Impurity-induced local modes in one-dimensional dusty plasma chains*

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(Received 22 March 2012; revised manuscript received 31 May 2012)

The effects of impurity on eigenmodes in one-dimensional dusty plasma lattices are studied. It is found that local modes can be excited besides lattice waves, due to the existence of an impurity particle. The dispersion relations of the modes are derived accordingly. Properties of the lattice and local modes are also analyzed and discussed, particularly for their symmetric features and conditions of the mode excitation.

Keywords: impurity, local mode, dusty plasma

PACS: 52.25.Vy, 52.27.Lw, 52.35.Lv **DOI:** 10.1088/1674-1056/21/11/115201

1. Introduction

Experimental and theoretical studies on dusty plasma lattices have been extensively carried out since their first realization in experiments.[1,2] Typical structures of dusty lattices are one-dimensional chains, two-dimensional hexagonal lattices, and three-dimensional cubic crystals. Dust lattice waves (DLWs) can be excited in the lattices, observed in many experiments,[3−10] and analyzed in theoretical treatments.[11−21] The simplest model for lattice modes is the one-dimensional chain.[11−18] A dispersion relation explicitly depending on the plasma parameters and the direction of the mode propagation in one-dimensional dust chain has been obtained.[16] When taking the size of the dust particles into consideration, each dust particle can be regarded as a point charge and a dipole moment.[17] Wang *et al.*[18] derived the general dispersion relations. For two-dimensional lattices, analytical treatments of longitudinal and transverse lattice waves have been presented recently, and the dispersion relations have been derived for two-dimensional hexagonal lattices and three-dimensional body-centered cubic (bcc) and face-centered cubic (fcc) lattices.[19] In the presence of a constant magnetic field, it has been found that the coupling of longitudinal and transverse modes leads to high-frequency and low-frequency branches in the dispersion relation.[20] For three-dimensional lattices, the general dispersion relation has been further discussed in bcc and fcc lattices.[21]

Early theoretical treatments only considered dusty lattices with perfect periodic structures. Eigenmodes in the periodic lattices are propagating waves. Experimentally, however, there are always defects and/or impurities with different mass and charge, the translation symmetry is broken locally, and new local vibration modes except the lattice waves can be excited. Thus, it is necessary to derive the local vibration mode dispersion relation in dusty plasma lattices induced by impurities.

In this paper, we study the simplest one-dimensional dust chain with an impurity particle. Since the translation symmetry is only locally broken, there also exist lattice wave solutions. Moreover, a new local vibration solution localized at the vicinity of the impurity and decaying in distance can be found, and formation conditions of the local modes are analyzed. We further analyze the properties of the lattice modes and local modes.

2. Lattice mode and local mode solutions

As shown in Fig. 1, we arrange the dust particles in the x direction and put the impurity in the origin. The particles have a uniform equilibrium space

*Project supported by the National Natural Science Foundation of China (Grant Nos. 40731056 and 10975012) and the President Fund for Undergraduate Research of Peking University, China.

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of a. The charge and mass of a dust particle are set as m and q, while the charge and mass of the impurity are M and Q. Without loss of generality, we only consider the interplay between neighbouring particles. Furthermore, the interparticle potential is of the Debye–Yukawa type, i.e., $\phi(r) = q\,\mathrm{e}^{-\kappa r}/r$, where the shielding parameter $\kappa = 1/\lambda_{\mathrm{D}}$, with λ_{D} being the Debye length. The impurity and dust particles are then strongly coupled to each other, and the coupling parameter $\Gamma_0 = qQ/aT \sim \Gamma = q^2/aT$ (T is the plasma temperature) is much greater than unity. For linear modes, the harmonic approximation can be applied. We only consider the longitudinal mode in the x direction to obtain the following equations for the oscillations of the impurity (at the origin) and the dust particle in the chain:

$$M\frac{\mathrm{d}^2\tilde{x}_0}{\mathrm{d}t^2} = F\left(\tilde{x}_1 + \tilde{x}_{-1} - 2\tilde{x}_0\right), \quad (1)$$

$$m\frac{\mathrm{d}^2\tilde{x}_{\pm1}}{\mathrm{d}t^2} = F\left(\tilde{x}_0 - \tilde{x}_{\pm1}\right) + f\left(\tilde{x}_{\pm2} - \tilde{x}_{\pm1}\right), \quad (2)$$

$$m\frac{\mathrm{d}^2\tilde{x}_n}{\mathrm{d}t^2} = f\left(\tilde{x}_{n+1} + \tilde{x}_{n-1} - 2\tilde{x}_n\right), |n| \geq 2, \quad (3)$$

