New assembly route for three-dimensional metamaterials obtained through effective medium theory*

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In this study, we illustrate the effective medium theories in the designs of three-dimensional composite metamaterials of both negative permittivity and negative permeability. The proposed metamaterial consists of random coated spheres with sizes smaller compared to the wavelength embedded in a dielectric host. Simple design rules and formulas following the effective medium models are numerically and analytically presented. We demonstrate that the revised Maxwell–Garnett effective medium theory enables us to design three-dimensional composite metamaterials through the assembly of coated spheres which are random and much smaller than the wavelength of the light. The proposed approach allows for the precise control of the permittivity and the permeability and guides a facile, flexible, and versatile way for the fabrication of composite metamaterials.

Keywords: metamaterials, effective medium theory

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1. Introduction

Recently, metamaterials have attracted a great deal of attention due to much experimental and theoretical interest and their large potential applications.[1−9] In metamaterials, two important parameters, effective permittivity $\varepsilon_{\rm eff}$ and effective permeability $\mu_{\rm eff}$ that determine their response to electromagnetic radiation, are simultaneously negative in a given frequency interval. Thus, they are also called left-handed materials or negative index materials. The composite metamaterials are usually fabricated through embedding the periodic scattering elements in a homogeneous dielectric medium to provide an effective permittivity $\varepsilon_{\rm eff}$ and an effective permeability $\mu_{\rm eff}$. One typical example is the split-ring resonator (SRR)/wire composite structure, in which SRRs and wires are constructed on either side of the circuit board substrate.[10,11] The SRR/wire structures have proven to be useful in demonstrating the underlying wave propagation behavior of negative index materials. Similar designs have also been experimentally and theoretically utilized and discussed to achieve negative permittivity and negative permeability.[11]

The initial planar metamaterial designs may have some limitations, and the estimation of effective parameters is also complicated, sometimes even the experimental data for the transmission cannot prove that both $\varepsilon_{\rm eff}$ and $\mu_{\rm eff}$ are indeed negative. Very recently, there has been a great deal of attention drawn to the three-dimensional metamaterials. The three-dimensional metamaterials allow the propagation of electromagnetic wave in all directions, have unique properties that are distinct from those of the conventional planar metamaterial structures, and thus offer more promising applications. In this work, we propose an interesting way to fabricate artificial three-dimensional composite negative index metamaterial structures. Electromagnetic waves interacting with a composite structure containing scattering elements are often well characterized by the effective medium theories (EMTs). The proposed composite structures obtained from assembling a mixture of binary coated spheres would enable both negative permittivity and negative permeability, where the assembly step is inherent to the EMTs and is expedient to analysis and control of $\varepsilon_{\rm eff}$ and $\mu_{\rm eff}$. We shall explain the principle of the composite metamaterial assembly method

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based on the EMTs and develop a general recipe for composite metamaterial designs, and thereby provide one assembly route for composite metamaterial structures with anticipated effective permittivity and effective permeability.

2. Effective medium theory

First of all, we would briefly sketch the most commonly used EMTs necessary for setting the basic idea for the assembly of composite structures. Each of the EMTs not only provides a better approach to characterize the effective physical parameters of composite materials, but more importantly also draws one way for the composite material assembly. The EMTs mostly considered for the applications include the simple effective medium theory (S-EMT) (assembly of thin parallel plates), the Maxwell–Garnett (MG) model (assembly of random small spheres), the revised MG (R-MG) model (assembly of random coated small spheres), the Bruggeman (BG) model (assembly of two self-consistent media), and the general effective medium theory (G-EMT) (assembly of complex mixed components).

