An investigation of merging and diverging cars on a multi-lane road using a cellular automation model

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In this paper, we have investigated two observed situations in a multi-lane road. The first one concerns a fast merging vehicle. The second situation is related to the case of a fast vehicle leaving the fastest lane back into the slowest lane and targeting a specific way out. We are interested in the relaxation time τ, i.e., which is the time that the merging (diverging) vehicle spends before reaching the desired lane. Using analytical treatment and numerical simulations for the NaSch model, we have found two states, namely, the free state in which the merging (diverging) vehicle reaches the desired lane, and the trapped state in which τ diverges. We have established phase diagrams for several values of the braking probability. In the second situation, we have shown that diverging from the fast lane targeting a specific way out is not a simple task. Even if the diverging vehicle is in the free phase, two different states can be distinguished. One is the critical state, in which the diverging car can probably reach the desired way out. The other is the safe state, in which the diverging car can surely reach the desired way out. In order to be in the safe state, we have found that the driver of the diverging car must know the critical distance (below which the way out will be out of his reach) in each lane. Furthermore, this critical distance depends on the density of cars, and it follows an exponential law.

Keywords: traffic flow, merging and diverging area, intelligent transportation system, cellular automation

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1. Introduction

Nowadays, traffic flow problems have attracted much attention. This is due to their harmful impact on the economy and environment. In addition to traffic managers, many researchers from different disciplines are working in this field.

In the past, the physical investigation of traffic flow was based mainly on fluid dynamic methods. In addition, there has been a great interest in studying traffic flow using the concepts and techniques of statistical mechanics (for review see Ref. [1]). In this microscopic description, traffic flow is considered as a system of interacting particles (vehicles) driven far away from the equilibrium. Thus, it offers the possibility to study various aspects of the dynamics of truly non equilibrium systems, which are of current interest in statistical mechanics. Within the conceptual framework of the microscopic approach, cellular automation (CA) models have been proposed. These models have become popular for modelling traffic because of their computer efficiency and simplicity to introduce basic phenomena. Much effort has been concentrated on the stochastic CA models of traffic flow first proposed by Nagel and Schreckenberg[2] and subsequently studied by the other authors using a variety of techniques.[3−5]

For a realistic description, the idealized single lane CA models are generalized to the case of multi-lane traffic. The main components required for this generalization are the lane changing rules. However, recent empirical findings show that lane changing appears to be responsible for many traffic instabilities, such as capacity drop[6] and the formation and propagation of stop-and-go waves.[7] Several investigations have been carried out on this aspect (for review see Refs. [1] and [8]).

Unfortunately, only a few papers have addressed the situation of weaving and merging lanes, because freeway merging areas have been among the most difficult aspects of simulation.[9,10] Hidas[9] collected data from video-recordings of the microscopic details of merging and weaving manoeuvres under the congested traffic conditions. Kim *et al.*[10] proposed a gap acceptance model to update the size of the critical trail-

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ing gap for a merging vehicle. Moreover, Laval and Leclercq[11] studied the relaxation phenomena commonly observed near congested ramps. Leclercq and Laval[12] validated the relaxation model proposed[11] using the microscopic data from NGSIM, and showed that the relaxation phenomena cannot be ignored near the congested ramps. On the other hand, Lei *et al.*[13] established the first CA model to simulate the weaving section. Recently, Bin *et al.*[14] proposed more realistic CA lane-changing rules to depict the merging and diverging behavior.

However, no paper has so far investigated the following two important situations: 1) The drive-in vehicle, which comes from the on ramp, is going to merge into the fastest lane. Under which conditions can he (or she) accomplish his (or her) goal, and how much time does it take? 2) For the exit vehicle, which would diverge from the fastest lane to leave the highway in a specific way, is it possible to exit the highway safely? If it is, at which location in the fastest lane must he (or she) begin to change lanes in order to reach the desired way out? It is well known that the mandatory lane-changing behavior performed by the drive-in vehicle and the exit vehicle leads to the apparition of gap acceptance phenomena, which causes the traffic breakdown and probably car accidents. We are not interested in these instabilities, such phenomena have been studied before.[9−14] Our aim is to study the conditions under which a vehicle merging (or diverging) into the fast lane (or from the fast lane) can realize its goal safely and quickly. We believe that many drivers have gone through these experiences. Indeed, to get out of the fast lane is not always a simple task, and of course to leave the highway in a specific way is sometimes not possible.