where $\tilde{x}_0$ and $\tilde{x}_n$ ($n = \pm1, \pm2, \ldots$) are the linear displacements of the impurity and dust particles from its equilibrium position $\bar{x}_0 = 0$ and $\bar{x}_n = na$, respectively, and

$$F = \frac{Qq}{a^3}\,\mathrm{e}^{-\kappa a}(2 + 2\kappa a + \kappa^2 a^2), \quad (4)$$

$$f = \frac{q^2}{a^3}\,\mathrm{e}^{-\kappa a}(2 + 2\kappa a + \kappa^2 a^2). \quad (5)$$

Here we have neglected the neutral gas damping effect to focus on the effect of impurity.

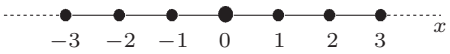


Fig. 1. Dust chain with an impurity in the origin.

The solution of Eqs. (1)–(3) clearly has a form of $\tilde{x}_n = (C\lambda^n + D\lambda^{-n})\,\mathrm{e}^{-\mathrm{i}\omega t}$, where C and D are arbitrary constants. Substituting it into Eq. (3), we obtain

$$\lambda^2 + \left(\frac{m}{f}\omega^2 - 2\right)\lambda + 1 = 0. \quad (6)$$

Note that for $\omega < 2\sqrt{f/m}$, λ is complex, which can then be written in a form of $\lambda \sim \mathrm{e}^{\mathrm{i}\alpha}$, where α is real. Without loss of generality, one can write $\alpha = ka$. It is clearly a propagating wave solution, i.e., the lattice mode. However, for $\omega > 2\sqrt{f/m}$, both roots of Eq. (6) are real and negative. For $|\lambda| > 1$, the solution is not stable, then the only possible solution is for $|\lambda| < 1$, i.e., $-1 < \lambda < 0$. Clearly, it is a mode decay with distance from the impurity particle, i.e., the local vibration mode. According to the symmetry of the chain, there should be two independent branches for both lattice waves and local vibrations, symmetric and antisymmetric. We now discuss them separately.

2.1. Symmetric lattice modes

For the lattice modes, the propagating solution has a form of $\lambda^{\pm n} = \mathrm{e}^{\pm \mathrm{i}k\bar{x}_n}$. In the symmetric case, $\tilde{x}_n = \tilde{x}_{-n}$. The two independent linear solutions are connected in the form of $\tilde{x}_n = A\cos(k\,|\bar{x}_n| + \delta_{\mathrm{S}})\,\mathrm{e}^{-\mathrm{i}\omega t}$, where δ_{S} is a phase shift for the symmetric mode caused by the impurity particle. The "boundary" condition at the origin can be set as $\tilde{x}_0 = B\,\mathrm{e}^{-\mathrm{i}\omega t}$ in the symmetric case. Substituting them into Eqs. (1)–(3), we have

$$-M\omega^2\frac{B}{A} = 2F\left[\cos\left(ka + \delta_{\mathrm{S}}\right) - \frac{B}{A}\right], \quad (7)$$

$$\begin{aligned} &-m\omega^2\cos\left(ka + \delta_{\mathrm{S}}\right) \\ &= F\left[\frac{B}{A} - \cos\left(ka + \delta_{\mathrm{S}}\right)\right] \\ &\quad + f\left[\cos\left(2ka + \delta_{\mathrm{S}}\right) - \cos\left(ka + \delta_{\mathrm{S}}\right)\right], \end{aligned} \quad (8)$$

$$-m\omega^2 = 2f\left(\cos ka - 1\right). \quad (9)$$

From Eq. (9), for the symmetric mode we further obtain

$$\begin{aligned} \omega_{\mathrm{S}} &= 2\sqrt{\frac{f}{m}}\left|\sin\frac{ka}{2}\right| \\ &= 2\sqrt{\frac{q^2}{ma^3}\,\mathrm{e}^{-\kappa a}\left(2 + 2\kappa a + \kappa^2 a^2\right)}\left|\sin\frac{ka}{2}\right| \\ &= \omega_{\max}\left|\sin\frac{ka}{2}\right|, \end{aligned} \quad (10)$$

where

$$\begin{aligned} \omega_{\max} &\equiv 2\sqrt{\frac{f}{m}} \\ &= 2\sqrt{\frac{q^2}{ma^3}\,\mathrm{e}^{-\kappa a}\left(2 + 2\kappa a + \kappa^2 a^2\right)}. \end{aligned} \quad (11)$$

We obtain the same dispersion relation as that in perfect lattices, with no dependence on the impurity.

