Charge oscillation and many-body effect in triangular quantum dots*

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We study the charge oscillation in the triangular quantum dots symmetrically coupled to the leads. A strong charge oscillation is observed even for a very small level difference. We attribute this oscillation behaviour to the many-body effect in the strongly correlated system instead of the physical scenarios based on the mean-field approach in the previous works for the two-level dot. The level difference induces the difference of the occupations between different dots, while the symmetry of the many-body states favours the homogeneous distribution of the charge density on the three dots. The interplay of these two factors results in the charge oscillation.

Keywords: charge oscillation, many-body effect, triangular quantum dots

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1. Introduction

Quantum dot (QD) devices provide well-controlled objects for studying quantum many-body physics. When the QD is weakly connected to two leads, with the increasing gate voltage, its charge increases in a steplike manner. The steplike behaviour is caused by the Coulomb blockade (CB), which has gained significant interest both theoretically and experimentally.[1−11] In a two-level QD or double quantum dots (DQD), the charging process can be more complicated. By varying the gate voltage ϵ, the charge occupation on each level shows the oscillation behaviour. The related phenomenon is the population switching:[10−15] near the center of the Coulomb valley, the electron population of the QD switches from one level to the other. It is also relevant to some experimentally observed effects, such as charge sensing[16,17] and correlated phase lapses[18,19] of the transmission through a QD.[20−26] This charge oscillation behaviour is affected considerably by the QD–leads coupling Γ, the interdot Coulomb interaction V, and the level difference Δ.[13−15,27,28] However, the mechanism of nonmonotonic filling has not been understood completely. The numerical renormalization group (NRG) results[15] showed that in the limit of $\Gamma/\Delta \ll 1$, the usual CB behaviour with a monotonic increase in occupation is expected; when $\Gamma/\Delta \approx 1$, a strengthening of the charge oscillation is observed; in the limit of $\Gamma/\Delta \gg 1$, the monotonic increase in occupation is recovered, although its behaviour is different from that for $\Gamma/\Delta \ll 1$. This oscillation behaviour obtained by the NRG was explained by using a simple self-consistent Hartree approximation (SCHA). However, the results of the SCHA based on the single-particle picture did not agree with the NRG result in the case of $\Gamma/\Delta \gg 1$. The nonmonotonic charge occupation in the DQD system was also studied by using the perturbation theory.[13,14] A mechanism based on the changes in the spectral weights and their positions ϵ and $\epsilon + V$ was proposed for the case $\Gamma/\Delta \approx 1$. This two-level model has been mapped into a generalized Anderson model with pseudospin and effective magnetic field.[27,28] The numerical solution of the Bethe ansatz equations showed a large charge oscillation for $\Gamma/\Delta \gg 1$. Since the CB effect originates from the electron interaction, we believe that the charge oscillation should exhibit the many-body effect.

Recently, the many-body effect in triple quantum dots (TQD) has attracted much attention.[29−40] In this paper, we study the charge oscillation in a triangular TQD system with interdot Coulomb repulsion and level difference Δ by using the NRG method. In contrast to the situation in the DQD system studied in the previous work, we obtain a much larger charge

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oscillation even for $\Gamma/\Delta \gg 1$. This result cannot be understood based on the simple SCHA. In fact, the level difference induces the difference of the occupations between different dots, while the symmetry of many-body states favours a homogeneous distribution of the charge density on the three dots. The interplay of these two factors results in the charge oscillation. We discuss in detail the effect of the system parameters on the charge oscillation. Generally, a large level width Γ increases the degeneracy of three levels and therefore enhances the symmetry of many-body states, while the level difference Δ has the reversal effect. The Coulomb interaction $V \gg \Gamma$ guarantees the system in the strongly correlated regime and exhibits the remarkable many-body effect. To obtain a large amplitude of the oscillation, we must choose an appropriate set of parameters.

This paper is organized as follows. In Section 2, we describe the model and the calculation algorithms. In Section 3, we present the NRG results for the charge density on each dot and the interdot charge correlation. We will give a physical picture based on the many-body effect for the charge oscillation. Finally, the discussion and the conclusion are given.