This paper is organized as follows. In the next section, we will explicit the two situations and present our CA model. In the third section, we will present our theoretical and numerical results. The last section will be devoted to the conclusion.

2. Model

For simplicity, the drive-in vehicle, which comes from the on ramp and is going to merge into the fastest lane, will be called vehicle M in this paper, whereas the diverging one (vehicle would diverge from the fast lane) will be called vehicle D. The two situations (merging into the fast lane and diverging from it) represent the most preoccupations for all divers in a multi lane high way. Let us begin with the first situation.

Consider the NaSch model in a two-dimensional CA lattice ($L \times N$) that represents a multi-lane road, where L is the length of the road, and N is the number of lanes. Each site (i, j), where $1 \leq i \leq L$ and $1 \leq j \leq N$, can be empty or used by at most one car. The $S(i, j)$ denotes the state of site (i, j), empty $S(i, j) = 0$, and used $S(i, j) = 1$. The drivers move forward according to the rules of the NaSch model.[2] Furthermore, the vehicles are filed according to their maximal velocities and their lanes, i.e, each type of vehicle can run only on its specific lane, and it is not allowed to change lane. More precisely, vehicles with $V_{\max 1} = 1$ move in the first lane, those with $V_{\max 2} = 2$ move in the second lane, and those with $V_{\max j} = j$ can run in lane j only. This situation means that the slow vehicles (trucks for example) run in the first and the second lanes, whereas the fast cars run in the third and the fourth lanes (for $N = 4$).

However, if vehicle M is a merging fast car, it can run with a maximal speed $V_{\max N}$. It will be injected into the first lane, and of course it will try to reach the fast lane N by changing lanes whenever possible. We do not take into account changing lanes for other vehicles, because this phenomenon has been well studied before.[15−26]

Using parallel dynamics, the updating of the model follows two steps.

Step 1 All kinds of vehicles move according to the NaSch rules without changing their lanes.

Step 2 When the system reaches its stationary state, vehicle M is injected into any free site $(i, 1)$ in the first lane, and moves ahead. This car is going to get into the faster lane. Thus, it will try to change lanes if some criteria are satisfied. Vehicle M in any site (i, j) will examine its environment before moving. Generally, a lane-changing vehicle will consider the leading vehicle (denoted as vehicle 1) as well as its following one (denoted as vehicle 2) in the adjacent lane. In other words, vehicle M estimates the gap between these two successive vehicles (1 and 2) in lane $j + 1$. The criteria of changing lanes are as follows.

C1 $S(i + 1, j + 1) = 0$, i.e, the desired site in the neighboring left lane must be free. This criterion means that vehicle M has just let vehicle 1 go past.

C2 Vehicle M estimates the velocity of vehicle 2 in the next time step $(t + 1)$ and calculates the number of free sites in front of vehicle 2 N_l. According to the NaSch model, the velocity of vehicle 2 in the next

time step $(t+1)$ is at most

$$V_{\text{vehicle2}}(t+1) = \min(V_{\text{vehicle2}}(t)+1, V_{\max j+1}), \quad (1)$$

where $V_{\text{vehicle2}}(t)$ is the velocity of vehicle 2 in the time step t, and $V_{\max j+1}$ represents its maximal velocity. Thus, if

$$N_l > V_{\text{vehicle2}}(t+1), \quad (2)$$

the changing lane is affordable.

C3 If the previous criteria are satisfied, then vehicle M will move to the left lane $j+1$ with the probability P_{c}. Otherwise, it will move ahead (according to the NaSch rules) while waiting for another opportunity.

Note that criteria C1, C2, and C3 mean that vehicle M accepts the gap between the leading vehicle (vehicle 1) and its follower (vehicle 2), and can move safely to the adjacent lane $j+1$.

The second situation in this study (vehicle D) duplicates the first situation in the sense that the criteria used by a vehicle to move from the slowest lane to the fastest one are the same as that needed by a vehicle to move from the fastest lane back to the slowest one targeting the way out.