The first case of considerable practice interest is that of a regular assembly of thin parallel plates, which leads to the expression of the S-EMT. We concentrate on periodic layered plates systems of the form shown in Fig. 1(a) and assume that each plate can be described by homogeneous and isotropic permittivity ε_1 or ε_2 and permeability μ_1 or μ_2. Suppose that the incident field has its electric vector parallel to the plates, if the layer is sufficiently thin compared to the wavelength, the field in the plate may be considered to be uniform. Since the tangential component of electric vector $\boldsymbol{E}$ is continuous across a discontinuity surface, the effective dielectric constant of the system is given by the ratio of the electric displacement $\boldsymbol{D}$ to the electric vector $\boldsymbol{E}$ as[12]

$$\begin{aligned}\varepsilon_{\text{eff}} &= \frac{\boldsymbol{D}}{\boldsymbol{E}} = \frac{t_1\varepsilon_1 + t_2\varepsilon_2}{t_1 + t_2} = f_1\varepsilon_1 + f_2\varepsilon_2 \\ &= f_1\varepsilon_1 + (1 - f_1)\varepsilon_2, \end{aligned} \tag{1}$$

where $f_1 = t_1/(t_1 + t_2)$ and $f_2 = t_2/(t_1 + t_2) = 1 - f_1$ are the volume fractions for the two different plates, respectively. This is the simplest case for the assembly of multilayered structures, and it can also be used for an assembly of parallel and similar thin cylindrical rods.

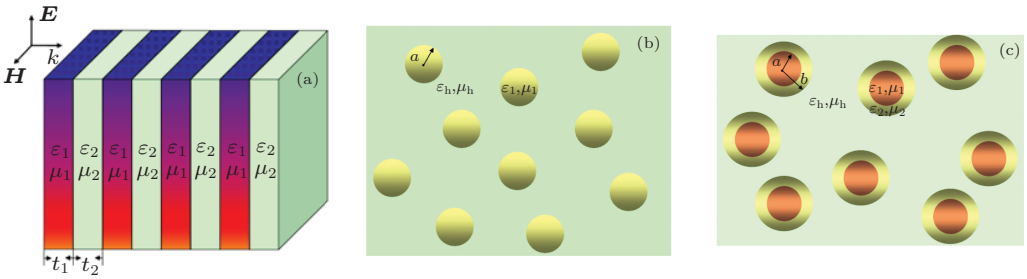


Fig. 1. (colour online) Sketches of assemblies of (a) thin parallel plates, (b) small spheres, and (c) coated small spheres.

The above mentioned S-EMT can be used to study the multilayered structures with some orders or some random air holes embedded into the dielectric substrate. For a homogeneous and isotropic medium of permittivity ε_{h} and permeability μ_{h} containing small random isotropic spheres with permittivity ε_1, permeability μ_1, and radius a, as shown in Fig. 1(b), we may need some other effective medium models. In this case, it should be noted that the order is not assumed by the effective medium theory, because it does introduce the chance of coherent scattering. Since that, here we emphasize a random system. For this case, the basic definition of an effective medium is that the random unit cell is a sphere embedded in an external homogeneous medium and the total inhomogeneous material possesses the macroscopic uniformity treated by the effective medium theory. The uniform static electric field distribution in a medium of ε_{h} will be distorted by the introduction of spheres, where a dipole moment is induced inside each sphere. The system of such a form containing sphere scatterings can be described by an effective medium permittivity ε_{eff}, which can be derived through the following relationship among the electric displacement $\boldsymbol{D}$, the electric

field $\boldsymbol{E}$, and the polarization vector $\boldsymbol{P}$:[13,14]

$$\boldsymbol{D} = \varepsilon_{\rm h}\boldsymbol{E} + \boldsymbol{P} = \varepsilon_{\rm h}\frac{1+2\alpha\, n_0/3\varepsilon_{\rm h}}{1-\alpha\, n_0/3\varepsilon_{\rm h}}\boldsymbol{E}, \tag{2}$$

where the polarizability of the sphere, α, is defined to be

$$\alpha = 4\pi\,\varepsilon_{\rm h}a^3\frac{\varepsilon_1-\varepsilon_{\rm h}}{\varepsilon_1+2\varepsilon_{\rm h}}, \tag{3}$$

and n_0 is the sphere number per unit volume. It follows that the effective permittivity is

$$\begin{aligned}\varepsilon_{\rm eff} &= \frac{\boldsymbol{D}}{\boldsymbol{E}} = \varepsilon_{\rm h}\frac{1+2\alpha\, n_0/3\varepsilon_{\rm h}}{1-\alpha\, n_0/3\varepsilon_{\rm h}}\\ &= \varepsilon_{\rm h}\frac{1+2f(\varepsilon_1-\varepsilon_{\rm h})/(\varepsilon_1+2\varepsilon_{\rm h})}{1-f(\varepsilon_1-\varepsilon_{\rm h})/(\varepsilon_1+2\varepsilon_{\rm h})}.\end{aligned} \tag{4}$$