2. Model and formulation

It is well known that the on-site Coulomb repulsion favours the homogeneous distribution of the charge while the interdot repulsion favours the charge fluctuation. The effect of the interdot repulsion on the charge oscillation has been investigated for double quantum dots.[13−15,27,28] In order to compare our results with those in the DQD, we consider the following model for the triangular TQD system with the interdot repulsion:

$$H = \sum_{jk\sigma} \epsilon_k c^{\dagger}_{jk\sigma} c_{jk\sigma} + \sum_i \epsilon_i n_i + V \sum_{j>i} n_i n_j + \sum_{jki\sigma} (V_k c^{\dagger}_{jk\sigma} d_{i\sigma} + \text{H.c.}), \quad (1)$$

where $c^{\dagger}_{jk\sigma}$ creates a spin-σ electron of wave vector k and energy ϵ_k in lead j (= L, R), $d^{\dagger}_{i\sigma}$ creates a spin-σ electron of energy ϵ_i in dot i (=1, 2, 3), $n_{i\sigma} = d^{\dagger}_{i\sigma} d_{i\sigma}$, $n_i = n_{i\uparrow} + n_{i\downarrow}$, and V is the interdot Coulomb repulsion. Here, we assume the tunnel matrix element V_k between the leads and the dots is symmetric with respect to the three dots and the two leads. We assume a dispersionless conduction band with a half bandwidth D, which is taken as the unit of energy. The tunnel coupling introduces a level width $\Gamma = 2\pi\rho_0 V_k^2$, where ρ_0 is the density of the state of the lead at the Fermi level $\epsilon_{\rm F} = 0$. Obviously, equation (1) does not include the on-site Coulomb repulsion between two electrons with different spins and can be mapped to a spinless model. However, in order to extend model (1) to a more general model including the on-site Coulomb repulsion in the further study, we still consider the spins on each dot in the following discussion.

We treat the model with the NRG method,[41,42] which is a nonperturbative approach to study the low-temperature properties of strongly correlated quantum dot systems. The key procedure of the NRG method consists of the logarithmic discretization of the conduction band, which is mapped onto a linear chain of fermions. The transformed chain can be iteratively diagonalized. In our NRG calculation, the discretization parameter is $\Lambda = 2.0$, and the number of states kept at each iteration is 1200.

3. Results and discussion

We study the charge oscillation in mode1 (1) using the NRG method. In order to avoid too many parameters, we assume an equal level difference $\Delta = \epsilon_2 - \epsilon_1 = \epsilon_3 - \epsilon_2$. However, our conclusion does not really require the level differences between the dots to be equal. Figure 1(a) shows the total charge $Q_{\rm dot} = \langle n_1 + n_2 + n_3 \rangle$ of the three dots for several level difference Δ. Obviously, $Q_{\rm dot}$ increases in a steplike manner with the decreasing ϵ_1. However, the behaviours of the individual level occupations are quite different. Figure 1(b) shows the charge density $\langle n_i \rangle$ on dot i as a function of level ϵ_1 for level difference $\Delta = 0.0005$, level width $\Gamma = 0.01$, and interdot Coulomb repulsion $V = 0.1$. With the decreasing ϵ_1, the charge on each dot exhibits an oscillation behaviour instead of the monotonical increasing. This nonmonotonic level dependence cannot be explained by the usual CB picture, in which the charge increases in a steplike manner when level $\epsilon_1 + V$ approaches the Fermi level. We attribute this oscillation to the many-body effects. Figure 1(c) presents the charge correlation $\langle n_i n_j \rangle$ between different dots. In the NRG calculation, the Fermi level is located at $\epsilon_{\rm F} = 0$. As the dot level ϵ_1 decreases to approaching the Fermi level, the three dots start to be occupied. In order to avoid the Coulomb repulsion and reduce the total energy, the many-body state $(n_1 n_2 n_3)$ has the configuration

(100), (010), or (001), in which one dot is occupied while the other two dots are empty. As ϵ_1 decreases further, the many-body state $(n_1n_2n_3)$ has the configuration (200), (020), or (002). In this process, the charge correlation $\langle n_in_j\rangle$ is zero (Fig. 1(c)).

