Spin-polarized current in double quantum dots*

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We analyze the transport through asymmetric double quantum dots with an inhomogeneous Zeeman splitting in the presence of crossed dc and ac magnetic fields. A strong spin-polarized current can be obtained by changing the dc magnetic field. It is mainly due to the resonant tunnelling. But for the ferromagnetic right electrode, the electron spin resonance also plays an important role in transport. We show that the double quantum dots with three-level mixing under crossed dc and ac magnetic fields can act not only as a bipolar spin filter but also as a spin inverter under suitable conditions.

Keywords: resonant tunnelling, spin-polarized current, spin filter, double quantum dots

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1. Introduction

Quantum transport in nanostructures has attracted much attention in the recent decades, because it deepens our understanding of fundamental quantum physics and has various applications. Many interesting phenomena have been found, such as the Coulomb blockade,[1] the Fano effect,[2,3] and the Kondo effect.[4] People have found an interesting relation between the optical and the transport properties. While on the other side, interesting phenomena in quantum optics may also appear in quantum transport, like dark states[5,6] in coupled quantum dots. And many interesting optical effects have been found in coupled quantum dots.[7,8] The spin degree of freedom of electrons has great potential as a carrier of classical information (spintronics)[9] and quantum information[10,11] (spin quantum bits). Spintronics is a new research area, in which the spin plays an important role in a lot of new physical phenomena. Controlling the spins of electrons in a mesoscopic system is an important task in spintronic devices, as well as in the fundamental understanding of spin relaxation and coherence. A key aim in spintronics is the realization of spin-based quantum information devices that can filter electrons according to their spin orientations (i.e., spin filters). The mixing of energy levels is a vital quantum property in the solid state. The charge transport through double quantum dots (DQD) with three-level mixing induced dark states has been studied experimentally[12] and theoretically.[13]

Recent studies indicate that the spin degree of freedom also plays an essential role in transport. Experiments recently carried out to study the transport in the DQD show that the Pauli exclusion principle is important in current rectification.[14] The spin blockade (SB) has been studied in the DQD[15−18] and the triple dots.[18,19] Theoretical and experimental efforts have also been devoted to the control of electron spin resonance (ESR) in single,[20] double,[21,22] and triple[23] quantum dots. Theoretical studies on spin filters have been done in single,[24,25] double,[18] and linear triple[18] quantum dots.

To explore the transport process with the spin polarized current, in this paper, we study the transport in an inhomogeneous asymmetry DQD with an inhomogeneous Zeeman splitting (due to different g factor for each dot) in the presence of crossed dc and ac magnetic fields. The tunnelling from one dot to the other is most likely when two equal spin levels are aligned. However, when the ac magnetic field is applied, it rotates the spin and allows the spin-flip pro-

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cesses along the tunnelling, which can lead to new features in the current. Diving electrons to perform coherent spin rotations by manipulating the ac magnetic field can be detected by the direct measurement of the tunnelling current.[17] Level mixing may lead to new states whose energies depend on the dc magnetic field as in Refs. [12] and [13], where one electron configuration was considered. While in our paper, we consider up to the two-electron process, in which the electron (spin) correlation is important. The ac magnetic field can be used for coherent electron spin manipulation in semiconductor quantum dots. In particular, we make use of the ESR, which is a fundamental component for the realization of spin-based quantum information devices in spintronics. We discuss the interplay of the ESR and resonant tunnelling in the transport. We also propose possible applications of the physics effects found in our systems.

2. Model and theoretical approach

Our system consists of two weakly coupled vertical quantum dots connected to two electric leads as schematically shown in Fig. 1. The first/left dot has an s-type orbit, and the second/right dot has three orbits as studied in experiment and theory.[12,13] The system in the presence of a static magnetic field along the Z (vertical) axis can be descried by the Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_{\rm LR} + \hat{H}_B(t) + \hat{H}_{\rm T} + \hat{H}_{\rm leads}, \qquad (1)$$