Here the fractional volume occupied by the spheres is $f = n_0(4\pi a^3/3)$. We can further write it in a more symmetric form

$$\frac{\varepsilon_{\rm eff}-\varepsilon_{\rm h}}{\varepsilon_{\rm eff}+2\varepsilon_{\rm h}} = f\frac{\varepsilon_1-\varepsilon_{\rm h}}{\varepsilon_1+2\varepsilon_{\rm h}}. \tag{5}$$

Equation (5) leads us to the well-known MG-EMT formula. The MG model usually describes isotropic spheres that are dispersed in a surrounding host matrix and isolated from each other. Although it accounts for the scattering effect caused by the small particles at the Rayleigh limit, the MG model is widely used for composite material analysis. Additionally, we can apply the MG approach to two or more inclusions of dielectric constants ε_1 and ε_2 with filling factors f_1 and f_2, respectively, in a host medium $\varepsilon_{\rm h}$ as[15,16]

$$\frac{\varepsilon_{\rm eff}-\varepsilon_{\rm h}}{\varepsilon_{\rm eff}+2\varepsilon_{\rm h}} = f_1\frac{\varepsilon_1-\varepsilon_{\rm h}}{\varepsilon_1+2\varepsilon_{\rm h}} + f_2\frac{\varepsilon_2-\varepsilon_{\rm h}}{\varepsilon_2+2\varepsilon_{\rm h}}. \tag{6}$$

The above equation provides one way to solve multiphase composites.

A more complicated case extending the results above for the small spheres is the effective medium theory for the coated spheres, and we call this the revised MG (R-MG) model. As shown in Fig. 1(c), we consider an inner or core sphere with permittivity ε_1, permeability μ_1, and radius a coated by a material of permittivity ε_2 and permeability μ_2. The random coated spheres are surrounded by an external medium of $\varepsilon_{\rm h}$ and $\mu_{\rm h}$. For such a form, the effective permittivity $\varepsilon_{\rm eff}$ of the whole volume medium can be obtained through a similar way as mentioned above. The scattered field can be attributed to the induced dipoles of the core spheres and the coating mantles, and thus the effective permittivity follows a similar formula to the MG model in Eq. (4) as[13]

$$\varepsilon_{\rm eff} = \frac{\boldsymbol{D}}{\boldsymbol{E}} = \varepsilon_{\rm h}\frac{1+2f\tilde{\varepsilon}}{1-f\tilde{\varepsilon}}, \tag{7}$$

where $\tilde{\varepsilon}$ is defined to be

$$\tilde{\varepsilon} = \frac{(\varepsilon_2-\varepsilon_{\rm h})(\varepsilon_1+2\varepsilon_2)\ +\eta(\varepsilon_1-\varepsilon_2)(\varepsilon_{\rm h}+2\varepsilon_2)}{(\varepsilon_2+2\varepsilon_{\rm h})(\varepsilon_1+2\varepsilon_2)+\eta(2\varepsilon_2-2\varepsilon_{\rm h})(\varepsilon_1-\varepsilon_2)}, \tag{8}$$

the fractional volume occupied by the coated spheres is $f = n_0(4\pi b^3/3)$, and $\eta = a^3/b^3$ is the fraction of the total sphere volume occupied by the inner core. Without coating, equation (7) degenerates into the MG model shown in Eq. (4).