When $\epsilon_1 < -0.03$ (point A in Fig. 1(b)), the total charge of the three dots approaches two electrons, and there is some possibility to be in the many-body configuration (210) or (201), which will increase the Coulomb energy by $2V$. In order to reduce the total energy, the system is more likely to be in the configuration of (200) instead of (210) and (201). Meanwhile, the possibilities of being in configurations (020) and (002) are also reduced. Therefore, the charge $\langle n_1\rangle$ on dot 1 increases to approaching $\langle n_1\rangle = 2$ at point B, while $\langle n_2\rangle$ and $\langle n_3\rangle$ decrease to nearly zero. In this process, the charge correlation $\langle n_in_j\rangle$ is still very small, since there is a very small possibility to be in configurations (210) and (201).

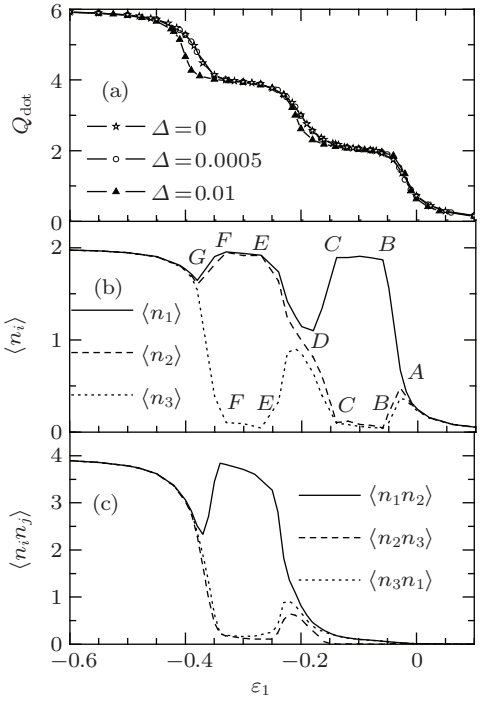


Fig. 1. (a) Total charge $Q_{\rm dot}$ of the three dots as a function of level ϵ_1 for different level difference Δ. (b) Charge density $\langle n_i\rangle$ on dot i (=1, 2 ,3) and (c) charge correlation $\langle n_in_j\rangle$ between dots i and j versus ϵ_1 for $\Delta = 0.0005$. The other parameters are $\Gamma = 0.01$ and $V = 0.1$. Letters A–G in panel (b) indicate some special points of ϵ_1.

As ϵ_1 decreases further, to add an additional electron to the configuration (200) will cost energy ϵ_1+2V, which is much higher than the Fermi energy $\epsilon_{\rm F} = 0$. Therefore, the total charge of the three dots nearly does not change in segment BC in Fig. 1(b). When $\epsilon_1 < -0.15$ (point C), the total charge of the three dots is more than two electrons, and the configuration (210) or (201) cannot be avoided. To reduce the total energy, the many-body system has a tendency to keep the completeness of the many-body Hilbert space, namely, the configurations such as (210), (201), (120), (021), (012), and (102) tend to be chosen with similar possibilities. As a result, the charge $\langle n_1\rangle$ on dot 1 decreases from $\langle n_1\rangle \approx 2.0$ to $\langle n_1\rangle \approx 1.0$, while $\langle n_2\rangle$ and $\langle n_3\rangle$ increase from $\langle n_2\rangle \approx \langle n_3\rangle \approx 0$ to $\langle n_2\rangle \approx \langle n_3\rangle \approx 1$. At the approximate particle–hole symmetric point $\epsilon_1 = -2V = -0.2$ (point D), the configurations such as (210), (201), (120), (021), (012), and (102) have almost the same possibilities to be occupied, while the charge on each dot has $\langle n_1\rangle \approx \langle n_2\rangle \approx \langle n_3\rangle \approx 1.0$. In this process, the charge correlation $\langle n_in_j\rangle$ increases to a value $\langle n_in_j\rangle \approx 2/3$.