3. Results and discussion

3.1. Analytic treatment

The deterministic NaSch's model (the braking probability $P = 0$) in one lane was studied previously.[23] Indeed, Nagel and Herrmann showed that there are two steady states depending on the density ρ. At a low density, there are enough empty sites, hence the system can self organize so that every car can move with its $V_{\max}$. This steady state is however possible only if

$$\rho \leq \rho_{\text{c}} = \frac{1}{V_{\max}+1}. \quad (3)$$

For

$$\rho > \rho_{\text{c}}, \quad (4)$$

the average distance headway is $1/\rho - 1$. Thus, the velocity of the car is limited by the density of holes; in other words, the available empty space decreases as ρ increases.

In our analysis, we will first consider the deterministic case, i.e., when $P_{\text{c}} = 1$ and $P = 0$. According to the previous study,[23] we can say that each lane j has its critical density

$$\rho_{\text{c}j} = \frac{1}{V_{\max j}+1}. \quad (5)$$

If vehicle M is moving on lane j, the change to the next lane $j+1$ will be possible when

$$\rho_{j+1} \leq \rho_{\text{c}j+1}. \quad (6)$$

This condition means that criteria C1 and C2 are satisfied, whereas for

$$\rho > \rho_{\text{c}j+1}, \quad (7)$$

the previous criteria can not be fulfilled. As a consequence, the change from lane 1 to lane 2 will be possible when

$$\rho_2 \leq \rho_{\text{c}2} = 0.33. \quad (8)$$

In the same way, the change from lane 2 to lane 3 requires that the density in lane 3 satisfies

$$\rho_3 \leq \rho_{\text{c}3} = 0.25, \quad (9)$$

whereas the change from lane 3 to lane 4 requires

$$\rho_4 \leq \rho_{\text{c}4} = 0.2. \quad (10)$$

We notice that

$$\rho_{\text{c}2} > \rho_{\text{c}3} > \rho_{\text{c}4}. \quad (11)$$

Suppose that the densities are the same in all lanes

$$\rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho, \quad (12)$$

then vehicle M may reach lane 4 when

$$\rho \leq \rho_{\text{c}4} = 0.2. \quad (13)$$

This conclusion leads to the conclusion that the relaxation time τ (the time that vehicle M spends before reaching the fourth lane) will diverge as long as $\rho \to \rho_{\text{c}4} = 0.2$. To quantify these phenomena, we can define two steady states depending on the density ρ. The free state (small values of τ), in which vehicle M can reach the desired lane 4,

$$\rho < \rho_{\text{c}4}. \quad (14)$$

The trapped state (infinite values of τ), in which vehicle M cannot reach lane 4,

$$\rho \geq \rho_{\text{c}4}. \quad (15)$$

In this case, τ is the order parameter, and ρ is the parameter of control. The system undergoes a phase transition form the free state to the trapped state at

$$\rho = \rho_{\text{c}4} = 0.2. \quad (16)$$

However, the situation in the highway is quite different. Indeed, each lane has its own density ρ_j, and the reactions of drivers are not the same, as a consequence, the two steady states (free and trapped) will depend on ρ_2, ρ_3, ρ_4, P_c, and P. These parameters will also influence the decision making of the driver of vehicle D. Unfortunately, such analyses can not be done analytically. We will use the numerical simulations to study this complicated situation.

3.2. Simulation results

In this section, we will address two situations in the general case (taking into account all parameters of the problem, ρ_j, P_c, and P). To do such numerical analyses, we have considered a two-dimensional lattice ($L \times N$) with periodic boundaries. Here $L = 1000$ is the length of the road, and $N = 4$ is the number of lanes, as shown in Fig. 1. In this case, the simulation begins with particles randomly distributed around the lattice. The system runs for more than 30000 MCS to ensure that the steady state is reached. In order to eliminate the numerical fluctuation, about 100 initial configurations are randomly chosen.

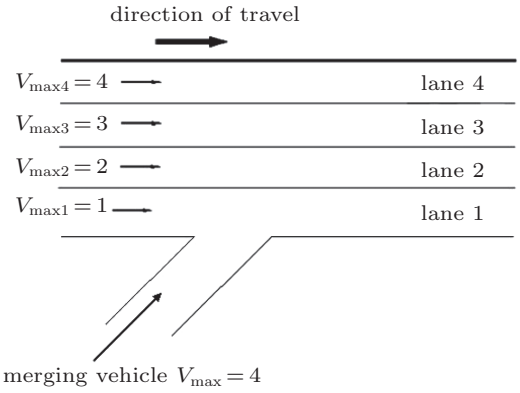


Fig. 1. A portion of the road, which is made up four lanes $N = 4$. Each vehicle moves ahead according to the rules of the NaSch model. The merging fast vehicle (vehicle M) enters the road and tries to reach the fourth lane.