where $\hat{H}_0 = \hat{H}_{\rm L} + \hat{H}_{\rm R}$ describes the uncoupled DQD. Here $\hat{H}_{\rm L} = \sum_\sigma \epsilon_{s\sigma}|s\sigma\rangle\langle s\sigma|$ and $\hat{H}_{\rm R} = (E_B + \Delta_{{\rm R},\sigma})|1\sigma\rangle\langle 1\sigma| + \Delta_{{\rm R},\sigma}|2\sigma\rangle\langle 2\sigma| - (E_B - \Delta_{{\rm R},\sigma})|3\sigma\rangle\langle 3\sigma| + L_{\rm c}(|1\sigma\rangle\langle 2\sigma| + |2\sigma\rangle\langle 1\sigma| + |2\sigma\rangle\langle 3\sigma| + |3\sigma\rangle\langle 2\sigma|)$ describe the left and the right QD, respectively, where $\epsilon_{s\sigma} = \epsilon_s + \Delta_{{\rm L},\sigma}$, $\Delta_{i,\sigma} = g_i\mu_{\rm B}B_{\rm dc}\sigma$ $(i = {\rm L}, {\rm R})$ is the Zeeman energy with $\sigma = \uparrow, \downarrow$ referring to the spin degree of freedom. Here we consider the DQD with an inhomogeneous Zeeman splitting, which could be due to a different g factor for each dot (in our calculation, we use $g_{\rm L} = -0.33$ and $g_{\rm R} = -0.89$, as in Ref. [26]). Then we are able to manipulate the electron spin differently in each dot. The energy $E_B = c_B(B_{\rm dc} - B_0)$ with c_B a constant describes the magnetic field dependence of the Fock–Darwin (FD) orbital levels, and $L_{\rm c}$ is the coupling strength between the levels due to the anisotropy of the right QD. The anisotropy of the right QD leads to the level mixing and interesting physics consequences.[12,13] The eigenstates of $\hat{H}_{\rm R}$ are $|\Psi_{0\sigma}\rangle$ and $|\Psi_{\pm\sigma}\rangle$, corresponding to eigenenergies $\epsilon_{0\sigma} = \Delta_{{\rm R},\sigma}$ and $\epsilon_{\pm\sigma} = \pm\sqrt{E_B^2 + 2L_{\rm c}^2} + \Delta_{{\rm R},\sigma}$. The energy $\epsilon_{+\sigma}$ plays a dominant role in charge transport. In this new basis, the interdot hopping can be written as $\hat{H}_{\rm LR} = \sum_{\alpha,\sigma}(\gamma_{\alpha\sigma}|s\sigma\rangle\langle\Psi_{\alpha\sigma}| + {\rm H.c.})$, $\alpha = 0, \pm$. The hopping rate of an electron between states $|s\sigma\rangle$ and $|\Psi_{\alpha\sigma}\rangle$ can be calculated as[13]

$$\gamma_{\alpha\sigma} = \gamma|\langle s\sigma|\Psi_{\alpha\sigma}\rangle|^2 L(|\epsilon_{s\sigma} - \epsilon_{\alpha\sigma}|; \Gamma_\sigma), \qquad (2)$$

where $\langle s\sigma|\Psi_{\alpha\sigma}\rangle$ $(\sigma = \{\uparrow, \downarrow\}; \alpha = 0, \pm)$ is the overlap between state $|s\sigma\rangle$ of the left QD and state $|\Psi_{\alpha\sigma}\rangle$ of the right QD, and $L(|\epsilon_{s\sigma} - \epsilon_{\alpha\sigma}|; \Gamma_\sigma)$ describes the Lorentzian broadening of the levels with $L(x, w) = (1 + (2x/w)^2)^{-1}$ and $\Gamma_\sigma \equiv \Gamma_{{\rm L},\sigma} + \Gamma_{{\rm R},\sigma}$ ($\Gamma_{{\rm L/R},\sigma} = \Gamma$ is the tunnelling rate of an electron with spin σ between the dot and the left/right lead). Here we make the simple assumption that all the overlaps of the $s\sigma$ levels with the left QD FD states $|i\sigma\rangle$ $(i = 1, 2, 3)$ are the same. We then set these overlaps to unity, since they can be subsumed into γ defined below. The squares of the relevant matrix elements are

$$|\langle s\sigma|\Psi_{0\sigma}\rangle|^2 = \frac{E_B^2}{E_B^2 + 2L_{\rm c}^2}, \qquad (3)$$

$$|\langle s\sigma|\Psi_{\pm\sigma}\rangle|^2 = 1 + \frac{L_{\rm c}^2}{E_B^2 + 2L_{\rm c}^2} \pm \frac{2L_{\rm c}}{\sqrt{E_B^2 + 2L_{\rm c}^2}}. \qquad (4)$$