For $\epsilon_1 < -0.2$ (point D), to add an electron to the configuration (210) will increase the energy by $\epsilon_1 + 2V < 0$. With further decreasing ϵ_1, the many-body state has the configuration (220), (022), or (202). As the total charge of the three dots approaches four electrons, there is some possibility for the configuration like (221), which will increase the Coulomb energy by $4V$. To reduce the total energy, the many-body states will have more possibilities to be in the configuration (220) until dots 1 and 2 are completely filled, while the charge $\langle n_3\rangle$ on dot 3 decreases to about zero at point E. In this process, the charge correlation $\langle n_1n_2\rangle$ increases to $\langle n_1n_2\rangle \approx 4.0$, while $\langle n_1n_3\rangle$ and $\langle n_2n_3\rangle$ nearly vanish. For $\epsilon_1 < -0.35$ (point F), the total charge of three dots is more than four electrons, and the many-body state such as (221) is dominated. Keeping the symmetry of the many-body states can reduce the total energy of the system, therefore configurations (122) and (212) tend to have similar possibilities with configuration (221). As a result, the charges on dots 1 and 2 decrease from about 2.0 to about 1.5 while the charge on dot 3 increases to about 1.5 at point G. In this region, the charge correlation $\langle n_1n_2\rangle$ decreases, while $\langle n_1n_3\rangle$ and $\langle n_2n_3\rangle$ increases.

It is noticeable that in a two-level QD system, the nonmonotonic charge oscillation is not visible in the limit $\Gamma/\Delta \gg 1$. Moreover, the SCHA results do not agree with the NRG ones.[15] However, in the present TQD model (1), a large oscillation is observed for $\Gamma/\Delta \gg 1$, as shown in Fig. 1(b). In fact, the SCHA based on the mean-field approximation presents a single-particle picture and cannot deal with the strongly correlated system for $V \gg \Gamma$. In

the limit $\Gamma/\Delta \gg 1$, all dots have nearly degenerate levels, and the symmetry of the many-body states is very important to minimize the total energy. In this case, the single-particle picture is invalid.

Figure 2 shows the charge oscillations for $\Delta = 0.01$ and 0.05. It is found that the oscillation for $\Gamma/\Delta \leq 1$ in Fig. 2 is much smaller than that for $\Gamma/\Delta \gg 1$ in Fig. 1. Because the level difference Δ is large, the many-body states are not symmetric, and the configuration (200) has a larger possibility than (020) and (002). With the decreasing ϵ_1, the charge density $\langle n_1 \rangle$ on dot 1 is larger than $\langle n_2 \rangle$ and $\langle n_3 \rangle$. As the total charge approaches two electrons at about $\epsilon_1 = -0.02$ (point A), the configuration (210) may appear, and $\langle n_2 \rangle$ and $\langle n_3 \rangle$ reach their maxima, which are smaller than those for $\Gamma/\Delta \gg 1$. Therefore, the charge oscillations on dots 2 and 3 are much smaller than those for $\Gamma/\Delta \gg 1$. For a similar reason, as the total charge approaches three electrons at about $\epsilon_1 = -0.2$ (point D), the configuration (210) has a larger possibility than those such as (021) and (012). In this process, the charge density $\langle n_1 \rangle$ decreases from $\langle n_1 \rangle \approx 2.0$ (point C) to a minimum (point D), which is much larger than that for $\Gamma/\Delta \gg 1$.

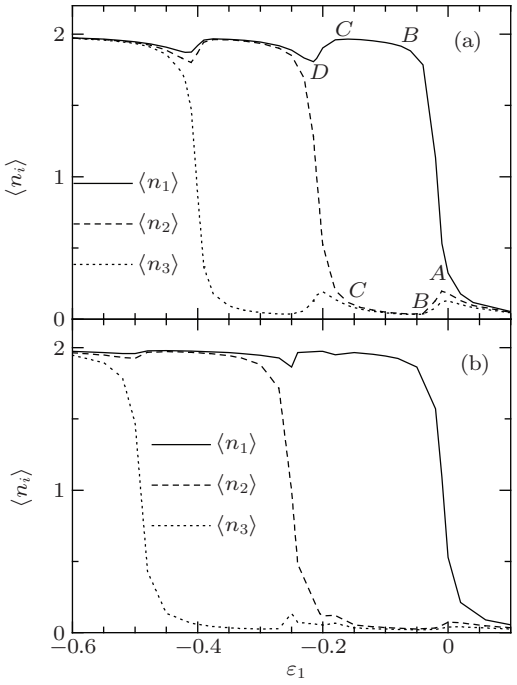


Fig. 2. Charge density $\langle n_i \rangle$ of dot i versus ϵ_1 for (a) $\Delta = 0.01$ and (b) $\Delta = 0.05$. The other parameters are $\Gamma = 0.01$ and $V = 0.1$.