3.2.1. The drive-in vehicle

To address this question, we inject vehicle M into the first lane. We compute time τ that vehicle M will spend to reach the fastest lane, lane 4.

Let us begin with the deterministic situation ($P_c = 1$ and $P = 0$) and a uniform density (ρ is the same in each lane). Figure 2 (down triangles) shows that actually the system undergoes a phase transition from the free state (small values of τ) to the trapped state (higher values of τ) at

$$\rho = \rho_c = 0.2, \tag{17}$$

which corresponds to the critical density of lane 4. These results are in agreement with the analytic predictions obtained in the previous section. Figure 3 shows an example of the trajectory of vehicle M.

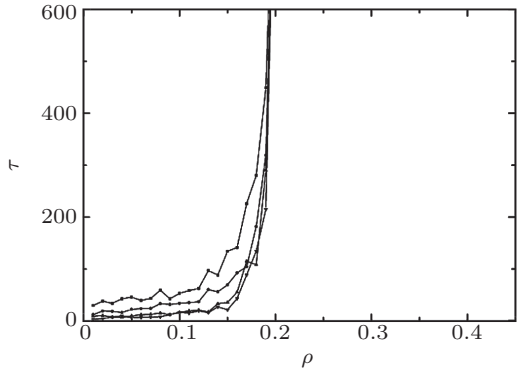


Fig. 2. The variation of the relaxation time τ with density ρ for different values of the probability of changing lane P_c in the deterministic NaSch rules ($P = 0$). Here all lanes have the same density ρ. Squares correspond to $P_c = 0.1$, circles correspond to $P_c = 0.2$, up triangles correspond to $P_c = 0.5$, and down triangles correspond to $P_c = 1$.

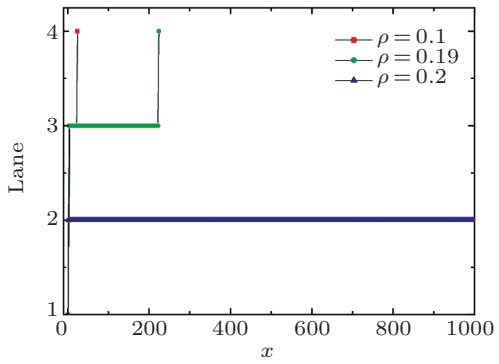


Fig. 3. (colour online) Trajectory of the merging car in the case of $P_c = 1$ and $p = 0$ for three values of the global density ρ. The $\rho = 0.1$ (squares) and $\rho = 0.19$ (circle dots) correspond to the free state, while $\rho = 0.2$ (up triangles) corresponds to the trapped state where the merging car can not reach the fourth lane.

It is found that P_c does not influence the critical density ρ_c, but it affects τ in the free state, as shown in Fig. 2. Indeed, situations with $P_c < 1$ would allow drivers to be indecisive or late to respond. Higher values of P_c mean aggressive drivers, whereas small values of P_c represent the prudent ones. Suppose that criteria C1 and C2 are satisfied, then, changing lane will be more probable if $P_c \to 1$, which leads to small values of τ.

On the other hand, figure 4 shows that $\rho_c(p \neq 0)$ increases as long as the braking probability increases, and the relaxation time τ varies continuously with ρ (contrary to situation $P = 0$). Such remarks can be

explained as follows. The values of ρ_c are governed by the application of criteria C1 and C2. For $P = 0$ and $\rho_c + \epsilon \quad (\epsilon \mapsto 0)$, the previous criteria can not be fulfilled. However, under the same situation but with $P \neq 0$, more empty space will be available due to the probability of deceleration of vehicle 2. Thus, the gap between the two vehicles (1 and 2) will be greater than that estimated by the merging driver (vehicle M) in the case $P = 0$. As a consequence,

$$\rho_c(p \neq 0) > \rho_c(p = 0). \tag{18}$$

According to these results, we can say that the behavior of the relaxation time τ versus the global density ρ is governed by parameters P_c and P, which affect the finite values of τ in the free state and control the critical density and the nature of the transition from the free state to the trapped state near ρ_c, respectively.