Substituting Eq. (9) into Eq. (8) leads to

$$\frac{B}{A} = \frac{(F - f)\cos\left(ka + \delta_{\mathrm{S}}\right) + f\cos\delta_{\mathrm{S}}}{F}, \quad (12)$$

and substituting Eqs. (9) and (12) into Eq. (7) leads to

$$\frac{\cos(ka + \delta_{\mathrm{S}}) - \cos\delta_{\mathrm{S}}}{\cos ka - 1}$$

$$= \frac{M}{m}\left[\left(1-\frac{f}{F}\right)\cos(ka+\delta_{\rm S})+\frac{f}{F}\cos\delta_{\rm S}\right]. \quad (13)$$

We then obtain

$$\begin{aligned}&\frac{\tan\delta_{\rm S}}{\tan(ka/2)}\\ &=\frac{(1-f/F)\cos ka+f/F-m/M}{2\,(1-f/F)\sin^2(ka/2)+m/M}\\ &=\frac{(1-q/Q)\cos ka+q/Q-m/M}{2\,(1-q/Q)\sin^2(ka/2)+m/M}.\end{aligned} \quad (14a)$$

Here we have also used Eqs. (4) and (5) for $F$and f. Clearly, the phase shift depends on the mass and charge of the impurity.

In the laboratory, the dust particle chain should be at the balance position of a confinement electrical field and the gravity field. This means that the condition $q/Q = m/M \equiv \eta$ should be satisfied. Then the phase shift can be simplified as

$$\tan\delta_{\rm S} = \frac{(1-\eta)\,(1-\cos ka)}{ankа-(1-\eta)\sin ka}. \quad (14b)$$

2.2. Antisymmetric lattice modes

In the antisymmetric case, $\tilde{x}_n = -\tilde{x}_{-n}$. Thus, the "boundary" condition at the origin is $\tilde{x}_0 = 0$. The lattice mode solution has the form of $\tilde{x}_n = A\sin(kx_n \pm \delta_{\rm A})\,{\rm e}^{-{\rm i}\omega t}$, where $\delta_{\rm A}$ is a phase shift for the anti-symmetric mode caused by the impurity particle. Substituting it into Eq. (3) leads to the dispersion relation

$$\begin{aligned}\omega_{\rm A} &= 2\sqrt{\frac{f}{m}}\left|\sin\frac{ka}{2}\right|\\ &= 2\sqrt{\frac{q^2}{ma^3}\,{\rm e}^{-\kappa a}\,(2+2\kappa a+\kappa^2a^2)}\left|\sin\frac{ka}{2}\right|\\ &= \omega_{\max}\left|\sin\frac{ka}{2}\right|,\end{aligned} \quad (15)$$

which is the same as that in the symmetric case, and the relation for the phase shift

$$\begin{aligned}\tan\delta_{\rm A} &= -\frac{(1-f/F)\tan ka}{1+[f(1-\cos ka)]/F\cos ka}\\ &= -\frac{(1-\eta)\sin ka}{\eta+(1-\eta)\cos ka}.\end{aligned} \quad (16)$$

We need to point out that, in principle, there is no restraint to the parameter η for the lattice modes.

2.3. Symmetric local modes

For the local mode, from its property of the solution, $-1 < \lambda < 0$, the solution has the form of $\lambda = -{\rm e}^{-Ka}$, where $1/K$ is the decay length of the mode. In the symmetric case, $\tilde{x}_n = \tilde{x}_{-n}$. The solution becomes $\tilde{x}_n = A\lambda^{|n|}\,{\rm e}^{-{\rm i}\omega t} = (-1)^n A\,{\rm e}^{-(K|\bar{x}_n|+{\rm i}\omega t)}$. Again, setting the "boundary" condition at the origin as $\tilde{x}_0 = B\,{\rm e}^{-{\rm i}\omega t}$, we have

$$-M\omega^2\frac{B}{A} = 2F\left(\lambda-\frac{B}{A}\right), \quad (17)$$

$$-m\omega^2\lambda = F\left(\frac{B}{A}-\lambda\right)+f\lambda(\lambda-1), \quad (18)$$

$$-m\omega^2\lambda = f(\lambda-1)^2. \quad (19)$$

From Eqs. (17)–(19), we can obtain

$$\left(1-\frac{f}{F}\right)\lambda^2+\left[2\left(\frac{f}{F}-\frac{m}{M}\right)-1\right]\lambda-\frac{f}{F}=0, \quad (20)$$

or

$$\left(1-\frac{q}{Q}\right)\lambda^2+\left[2\left(\frac{q}{Q}-\frac{m}{M}\right)-1\right]\lambda-\frac{q}{Q}=0. \quad (21)$$