It is interesting to note that a coated sphere will become invisible if $\tilde{\varepsilon}$ equals zero. When $\tilde{\varepsilon}$ is zero, the polarizability $\alpha = 4\pi\,\varepsilon_{\rm h}\tilde{\varepsilon}b^3$ is zero, and thereby the scattered field results in zero rendering the coated particle invisible. In this case, the numerator in $\tilde{\varepsilon}$ vanishes

$$(\varepsilon_2-\varepsilon_{\rm h})(\varepsilon_1+2\varepsilon_2)\ +\eta(\varepsilon_1-\varepsilon_2)(\varepsilon_{\rm h}+2\varepsilon_2)=0. \tag{9}$$

Equation (9) further leads to the relationship between the core permittivity ε_1 and the mantle ε_2 as

$$\varepsilon_1 = \varepsilon_2\frac{(2+\eta)\varepsilon_{\rm h}+2(\eta-1)\varepsilon_2}{(\eta-1)\varepsilon_{\rm h}+(2\eta+1)\varepsilon_2}. \tag{10}$$

We only need an appropriate choice for the permittivities of the core sphere, the outer coating mantle, and the external surrounding medium, and then we can take $\tilde{\varepsilon}$= 0. The basic premise for this special case of the invisible effect is that the coated sphere is much smaller compared to the wavelength, and also it is worthy noting that all permittivity constants are treated as real numbers neglecting the absorption, although the R-MG model denoted in Eq. (7) is valid for the complex permittivity situation. On the other hand, if we let the denominator of $\tilde{\varepsilon}$ equal zero, it would lead to the well known condition for the excitation of the Fröhlich mode, which plays a significant role in the investigations of the resonance of small particles. For this case, the relationship between ε_1 and ε_2 can be expressed as

$$\varepsilon_1 = -2\varepsilon_2\frac{(2+\eta)\varepsilon_{\rm h}+(1-\eta)\varepsilon_2}{2(1-\eta)\varepsilon_{\rm h}+(2\eta+1)\varepsilon_2}. \tag{11}$$

Numerous discussions relating to the surface excitation effect of small particles would be involved with this expression, and it is essential to the study of surface modes of small particles.[13]

Another situation often encountered is that it is not possible to distinguish precisely between inclusions and the host matrix, where both components are treated symmetrically. When we consider an assembly

in which both media of the mixture intersperse with each other with equal footing, the BG model is often used.[13,15,17] The assembly route for the metamaterial based on the BG model was also discussed recently.[18] In the BG model, the two elements are treated equally, and their properties are determined self-consistently. The effective dielectric function for a two-phase system is obtained by solving the BG equation

$$f\left(\frac{\varepsilon_1-\varepsilon_{\rm eff}}{\varepsilon_1+2\varepsilon_{\rm eff}}\right)+(1-f)\left(\frac{\varepsilon_{\rm h}-\varepsilon_{\rm eff}}{\varepsilon_{\rm h}+2\varepsilon_{\rm eff}}\right)=0, \quad (12)$$

where ε_1 and $\varepsilon_{\rm h}$ are the permittivities of the two mixed elements, respectively, and f is the volume fraction of medium ε_1.

Detailed discussions about these four traditional or classical EMT models, for instance, their validity or applications, have been well described previously.[13–19] Although some of them may in fact be questioned or improved through modern advancements, the EMTs described above have been widely used to characterize many composite systems and presented a better agreement with the measured results. It is important to notice that at filling fraction $f=0$ or $f=1$, the S-EMT, the MG model, and the BG model lead to the same result. Of course, if the topology is well specified, it is possible to determine the effective dielectric constant of composite materials directly through solving the Maxwell equations numerically.

To overcome the limitations of the above mentioned classical EMTs, a significant improvement has been recently made to the EMTs by McLachlan *et al.*,[20] and a general effective medium theory has been proposed. In that model, the effective dielectric function of the composite system has the expression

$$f\frac{\varepsilon_1^{1/t}-\varepsilon_{\rm eff}^{1/t}}{\varepsilon_1^{1/t}+A\varepsilon_{\rm eff}^{1/t}}+(1-f)\frac{\varepsilon_{\rm h}^{1/s}-\varepsilon_{\rm eff}^{1/s}}{\varepsilon_{\rm h}^{1/s}+A\varepsilon_{\rm eff}^{1/s}}=0, \quad (13)$$

where t and s are the percolation exponents, $A=(1-p_{\rm c})/p_{\rm c}$, and $p_{\rm c}$ is the percolation threshold. The other notations have the same meanings as those in Eq. (4). This G-EMT model combines most aspects of both the percolations and the classical EMTs, and thus can be widely used, not limiting to the well-defined morphologies. The G-EMT has been used to successfully fit the experimental data for a large number of binary composite media. The most detailed explanation of this model is given in Ref. [20].