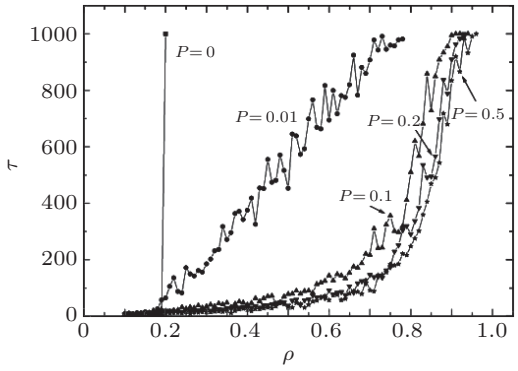


Fig. 4. The relaxation time τ versus the global density ρ (the densities are the same in all lanes) for $P_c = 1$ and different values of braking probability P. We notice that the critical density ρ_{cj} increases as P increases, and the transition from the free state to the trapped state at $\rho = \rho_c$ is discontinued for $P = 0.0$ and continues for $P \neq 0$.

Note that, the density in the multi-lane is not uniform, i,e, each lane has its own density of cars. As a consequence, we should take into consideration this important aspect in order to complete our analysis. We will first address the deterministic case ($P_c = 1$ and $P = 0$), and study the behavior of τ versus the density ρ_j in lane j for fixed and small values of the densities $\rho_{i\neq j}$ in the other lanes $i \neq j$. Indeed, the densities $\rho_{i\neq j}$ are fixed and small, because we are interested in the application of criteria C1 and C2 in lane j; whereas in the other lanes $i \neq j$, vehicle M can change the lane safely.

Figure 5 shows that each lane has its own critical density $\rho_c = \frac{1}{V_{max}+1}$ in agreement with the analytic description obtained in the previous section.

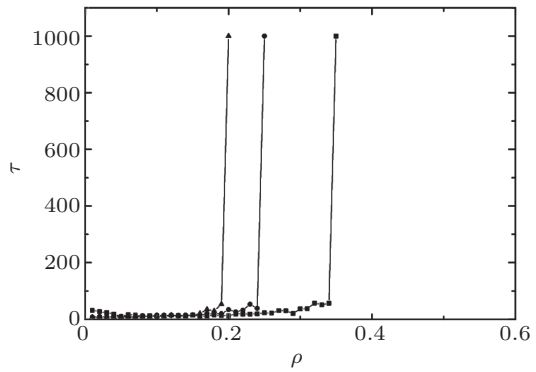


Fig. 5. The relaxation time τ versus the density ρ_j in lane j, where the densities in the other lanes $i \neq j$ are equal ($\rho_i = 0.1$). Hence, square dots correspond to the variation of τ versus ρ_2 with $\rho_1 = \rho_3 = \rho_4 = 0.1$, we show that τ diverges at $\rho_{2c} = 0.33$. The same remark holds for the case $j = 3$ (circle dots) at $\rho_{3c} = 0.25$. And in the case of $j = 4$ (up triangles), τ diverges at $\rho_{4c} = 0.2$.

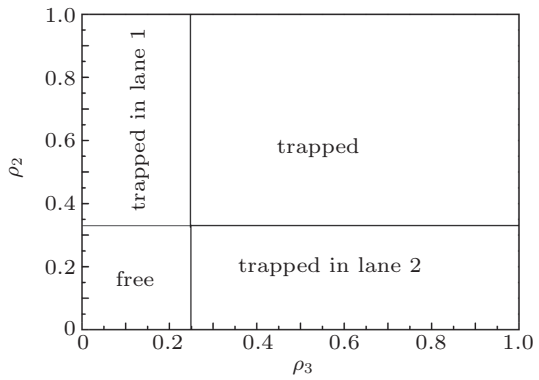


Fig. 6. The phase diagram (ρ_3, ρ_2) for $\rho_4 = 0.1$ and $\rho_1 = 0.3$ in the deterministic case ($P_c = 1$ and $P = 0$). The choice $\rho_4 = 0.1$ is to ensure that the merging vehicle can reach the last lane when it is in lane 3. In other words, we are interested in both lanes 2 and 3, because for $\rho_4 > \rho_{4c} = 0.2$, the system must be in the trapped state whatever ρ_1, ρ_2, and ρ_3 are. Furthermore, ρ_1 does not affect these results, since the merging car begins its trajectory from lane 1.