The $\hat{H}_{\rm T} = \sum_{l\in\{{\rm L,R}\alpha\}k\sigma}(\gamma_l|lk\sigma\rangle\langle l\sigma| + {\rm H.c.})$ gives the tunnelling between the DQD and the leads described by $\hat{H}_{\rm leads} = \sum_{lk\sigma}\epsilon_{lk}|lk\sigma\rangle\langle lk\sigma|$. We now add an additional ac magnetic field in the XY plane, which will lead to interesting phenomena, for instance the ESR and the spin polarized current. This part Hamiltonian is

$$\hat{H}_B(t) = \sum_i g_i\mu_{\rm B}B_{\rm ac}(\hat{s}_x^i\cos\omega t + \hat{s}_y^i\sin\omega t), \qquad (5)$$

where $\hat{s}_i = (1/2)\sum_{\sigma\sigma'}|i\sigma\rangle\sigma_{\sigma\sigma'}\langle i\sigma'|$ is the spin operator in each dot. The ac magnetic field leads to the spin-flip transition. The transition rate can be calculated using the Fermi golden rule[15]

$$\left[\frac{1}{\tau_{\rm sf}}\right]_{\rm L(R)} = \frac{\pi}{2}(g_{\rm L(R)}\mu_{\rm B}B_{\rm ac})^2\frac{\Gamma}{(\Delta_{\rm L(R)} - \hbar\omega)^2 + \Gamma^2}. \qquad (6)$$

We can see that $\Delta_{\rm L(R)} = \hbar\omega$ leads to the maximum spin-flip rate, i.e., the ESR.

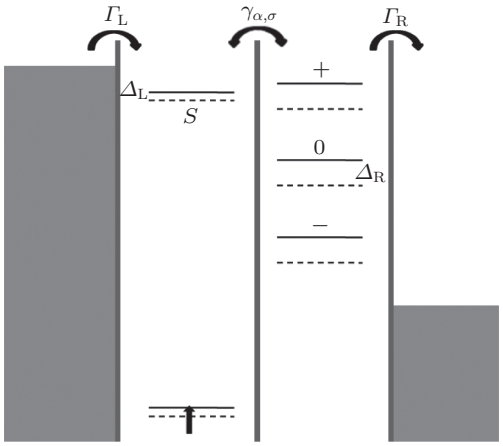


Fig. 1. Schematic diagram of a system exposed to crossed dc ($B_{\rm dc}$) and ac ($B_{\rm ac}$) magnetic fields. One electron is confined in the left dot, so only one electron with the opposite spin can enter the DQD. The dots are coupled coherently with tunnelling amplitude $\gamma_{\alpha\sigma}$ and incoherently to the leads with rates $\Gamma_{\rm L}$ and $\Gamma_{\rm R}$.

We consider the case with one electron always confined in the left dot and an additional electron participating in the transport process,[18] which may be realized in the coupled QDs by applying a suitable gate voltage. The relevant basis states are

- 1-electron states: $|1\rangle = |\uparrow, 0\rangle$, $|2\rangle = |\downarrow, 0\rangle$;
- 2-electron states: $|3\rangle = |\uparrow\downarrow, 0\rangle$, $|4\rangle = |\uparrow, \downarrow_+\rangle$, $|5\rangle = |\uparrow, \downarrow_0\rangle$, $|6\rangle = |\uparrow, \downarrow_-\rangle$, $|7\rangle = |\uparrow, \uparrow_+\rangle$, $|8\rangle = |\uparrow, \uparrow_0\rangle$, $|9\rangle = |\uparrow, \uparrow_-\rangle$, $|10\rangle = |\downarrow, \uparrow_+\rangle$, $|11\rangle = |\downarrow, \uparrow_0\rangle$, $|12\rangle = |\downarrow, \uparrow_-\rangle$, $|13\rangle = |\downarrow, \downarrow_+\rangle$, $|14\rangle = |\downarrow, \downarrow_0\rangle$, $|15\rangle = |\downarrow, \downarrow_-\rangle$.