In order to explore the effect of Coulomb repulsion on the charge oscillation, we present the charge density and the charge correlation for $\Delta = 0.0005$ and $V = 0.03$ in Fig. 3. It can be seen that the oscillation is smaller than that for $V = 0.1$ in Fig .1. This result can be explained by the following physical picture. With the decreasing ϵ_1, the total charge of the three dots approaches two electrons at about $\epsilon_1 = -0.01$ (point A). With the further decreasing ϵ_1, in order to avoid the Coulomb energy and reduce the total energy, the system will have more possibilities in configuration (200) instead of (210) or (201). In contrast to the case of $V = 0.1$ in Fig 1, the Coulomb repulsion $V = 0.03$ and the level width $\Gamma = 0.01$ are in the same order so that a new configuration (210) or (201) is allowed. Meanwhile, the possibilities of having configuration (200) is less that that for $V = 0.1$. Therefore, the maximum of $\langle n_1 \rangle$ at point B is less than that for $V = 0.1$, while the minima of $\langle n_2 \rangle$ and $\langle n_3 \rangle$ at point B are larger than those for $V = 0.1$. For $\epsilon_1 < -0.03$, the number of electrons on the three dots is greater than two, and the configuration (210) cannot be avoided. Because of a small Coulomb repulsion $V = 0.03$, by keeping the symmetry of the many-body states, the system gains less energy than that for the case of $V = 0.1$. Therefore, configuration (210) is more popular than configuration (021) or (012), and the minimum of $\langle n_1 \rangle$ at point D in Fig. 3 is larger than that for $V = 0.1$, while the maximum of $\langle n_3 \rangle$ at point D in Fig. 3 is smaller than that for $V = 0.1$.

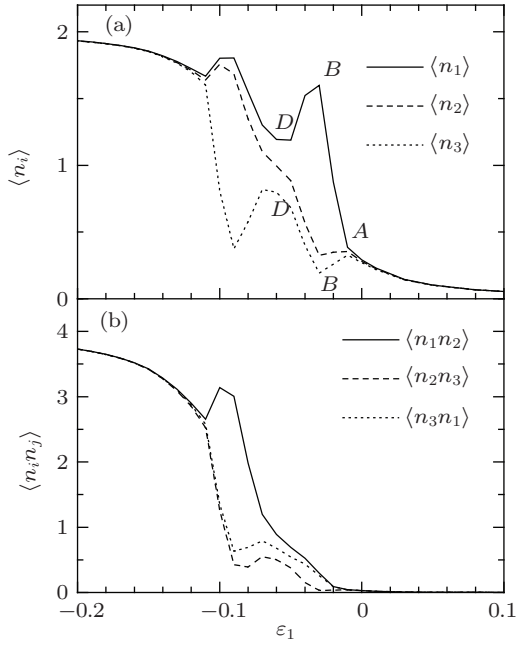


Fig. 3. (a) Charge density $\langle n_i \rangle$ of dot i and (b) charge correlation $\langle n_i n_j \rangle$ between dots i and j versus ϵ_1 when $V = 0.03$, $\Delta = 0.0005$, and $\Gamma = 0.01$.

The tunnel coupling linewidth Γ also plays an important role in the charge oscillation. We show the

results for $\Gamma = 0.001$ and 0.03 in Fig. 4. It is found that for $\Gamma = 0.001$, the oscillation is not visible, and a simple CB effect is observed. While for $\Gamma = 0.03$, the oscillation is stronger than that for $\Gamma = 0.01$ in Fig. 3. This behaviour can be explained as follows. For $\Gamma/\Delta \ll 1$, the three levels of the TQD are not degenerate, the symmetry of the many-body states is completely broken. The electrons start to fill dot 1 because of its lowest level. As $\epsilon_1 + 2V < 0$, dot 2 is filled, and for $\epsilon_1 + 4V < 0$, dot 3 is filled. For larger Γ/Δ, the level width covers the level difference, and the degeneracy of the three levels and the symmetry of the many-body states are enhanced. This feature is favourable of all the oscillations, according to the above discussion.