The deduction for the MG or R-MG model actually starts from the Rayleigh limit, where the sphere or the coated sphere is required to be much smaller than the wavelength. The MG, the BG, and the R-MG effective medium theories are derived by considering the scattering amplitude in the small sphere limit. For particles of a larger size, higher-order terms should be taken into account. Meanwhile, as emphasized above, the effective medium theories are mainly used to the random systems where the coherent scattering is ignored.

Although all these commonly used EMT models are derived from a dielectric viewpoint, it is worth noting that all these models can apply not only to the permittivity ε, but also to the electrical conductivity σ, the thermal conductivity K, the gaseous diffusion D, and the magnetic permeability μ of the mixture.[20,21] This is especially important to allow for the assembly of composite metamaterials based on EMTs. For example, following the analogical approach in the R-MG model, we can obtain an expression of the effective permeability $\mu_{\rm eff}$ for coated spheres. With each permittivity replaced with its corresponding permeability, the R-MG model for the effective permeability can be written as

$$\mu_{\rm eff}=\mu_{\rm h}\frac{1+2f\tilde{\mu}}{1-f\tilde{\mu}}, \quad (14)$$

with

$$\tilde{\mu}=\frac{(\mu_2-\mu_{\rm h})(\mu_1+2\mu_2)\ +\eta(\mu_1-\mu_2)(\mu_{\rm h}+2\mu_2)}{(\mu_2+2\mu_{\rm h})(\mu_1+2\mu_2)+\eta(2\mu_2-2\mu_{\rm h})(\mu_1-\mu_2)}, \quad (15)$$

where the permeabilities of the inner core sphere, the outer coating layer, and the external surrounding medium are μ_1, μ_2, and $\mu_{\rm h}$, respectively; $\mu_{\rm eff}$ is the effective permeability of the composite, and f and η have the same meanings as those in Eqs. (7) and (8).

In this work, we shall embark on the study of coated spheres based on the above mentioned R-MG model for the effective permittivity and the effective permeability denoted by Eqs. (7), (8), (14), and (15), and show how metamaterial structures with both negative permittivity and negative permeability can be achieved from the assembly of coated spheres through an appropriate design. The composite structure containing the binary coated spheres starting with the R-MG model is essential for us to guide a unique assembly route for metamaterials. Although in the following sections we will focus on the coated sphere assembly configuration subject to the three-dimensional artificial composite metamaterials based on the R-MG model, it is noteworthy that each of the EMT models presented above would actually let us establish one

possible assembly way to design effective permittivity and effective permeability.

3. Simulations

The numerical evidence to better underline the performance of the R-MG model is now available through the full-wave numerical simulations. Figure 2 shows the electric-field distributions and the electromagnetic power-flow lines of the two-dimensional full-wave simulations for three cases: (i) a small sphere as designed in Fig. 1(b); (ii) a small sphere of the same geometry as that in case (i) but with a coating mantle; (iii) the same geometry as that in case (ii) but with a proper design in permittivity to show the invisible effect. In these cases, the full wave electromagnetic simulations are performed using the finite element method. A 2 GHz transverse-electric (TE) plane wave is incident along the x direction upon the small sphere or the coated sphere of a diameter less than the wavelength. The computational domain is terminated by perfectly matched layers (PMLs). The color map depicts the spatial distribution of the electric field oriented along the z direction.