To approach the real situation, we have studied the case where all ρ_j are variable. For the deterministic ($P_c = 1$ and $P = 0$) case and the stochastic ($P_c = 1$ and $P \neq 0$) case, the results are shown in Figs. 6 and 7, respectively. These two figures summarize the states of vehicle M depending on the situation (density and the behavior of drivers) in each lane. We note that for a small value of the braking probability, $P = 0.1$, the area of the free state in the phase diagram increases significantly because ρ_c increases with increasing P as explained before. On the other hand, we have not taken into account the case $P_c < 1$, because we know

that P_c influences the finite value of τ in the free state.

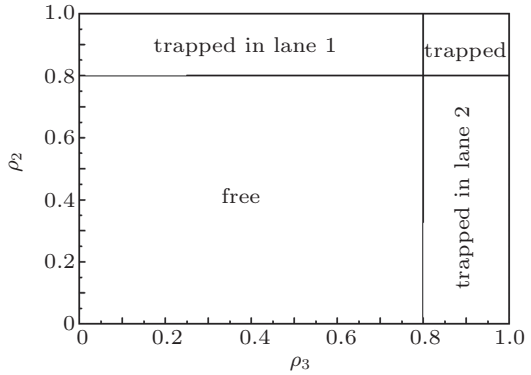


Fig. 7. The same phase diagram as that in Fig. 6 for the stochastic case ($P_c = 1$ and $P = 0.1$). We notice that for $P \neq 0$, the area of the trapped state in the phase diagram increases, since we have shown that the values of ρ_{cj} depend strongly on the braking probability (see Fig. 4).

3.2.2. The exit vehicle

This situation corresponds to a fast vehicle (vehicle D) in the fastest lane that wants to leave the highway at a specific site i_e of the slowest lane. According to the previous results, the change from lane $j+1$ to lane j is possible only when $\rho_j < \rho_{cj}$. However, under this condition, no one can be sure that vehicle D can reach the desired site i_e in the slowest lane.

For simplicity, we will first consider the situation of two lanes, lane 3 ($\rho_3 = 0.3$) and lane 2 ($\rho_2 = 0.25$). Vehicle D runs in lane 3 and wants to reach a specific site i_e in lane 2. It seems that it is no problem to change from lane 3 to lane 2, since

$$\rho_2 < \rho_{c2}. \tag{19}$$

However, figure 8 shows that reaching i_e in lane 2 is not a simple task. Figure 8 displays the probability P_e that vehicle D (which comes from lane 3) reaches position $i_e = 900$ in lane 2 versus distance d. Here

$$d = i_e - i_d \tag{20}$$

is the distance between position $i_e = 900$ and i_d (the position in lane 3 where vehicle D has decided to change lane). According to this figure, we can define two states. The critical state, in which vehicle D can reach i_e but with probability $P_e < 1$. Figure 9 shows clearly that there is a critical distance d_c, below which vehicle D may not reach the desired way out i_e. Indeed, the inset of Fig. 9 indicates that for

$$d < d_c = 200, \tag{21}$$

i.e,

$$i_d > i_e - d_c, \tag{22}$$

there is a probability $q \neq 0$ ($q = 1 - P_e$) that the fast car can not reach i_e. Furthermore, q increases as long as $d \to 0$, which means that the situation will be more critical ($q \to 1$) when $i_d \to i_e$. The safe state, in which vehicle D can surely reach i_e, since $P_e = 1$ ($q = 0$). This is possible only when

$$d \geq d_c, \tag{23}$$

i.e.,

$$i_d \leq i_e - d_c. \tag{24}$$

We note that the probability P_e undergoes a continuous transition from the critical state to the safe state at

$$d = d_c. \tag{25}$$

We conclude that the possible change from lane 3 to lane 2 ($\rho_2 < \rho_{c2}$) does not mean that vehicle D can leave the highway at any site in lane 2 (if we consider that the highway contains only lanes 3 and 2). Indeed, the result in Fig. 8 indicates that vehicle D should decide to change lane at a better position i_d in the fastest lane, i.e., it should estimate the distance $d \geq d_c$, otherwise, the way out will be out of its reach. This conclusion leads us to ask the following question, what are the parameters that govern the estimation of distance d?