The dynamics of our system may be described by the equations of motion for state occupation probabilities ρ_s including the scattering in and the scattering out contributions

$$\dot{\rho}(t)_s = \sum_{m\neq s} W_{sm}\rho_m - \sum_{k\neq s} W_{ks}\rho_s. \tag{7}$$

The neglection of the off-diagonal elements of the density matrix is appropriate for weakly coupled QDs.[13,15] Here W_{ij} is the transition rate from state j to state i. The expressions for transition rates W_{ij} depend on the different processes, for example,

$$|\uparrow, 0\rangle \longrightarrow |\uparrow\downarrow, 0\rangle \Longrightarrow W_{3,1} = \Gamma_{{\rm L},\downarrow}, \tag{8}$$

$$|\downarrow, \downarrow_+\rangle \longrightarrow |\uparrow, \downarrow_+\rangle \Longrightarrow W_{4,13} = \left[\frac{1}{\tau_{\rm sf}}\right]_{\rm L}, \tag{9}$$

$$|\uparrow\downarrow, 0\rangle \longrightarrow |\uparrow, \downarrow_+\rangle \Longrightarrow W_{4,3} = \gamma_{+\downarrow}. \tag{10}$$

The other transition rates can be calculated in the same way. With the inclusion of the spin-flip interactions, the rate equations for the occupation probabilities of $|1\rangle = |\uparrow, 0\rangle$ and $|3\rangle = |\uparrow\downarrow, 0\rangle$ are

$$\begin{aligned}\dot{\rho}_1 &= W_{1,2}\rho_2 + W_{1,4}\rho_4 + W_{1,5}\rho_5 + W_{1,6}\rho_6 + W_{1,7}\rho_7 \\ &\quad + W_{1,8}\rho_8 + W_{1,9}\rho_9 - W_{2,1}\rho_1 - W_{3,1}\rho_1, \end{aligned}\tag{11}$$

$$\begin{aligned}\dot{\rho}_3 &= W_{3,1}\rho_1 + W_{3,2}\rho_2 + W_{3,4}\rho_4 + W_{3,5}\rho_5 + W_{3,6}\rho_6 \\ &\quad + W_{3,10}\rho_{10} + W_{3,11}\rho_{11} + W_{3,12}\rho_{12} \\ &\quad - W_{4,3}\rho_3 - W_{5,3}\rho_3 - W_{6,3}\rho_3 \\ &\quad - W_{10,3}\rho_3 - W_{11,3}\rho_3 - W_{12,3}\rho_3. \end{aligned}\tag{12}$$

We are mainly interested in the steady current. The spin-resolved steady current going from the right dot to the right lead can be calculated as

$$I_\uparrow = e\Gamma_{{\rm R}\uparrow}(\rho_7 + \rho_8 + \rho_9 + \rho_{10} + \rho_{11} + \rho_{12}), \tag{13}$$

$$I_\downarrow = e\Gamma_{{\rm R}\downarrow}(\rho_4 + \rho_5 + \rho_6 + \rho_{13} + \rho_{14} + \rho_{15}). \tag{14}$$

The spin polarization of the current is defined as

$$P = \frac{I_\uparrow - I_\downarrow}{I_\uparrow + I_\downarrow}, \tag{15}$$

where $I_\uparrow$ ($I_\downarrow$) is the current of electrons with spin up (down).

3. Results and discussion

We now consider the case of a large ϵ_s (the case of a small ϵ_s has been studied in Ref. [27]), which can be achieved by adjusting the gate voltage on the left QD. In this case, the large ϵ_s leads to s-orbit resonating close with the "+"-channel. The key parameter determining the charge transport in our system is the dc magnetic field-dependent and spin-dependent energy level $\epsilon_{+\sigma}$. We can see that resonant tunnelling occurs when the energy levels in the two dots are aligned, i.e., $\epsilon_{s\sigma} = \epsilon_{+\sigma}$. The resonant tunnelling results in interesting spin-dependent hopping rates, as shown in Fig. 2. Here the peaks are due to the resonance tunnelling as a consequence of the energy levels' alignment. The spin dependent tunnelling rates $\gamma_{+\sigma}$ ($\sigma = \uparrow, \downarrow$)