In Fig. 2(a), we consider a small nonmagnetic sphere of permittivity $\varepsilon_1 = 5.0$ in the air. As can be seen, the electromagnetic waves are scattered upon the small sphere. Figure 2(b) extends the situation above from the small sphere to a coated sphere, where the coating shell has a permittivity of $\varepsilon_2 = 2.0$, and the core is the same as that in Fig. 2(a). It is noticeable that the presence of the coating shell enhances the total scattering of the plane-wave field. In particular, figure 2(c) presents how the performance of the coated sphere drastically reduces the total scatting of the incident plane wave and thereby renders a homogeneous coated spherical particle invisible through a proper design according to the R-MG model. Supposing the surrounding medium $\varepsilon_h = 1.0$ and the coating mantle $\varepsilon_2 = 2.0$, we have core permittivity $\varepsilon_1 = -1.69$ according to Eq. (10). As shown in Fig. 2(c), the invisible effect is clear with minimum scattering as expected, and it is also evident from this figure that the EMTs can give desired effective physical parameters through proper designs. Compared with the recently proposed cloaking,[22−24] where the electromagnetic fields can be bended and stretched through a specified coordinate transformation, figure 2(c) demonstrates another route to cloaking through the assembly of coated spheres. Actually, the scattered wave is attributed to the induced dipole of the coated sphere, but this does not matter whether one assigns this scattering to permittivity or to permeability. In fact, if we replace all the permittivities in Fig. 2 to the corresponding permeabilities, the same simulated profiles can be obtained. For the whole volume medium containing coated spheres, the scattering accounts for its effective permittivity and effective permeability.

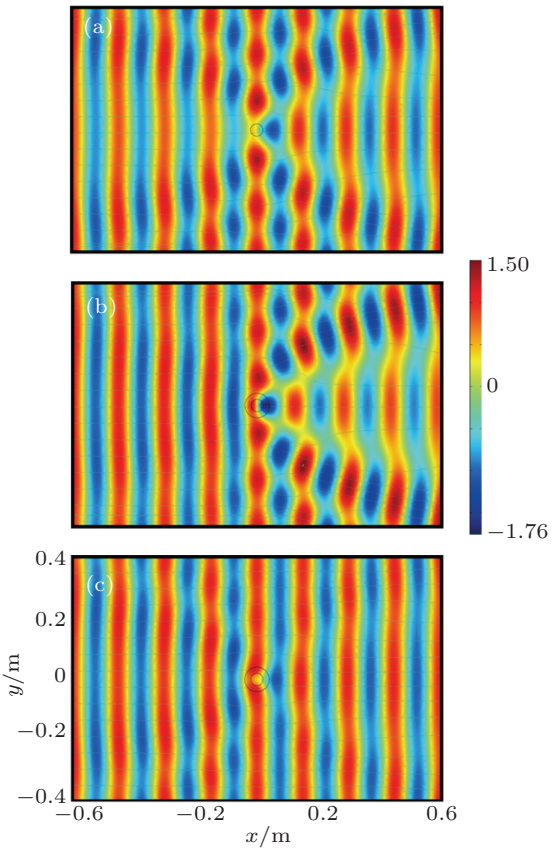


Fig. 2. (colour online) Simulated electric field distributions in the external surrounding media ($\varepsilon_h = 1.0$) containing (a) small dielectric sphere of permittivity $\varepsilon_1 = 5.0$, (b) coated sphere with permittivity of the core sphere $\varepsilon_1 = 5.0$ and permittivity of the outer coating shell $\varepsilon_2 = 2.0$, and (c) invisible coated sphere with permittivity of the core sphere $\varepsilon_1 = -1.69$ and permittivity of the outer coating shell $\varepsilon_2 = 2.0$. The power flow lines (grey lines) are directed in the x direction.

Since the effective permittivity and the effective permeability of the composite containing coated spheres are adjustable in terms of the R-MG model, we are able to construct a composite metamaterial structure with both negative permittivity and negative permeability through a proper design in f, η, permittivities, and permeabilities of surrounding medium, core sphere, and mantle material. For instance, with fixed filling fraction f and η, the effective permittiv-

ity and the effective permeability are calculated using the R-MG model denoted in Eqs. (7) and (14) for different ratios of the core sphere permittivity or permeability to the coating's values, as well as different ratios of the coated sphere permittivity or permeability to the surrounding medium's. Figure 3 plots the calculated results for these different configurations of various material parameters to explore the sensitivity and the reliability of the metamaterial composite by assembling coated spheres. It is seen that the necessary condition of realizing negative permittivity/permeability includes that the coating shell and the core sphere at least have the opposite signs in the permittivity/permeability, if we assume that the surrounding medium has a positive magnitude.