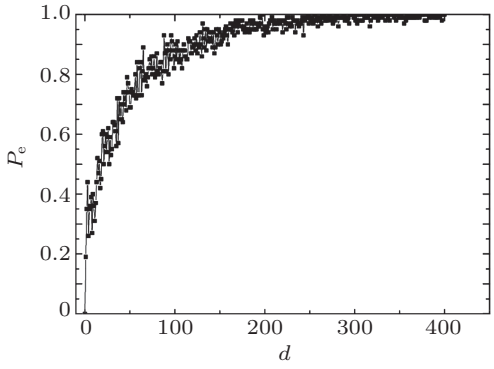


Fig. 8. The probability P_e to reach a specific site i_e in lane 2 ($\rho_2 = 0.25$ and $V_{max2} = 2$) versus distance d between i_e and site i_d in lane 3 ($\rho_3 = 0.3$, $V_{max3} = 3$). We have chosen $\rho_2 = 0.25 < \rho_{c2} = 0.33$ to ensure the safety of changing lanes, and $P_c = 1$. Each point in the figure is obtained by choosing 100 random initial configurations. For example, for $d = 100$, $P_e = 0.9$, which means that in 90 different initial configurations, the fast driver reaches site i_e in lane 2, whereas he does not in the other 10 configurations.

It is found that the shape of curve P_e versus d (Fig. 9) depends strongly on the density ρ_2 of lane 2, and the critical distance d_c follows an exponential law

$$d_c \approx 0.52 \exp(\rho_2/0.04). \tag{26}$$

This result indicates that the situation of the diverging vehicle will be worse when $\rho_2 \to \rho_{c2}$.

On the other hand, curve P_e versus d does not depend on the probability P_c, as shown in Fig. 10. Such a result confirms the discussion in the previous section, where we have shown that P_c does not affect the critical densities.

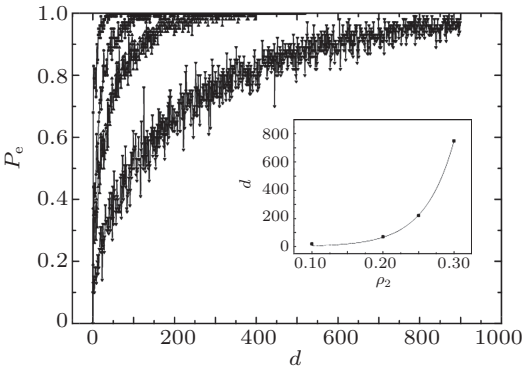


Fig. 9. The probability P_e versus distance d for the same conditions as those in Fig. 8 ($i_e = 900$, $P_c = 1$, $\rho_3 = 0.3$, $V_{max3} = 3$, and $V_{max2} = 2$), but for different values of ρ_2. Squares correspond to $\rho_2 = 0.1$, circles $\rho_2 = 0.2$, up triangles $\rho_2 = 0.25$, and down triangles correspond to $\rho_2 = 0.3$, i.e, near $\rho_{c2} = 0.33$. The inset shows d_c versus density ρ_2, the solid line is an exponential fit.

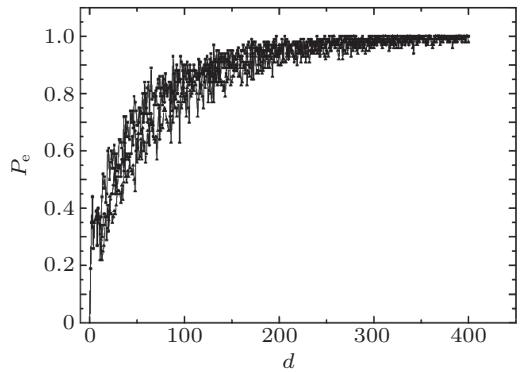


Fig. 10. The probability P_e versus distance d for different values of P_c, (squares $P_c = 1$, circles $P_c = 0.5$, and up triangles $P_c = 0.3$). Here the highway contains only two lanes (lane 3, $\rho_3 = 0.1$, $V_{max3} = 3$; lane 2, $\rho_2 = 0.25$, $V_{max2} = 2$).