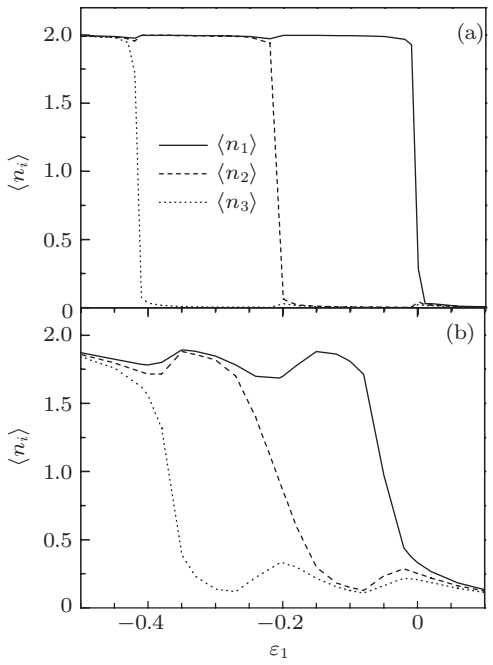


Fig. 4. Charge density $\langle n_i\rangle$ of dot i versus ϵ_1 for (a) $\Gamma = 0.001$ and (b) $\Gamma = 0.03$. The other parameters are $\Delta = 0.01$ and $V = 0.1$.

4. Conclusion

In conclusion, we study the charge oscillation in a TQD system symmetrically coupled to the leads. We attribute the oscillation behaviour to the many-body effect in the strongly correlated system instead of the physical scenarios based on the mean-field approach in the previous work for the two-level dot or the DQD system.[15] The level difference induces the difference of the occupations between different dots, while the symmetry of the many-body states favours the homogeneous distribution of the charge density on the three dots. The interplay of these two factors results in the charge oscillation. In the region where adding an electron to the three dots needs overcoming a large Coulomb energy (segments AB and DE in Fig. 1), the electrons tend to fill the lower levels to avoid the configuration with high Coulomb energy. In the region where the reduction of the level ϵ_1 can almost compensate the Coulomb energy (segments CD and FG in Fig. 1), the total energy can be reduced by keeping the symmetry of the many-body states.

In order to obtain a large oscillation, we should choose the appropriate parameters. A large Γ/Δ increases the degeneracy of the three levels and the symmetry of the many-body states. However, $\Gamma/\Delta \to \infty$ will reduce the difference of the occupations and weaken the oscillation. On the other hand, in the case of the large level difference Δ, although a large difference of the charge density can be kept, the oscillation may be very small due to the lack of symmetry of the many-body states (see Fig. 2). The NRG result also shows that to observe a large oscillation, the system should be in the strongly correlated region $V/\Gamma \gg 1$, since this oscillation results from the many-body effect.

References

[1] Park J, Pasupathy A N, Goldsmith J I, Chang C, Yaish Y, Petta J R, Rinkoski M, Sethna J P, Abruña H D, McEuen P L and Ralph D C 2002 *Nature* **417** 723

[2] Field M, Smith C G, Pepper M, Ritchie D A, Frost J E F, Jones G A C and Hasko D G 1993 *Phys. Rev. Lett.* **70** 1311

[3] Leobandung E, Guo L and Chou S Y 1995 *Appl. Phys. Lett.* **67** 7

[4] Jiang Z T 2010 *Chin. Phys.* B **19** 077307

[5] Wen R, Zhang D P and Tian G S 2012 *Chin. Phys.* B **21** 037401

[6] Girvin S M, Glazman L I, Jonson M, Penn D R and Stiles M D 1990 *Phys. Rev. Lett.* **64** 3183

[7] Stafford C A and Sarma S D 1994 *Phys. Rev. Lett.* **72** 3590

[8] Simmel F, Heinzel T and Wharam D A 1997 *Europhys. Lett.* **38** 123

[9] Aleiner I L, Brouwer P W and Glazman L I 2002 *Physics Reports* **358** 309

[10] Hackenbroich G, Heiss W D and Weidenmuller H A 1997 *Phys. Rev. Lett.* **79** 127