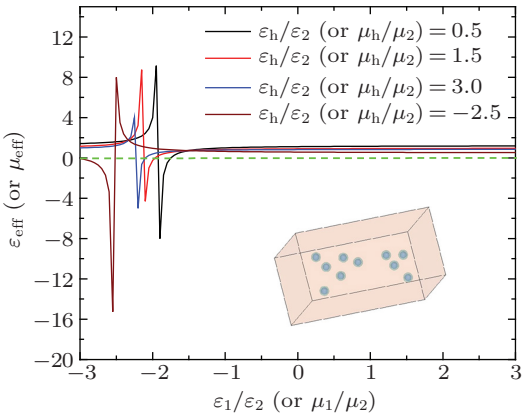


Fig. 3. (colour online) Effective permittivity or effective permeability for different configurations with various permittivity or permeability components calculated by the R-MG model for the assembly of coated spheres.

The R-MG model provides a conceptually simple approach to the design of metamaterials, and it is perceivable that one can discuss various complex metamaterial configurations. As a typical example, let us consider a fairly interesting case that a metamaterial structure is fabricated by embedding coated spheres into a transparent medium of ε_h, where a magnetic core is coated by one metal shell. Figure 4 shows the simulated distribution of the electric field, where we assume that the core magnetic sphere has $\varepsilon_1 = 2.0$ and $\mu_1 = -50.0 + \mathrm{i}100.0$, and the coating metal has $\varepsilon_2 = -24.0 + \mathrm{i}0.5$ and $\mu_2 = 1.0$. As shown in Fig. 4, the scattering effect is seen clearly due to the induced dipoles of the magnetic core and the metal shell. However, a general problem is that the permeability of the magnetic particle and the permittivity of the metal mantle are mostly frequency dependent. Thus, we need to take the frequency dependence into account.

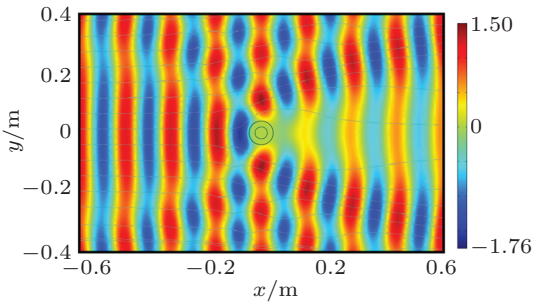


Fig. 4. (colour online) Electric field distribution in the external surrounding medium of $\varepsilon_h = 1.0$ and $\mu_h = 1.0$ containing a coated magnetic sphere. The core magnetic sphere of $\varepsilon_1 = 2.0$ and the outer metal shell of $\mu_2 = 1.0$ are assumed in the calculation.

For the outer metal coating layer, it is known that the permittivity as a function of frequency can be well described by the Drude model as

$$\varepsilon_2 = 1 - \frac{\omega_p^2}{\omega^2 + \mathrm{i}\gamma\omega}, \tag{16}$$

where the key parameters describing the dynamics of free electrons or plasmons in a metal are the plasma frequency $\omega_p = \sqrt{Ne^2/(\varepsilon_0 m^*)}$ and the carrier damping constant γ, N is the carrier density, m^* is the electron effective mass, and ε_0 is the free-space permittivity constant with a value of 8.854×10^{-12} F/m. On the other hand, the frequency-dependent permeability of the inner magnetic particle is given by

$$\mu_1 = 1 - \frac{\omega^2}{\omega^2 - \omega_m^2 + \mathrm{i}\Gamma\omega}, \tag{17}$$

where ω_m is the magnetic resonance frequency, and Γ is the damping constant.

