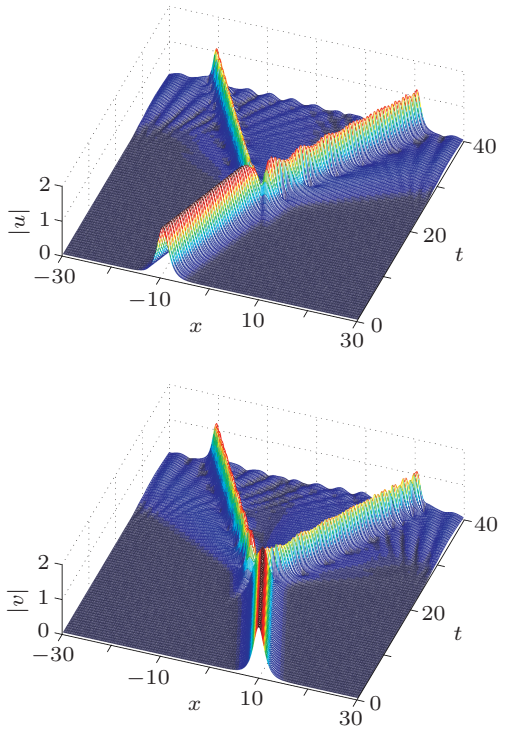


Fig. 3. (color online) Inelastic collision of two solitons by the MSWS method with $\beta = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1/2$, $\gamma = 0$, $\Gamma = 0$.

Figure 3 illustrates that two solitary waves are generated after collision. We see that after the interaction, the solitary waves dispersive oscillations and their amplitudes are altered. Both waves change their shape and the collision is inelastic. Figure 4 represents the global energy conservation. The results found by using the MSWS method are similar to those obtained by the MSFP method and the method in Ref. [4]. However, the CPU time using the MSWS method is 356 s, which is about 40% less than that using the MSFP method.

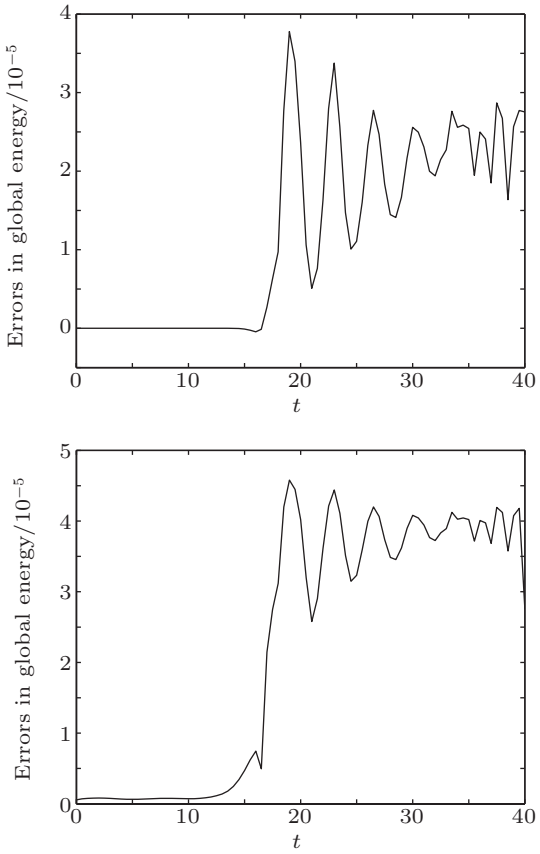


Fig. 4. The error of global energy for the inelastic collision of two solitons. Up: MSWS method. Down: MSFP method.

5. Conclusions

In this paper, we constructed a new multi-symplectic wavelet splitting scheme for the SCNLS equations. First, we split the SCNLS equations into one linear subsystem and one nonlinear subsystem. For the linear subsystem, we proposed an efficient multi-symplectic scheme based on the wavelet collocation method and the symplectic Euler method. The

nonlinear subproblem is integrated by symplectic integrators. Then, the composition method is used to get the solutions. Although this scheme is not explicit entirely for the SCNLS equations, it is shown to be efficient in practice, because the costs of solving nonlinear algebraic equations at every time step are saved in part. In addition, the obtained spatial differentiation matrix is a sparse circulant matrix with limited bandwidth, which also leads to less computations. Numerical results show that the proposed method is effective for the SCNLS equations. Furthermore, this MSWS method provides new choices for simulating the nonlinear Schrödinger equation especially with high spatial dimension and strongly coupled conditions.

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