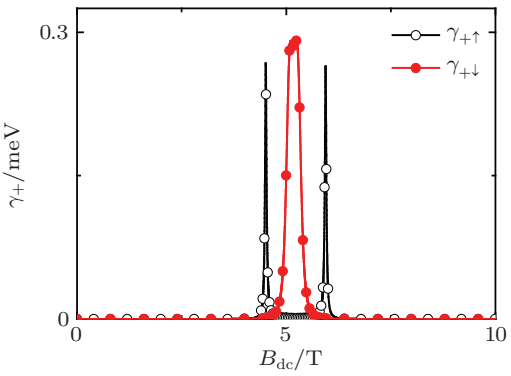


Fig. 2. (colour online) The spin-dependent hopping rates $\gamma_{+\sigma}$ versus magnetic field $B_{\rm dc}$. Parameters used are $L_{\rm c} = 1$ meV, $\Gamma = 10$ μeV, $\gamma = 100$ μeV, $E_s = 1.5$ meV, and $B_0 = 5.2$ T.

will play determine roles in current and spin polarization, as shown in the following discussion. From formula (2), it is clear that $\gamma_{+\sigma}$ will reach the maximum when $\epsilon_{s\sigma} = \epsilon_{+\sigma}$.

3.1. Current

The calculation under small spin-flip rates leads to an approximate formula for the current

$$\langle I\rangle \approx \frac{2\Gamma\gamma_{+\uparrow}\gamma_{+\downarrow} + \Gamma^2(\gamma_{+\uparrow}+\gamma_{+\downarrow})}{5\gamma_{+\uparrow}\gamma_{+\downarrow} + 3\Gamma(\gamma_{+\uparrow}+\gamma_{+\downarrow}) + \Gamma^2}. \quad (16)$$

Figure 3 shows the current through the system as a function of the static magnetic field intensity. We find very good agreement between the numerical and the analytical results. We should notice that the current peaks here have nothing to do with the ESR. Instead, it is because of the resonant tunnelling when the energy levels in the two dots are aligned, i.e., $\epsilon_{s\sigma} = \epsilon_{+\sigma}$. The energies of $|\Psi_{+\sigma}\rangle$ states depend complicatedly on the dc magnetic field, which results in that we get level alignments at different $B_{\rm dc}$. The hopping rate $\gamma_{+\downarrow}$ is proportional to the matrix element $|\langle s\downarrow|\Psi_{+\downarrow}\rangle|^2$, which (from Eq. (4)) reaches the maximum at $E_B = 0$, i.e., $B_{\rm dc} = B_0$. We should also notice that $\epsilon_{s\downarrow} \approx \epsilon_{+\downarrow}$ at B_0. So the hopping rate $\gamma_{+\downarrow}$ reaches the maximum at B_0, as shown in Fig. 2. We can ignore the contribution from $\gamma_{+\uparrow}$ due to $\gamma_{+\uparrow} \ll \gamma_{+\downarrow}$ around B_0. As seen from Eq. (16), the current through the system can be given approximately by $\langle I\rangle \approx \Gamma/(3+\Gamma/\gamma_{+\downarrow})$. Therefore, the current reaches an extreme at B_0, as shown in Fig. 3. The other two peaks adjacent to B_0 are because of the resonant tunnelling too. The hopping rate $\gamma_{+\uparrow}$ reaches the maxima respectively both due to $\epsilon_{s\uparrow} = \epsilon_{+\uparrow}$. In this case, the current through the system can be given approximately by $\langle I\rangle \approx \Gamma/(3+\Gamma/\gamma_{+\uparrow})$, so the current reaches the maximum too.

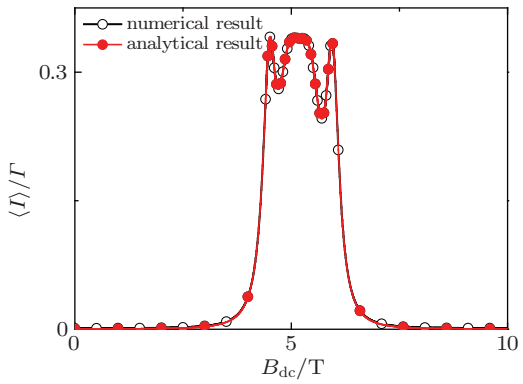


Fig. 3. (colour online) Current I versus magnetic field $B_{\rm dc}$. The line with circles is the exact numerical result, and the line with solid dots is the analytical result of Eq. (16). Here $\hbar\omega = 100$ μeV, $B_{\rm ac} = 0.003$ T, the other parameters are the same as those in Fig. 2.