Let us consider now the situation of three lanes. Vehicle D runs in lane 3 and wants to leave the highway in site $i_{e(1)}$ of the slowest lane 1. First, the driver of vehicle D should take into consideration the situation in lane 1. Indeed, knowing the function

$$d_{c(1)} \approx A_1 \exp(\rho_1/B_1), \tag{27}$$

he (or she) will be able to estimate d_1 well. Since

$$d_1 > d_{c(1)} \tag{28}$$

and

$$d_1 = i_{e(1)} - i_{d(2)}, \tag{29}$$

hence, the good positions in lane 2 are those which satisfy

$$i_{d(2)} < i_{e(1)} - d_{c(1)}. \tag{30}$$

The change from lane 3 to lane 2 also depends on

$$d_{c(2)} \approx A_2 \exp(\rho_2/B_2), \tag{31}$$

where

$$d_2 = i_{e(2)} - i_{d(3)} > d_{c(1)}, \tag{32}$$

which leads to

$$i_{d(3)} < i_{e(2)} - d_{c(2)}. \tag{33}$$

However, $i_{e(2)}$ and $i_{d(2)}$ are the same. As a consequence,

$$i_{d(3)} < i_{e(1)} - d_{c(1)} - d_{c(2)}. \tag{34}$$

This relationship indicates the choice of the available positions for vehicle D in lane 3 to leave the highway in position $i_{e(1)}$.

Generally, to realize a safe and comfortable exit from the highway (which contains K lanes) in a specific position $i_{e(1)}$, the driver of vehicle D in lane K should know $d_{c(j)}$ in each lane j, where $1 \le j < (K-1)$. After that, the positions at which he (or she) can begin to change lane are those which satisfy the relationship

$$i_{d(K)} < i_{e(1)} - \sum_{j=1}^{K-1} d_{c(j)}. \tag{35}$$

We note that the estimation of $i_{d(K)}$ based on Eq. (35) is difficult for the driver of vehicle D, and impossible when the number of lanes K is big.

We recall that the establishment of an effective traffic model to reveal fundamental traffic characteristics is an essential requirement in the design, planning, and operation of transportation systems. TRANSIMS[24,25] is an example of such a system, it is a transportation modelling package that has a microsimulator component built on the CA network representation. Hence, we propose that the previous analysis (Eq. (35)) can be done using an intelligent transportation system, which can guide the movement of vehicle D.

On the other hand, we have not treated the stochastic case $P \neq 0$. We recall that P is a parameter which quantifies human behavior and difficult to measure. Despite this, we can easily prove that $d_{c(j)}(P \neq 0)$ is smaller than $d_{c(j)}(P = 0)$. As a consequence, it is necessary to consider the deterministic case, because it represents the safe situation.

If vehicle D cannot accomplish his goal, what will he do? Will he stop and wait or go ahead to exit at downstream of the ramp? In our investigation, we have not given the answers to this question, because our aim is to establish Eq. (35). In reality, a diverging driver who fails to realize his (or her) goal must not stop (so as not to disturb the traffic and cause an accident). He (or she) must go ahead to exit at another off ramp.

4. Conclusion

In this paper, we have investigated two commonly observed situations when the road has multiple lanes. The first situation concerns a fast diver who enters the road from the on ramp and wants to merge into the fastest lane. The second situation is related to the case of a fast vehicle which would diverge from the fast lane, and exit from the highway at a specific point. We believe that these two situations represent the most preoccupations for merging and diverging divers in a multi-lane road. Our proposed model is an extension to the NaSch model for the multi-lane road. The analytical and numerical simulation results lead us to conclude that merging into the main road (or diverging from it) is not so easy. Indeed, in both cases, our investigation shows the existence of two steady states, the free state in which the merging (or diverging) car can move from lane to lane without any hindrance, and the trapped state in which it can not reach the desired lane. These two states depend strongly on the density of cars in each lane and the braking probability. Furthermore, the case of the diverging car is more complicated in the sense that the driver of the diverging vehicle should know the critical distance in each lane. After that, he (or she) should estimate the departure site in the fast lane. Such an analysis is impossible for the driver of the diverging car. Hence, we believe that this analysis can be realized by using an intelligent transportation system, which is able to evaluate the density of cars in each lane and the critical distances. Based on our model, this system can do the necessary simulations, and find out the optimum path for the diverging car. The implementation of such a system can avoid the instabilities caused by the mandatory lane changing behavior performed by merging (diverging) vehicles and car accidents.

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