Because the size of a particle in wavelength is more fundamental for light scattering than that in absolute terms, we introduce a scaling parameter $\Delta = b/c$, where b is the radius of the whole coated particle, and c is the light speed in vacuum. All the physical parameters ω_p, ω_m, γ, Γ, and ω used in the following calculation are normalized by Δ. Figure 5(c) illustrates explicitly how the effective permittivity and the effective permeability of the composite medium change as functions of the normalized frequency, where the magnetic particle coated with a metal layer is embedded into one transparent medium of $\varepsilon_h = 2.33$ with fixed filling factor parameters. Figures 5(a) and 5(b) show the corresponding frequency-dependent permittivities and permeabilities of the core magnetic particle and the outer coating metal layer, respectively. It can be

found from Fig. 5(c) that the composite structure possesses both negative permittivity and negative permeability when the normalized frequency is between 2.34 and 2.51. The optimization might be achieved, for instance, by changing the filling fraction of the coated spheres, the ratio of radius between the coating particle and the core, or the surrounding medium. In this configuration, the composite metamaterial with flexible permittivity and permeability can be obtained. The figure shows a practical strategy to use magnetic particles coated with metal shells in a proper design to realize real three-dimensional composite metamaterials.

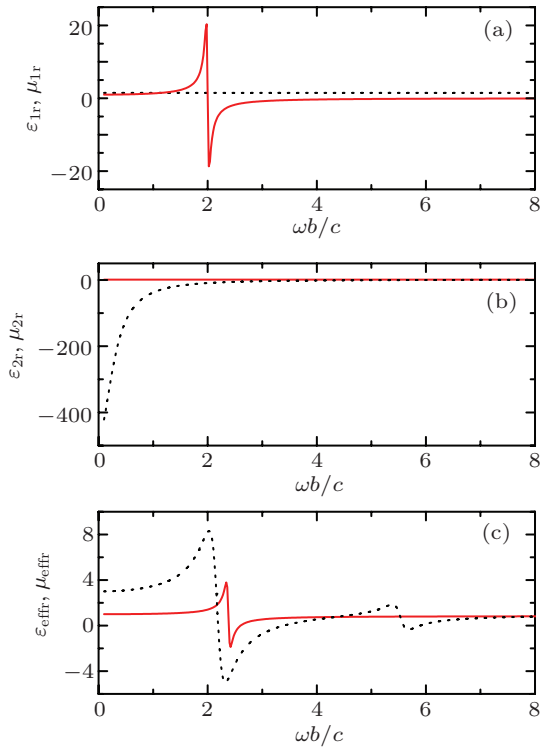


Fig. 5. (colour online) Illustration of the assembly of composite metamaterials by embedding coated spheres into a transparent dielectric medium of $\varepsilon_h = 2.33$. (a) The permittivity (dashed line) and the permeability (solid line) of the magnetic inner sphere each as a function of the normalized frequency. The frequency-dependent permeability μ_1 is calculated by Eq. (17) with $\omega_m \ \Delta = 2.0$ and $\Gamma\Delta = 0.05$. The permittivity is fixed at $\varepsilon_1 = 1.5$. (b) The permittivity and the permeability of the outer metal shell each as a function of the normalized frequency. The frequency-dependent permeability ε_2 is calculated by Eq. (16) with $\omega_p\Delta = 8.5$ and $\gamma\Delta = 0.6$. The permeability is fixed at $\mu_2 = 1.0$. (c) The effective permittivity and the effective permeability of the composite each as as a function of the normalized frequency calculated by the R-MG model. The filling fraction parameters are $f = 0.4$ and $\eta = 0.3$.

4. Conclusion

In summary, we have proposed a method to fabricate three-dimensional composite metamaterials through the assembly of small random coated spheres based on the effective medium theories. During the process, it becomes obvious that the permittivity and the permeability properties of composite structures are much better controlled and obtained, and may provide a comparable flexibility. Therefore, it is a promising route for the realization of real three-dimensional metamaterials of both negative permittivity and negative permeability. Apart from the metamaterial design, a further understanding of all presented EMT models will immediately yield new ideas for novel structure assembly and device applications.

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