3.2. Spin polarization

It is interesting to examine the spin polarization of the current, which is shown in Fig. 4. Here the analytical result is based on the approximate solution

$$P \approx \frac{\gamma_{+\uparrow} - \gamma_{+\downarrow}}{\gamma_{+\uparrow} + \gamma_{+\downarrow}}. \quad (17)$$

The analytical result agrees well with the exact numerical one. The nice spin polarized (spin-down) current in Fig. 4 around $B_{\rm dc} \sim 5.2$ T is because of the spin-dependent effective tunnelling between the two dots, as shown in Fig. 2. In this case, the $s\downarrow$ level in the left QD is aligned with state $|\Psi_{+\downarrow}\rangle$ in the right QD, so a ↓-polarized current is produced due to $\gamma_{+\downarrow} \gg \gamma_{+\uparrow}$. At $B_{\rm dc} \sim 4.5$ T and 5.9 T, we can find overall spin-up currents. This is because the $s\uparrow$ level is close to resonance with the right QD state $|\Psi_{+\uparrow}\rangle$, i.e., $\gamma_{+\uparrow} \gg \gamma_{+\downarrow}$, so that a ↑-polarized current is produced. Thus, our system can act as a bipolar spin filter through changing the dc magnetic field. And we can easily notice that the nice spin polarized currents are achieved just when the currents reach the maxima.

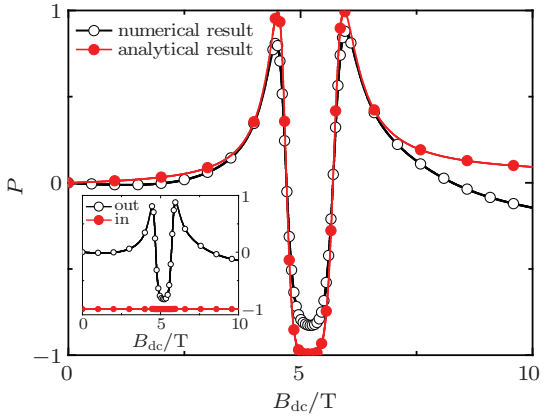


Fig. 4. (colour online) The spin polarization of current versus magnetic field $B_{\rm dc}$. The line with circles is the exact numerical result, and the line with solid dots is the analytical result of Eq. (17). The inset shows that the polarization of current for the right ferromagnetic electrode allows only incoming electrons with spin down, where the line with circles is the out current's polarization, and the line with solid dots is the incoming current's polarization. The parameters are the same as those in Fig. 3.

As shown in the insert of Fig. 4 (for the right ferromagnetic electrode), the out current has a good spin polarization too, under suitable conditions. In this

case, the ESR also plays an important role in transport. The SB can be suppressed due to the ESR. Under the conditions that $\hbar\omega \approx \Delta_{\rm L}$ (ESR occurs) and the ac frequency in the right dot is far off resonance (the spin flip is less effective) near B_0, the right QD can act as a spin filter.[16] At $B_{\rm dc} \sim 4.5$ T and 5.9 T, a spin-down polarized incoming current is converted into a spin-up polarized current, which is a consequence of the spin-flip transition and the spin-dependent inter-dot tunnelling (see Fig. 2). Our result shows that our system can also work as a spin inverter.

4. Conclusion

We have studied the quantum transport in an asymmetric DQD with an inhomogeneous Zeeman splitting in the presence of crossed ac and dc magnetic fields. The resonant tunnelling leads to rich physics. The dc magnetic field induces tunable and spin dependent inter-dot tunnelling rates. And the energy levels of $|\Psi_{+\sigma}\rangle$ states also depend on the dc magnetic field. A nice spin polarized current can be achieved only by changing the dc magnetic field. In particular, by changing the dc magnetic field, our system can act not only as a bipolar spin filter but also as a spin inverter.

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