[11] Silvestrov P G and Imry Y 2000 *Phys. Rev. Lett.* **85** 2565

[12] Silvestrov P G and Imry Y 2001 *Phys. Rev.* B **65** 035309

[13] König J and Gefen Y 2005 *Phys. Rev.* B **71** 201308 (R)

[14] Berkovits R, Oppen F V and Gefen Y 2005 *Phys. Rev. Lett.* **94** 076802

[15] Sindel M, Silva A, Oreg Y and Delft J V 2005 *Phys. Rev.* B **72** 125316

[16] Field M, Smith C G, Pepper M, Ritchie D A, Frost J E F, Jones G A C and Hasko D G 1993 *Phys. Rev. Lett.* **70** 1311
[17] Goldstein M, Berkovits R and Gefen Y 2010 *Phys. Rev. Lett.* **104** 226805
[18] Schuster R, Buks E, Heiblum M, Mahalu D, Umansky V and Shtrikman H 1997 *Nature* **385** 417
[19] Avinun-Kalish M, Heiblum M, Zarchin O, Mahalu D and Umansky V 2005 *Nature* **436** 529
[20] Silva A, Oreg Y and Gefen Y 2002 *Phys. Rev.* B **66** 195316
[21] Meden V and Marquardt F 2006 *Phys. Rev. Lett.* **96** 146801
[22] Golosov D I and Gefen Y 2006 *Phys. Rev.* B **74** 205316
[23] Karrasch C, Hecht T, Weichselbaum A, Oreg Y, Delft T V and Meden V 2007 *Phys. Rev. Lett.* **98** 186802
[24] Goldstein M and Berkovits R 2007 *New J. Phys.* **9** 118
[25] Goldstein M, Berkovits R, Gefen Y and Weidenmuller H A 2009 *Phys. Rev.* B **79** 125307
[26] Lee H W and Kim S 2007 *Phys. Rev. Lett.* **98** 186805
[27] Kashcheyevs V, Schiller A, Aharony A and Entin-Wohlman O 2007 *Phys. Rev.* B **75** 115313
[28] Kashcheyevs V, Karrasch C, Hecht T, Weichselbaum A, Meden V and Schiller A 2009 *Phys. Rev. Lett.* **102** 136805
[29] Mitchell A K, Jarrold T F and Logan D E 2009 *Phys. Rev.* B **79** 085124
[30] Mitchell A K and Logan D E 2010 *Phys. Rev.* B **81** 075126
[31] Kuzmenko T, Kikoin K and Avishai Y 2006 *Phys. Rev. Lett.* **96** 046601
[32] Kuzmenko T, Kikoin K and Avishai Y 2006 *Phys. Rev.* B **73** 235310
[33] Zitko R and Bonca J 2007 *Phys. Rev. Lett.* **98** 047203
[34] Ingersent K, Ludwig A W W and Affleck I 2005 *Phys. Rev. Lett.* **95** 257204
[35] Zitko R, Bonca J, Ramsak A and Rejec T 2006 *Phys. Rev.* B **73** 153307
[36] Wang W Z 2007 *Phys. Rev.* B **76** 115114
[37] Delgado F, Shim Y P, Korkusinski M, Gaudreau L, Studenikin S A, Sachrajda A S and Hawrylak P 2008 *Phys. Rev. Lett.* **101** 226810
[38] Delgado F, Shim Y P, Korkusinski M, and Hawrylak P 2007 *Phys. Rev.* B **76** 115332
[39] Korkusinski M, Gimenez I P, Hawrylak P, Gaudreau L, Studenikin S A and Sachrajda A S 2007 *Phys. Rev.* B **75** 115301
[40] Saraga D S and Loss D 2003 *Phys. Rev. Lett.* **90** 166803
[41] Krishna-murthy H R, Wilkins J W, and Wilson K G 1980 *Phys. Rev.* B **21** 1003
[42] Krishna-murthy H R, Wilkins J W, and Wilson K G 1980 *Phys. Rev.* B **21** 1044