There are two solutions of Eq. (21) under the condition $q/Q = m/M \equiv \eta$

$$\lambda_1 = \frac{1-\sqrt{1+4\eta-4\eta^2}}{2\,(1-\eta)}, \quad (22a)$$

$$\lambda_2 = \frac{1+\sqrt{1+4\eta-4\eta^2}}{2\,(1-\eta)}. \quad (22b)$$

As we have discussed above, $-1 < \lambda < 0$. Then only λ_1 is possible to satisfy the condition, if

$$-1 < \frac{1-\sqrt{1+4\eta-4\eta^2}}{2\,(1-\eta)} < 0. \quad (23)$$

This inequality can always be satisfied for $0 < \eta < 1$. Thus, if the impurity particle has a larger mass and charge than that of the other dust particles, i.e., $|Q| > |q|$, and $M > m$, the symmetric local mode can be excited.

Substituting λ_1 into Eq. (19), we can obtain the dispersion relation for this symmetric local mode

$$\begin{aligned}\omega_{\rm LS} &= \sqrt{\frac{f}{m|\lambda_1|}}(1+|\lambda_1|)\\ &= \omega_{\max}\frac{1+|\lambda_1|}{2\sqrt{|\lambda_1|}},\quad 0<|\lambda_1|<1,\end{aligned} \quad (24)$$

where

$$\lambda_1 = \frac{1-\sqrt{1+4\eta-4\eta^2}}{2\,(1-\eta)}, \quad (25)$$

and the decay length

$$\begin{aligned}\frac{1}{K} &= \frac{a}{\ln(1/|\lambda_1|)}\\ &= \frac{a}{\ln\left[2\,(1-\eta)\,/\left(\sqrt{1+4\eta-4\eta^2}-1\right)\right]}.\end{aligned} \quad (26)$$

2.4. Antisymmetric local modes

In the antisymmetric case, $\tilde{x}_n = -\tilde{x}_{-n}$, the "boundary" condition at the origin is $\tilde{x}_0 = 0$. Then, the local mode has the form of $\tilde{x}_n = \pm A\lambda^{|n|}\,\mathrm{e}^{-\mathrm{i}\omega t} = \pm(-1)^n A\,\mathrm{e}^{-(\mathrm{i}\omega t + K|\bar{x}_n|)}$. Substituting it into Eqs. (2) and (3) leads to

$$-m\omega^2\lambda = -F\lambda + f\lambda(\lambda-1) = f(\lambda-1)^2, \quad (27)$$

and therefore

$$\lambda = \frac{f}{f-F} = \frac{q/Q}{q/Q-1} = -\frac{\eta}{1-\eta}. \quad (28)$$

Then from $-1 < \lambda < 0$, we have the condition for the stable antisymmetric local mode, $0 < \eta < 1/2$ or $|Q| > 2|q|$.

Substituting Eq. (28) into Eq. (27) leads to the dispersion relation

$$\omega_{\mathrm{LA}} = \sqrt{\frac{fQ^2}{mq\,(Q-q)}} = \sqrt{\frac{f}{m\eta\,(1-\eta)}} = \frac{\omega_{\max}}{2\sqrt{\eta(1-\eta)}}, \quad (0<\eta<1/2), \quad (29)$$

and the decay length

$$\frac{1}{K} = \frac{a}{\ln(1/|\lambda|)} = \frac{a}{\ln\left[(1-\eta)/\eta\right]} \quad (30)$$

for the antisymmetric local mode.

3. Properties of the lattice modes and local modes

Based on the dispersion relations derived in the last section, we then discuss the general properties of the lattice modes and local modes, as well as the relations between them.

3.1. The lattice modes

From dispersion relations (10) and (15), it can be seen that the symmetric and antisymmetric lattice modes have the same form, as shown in Fig. 2. Both modes are "forbidden" in the frequency band of $\omega > \omega_{\max}$. Nevertheless, they have different phase shifts as shown in Figs. 3 and 4. We then discuss the features of these phase shifts.

In the "positively dispersive" range of $0 < ka < \pi$, where the group velocity of the modes is always slower than the phase velocity, the phase shift for the lattice mode is small in the long wavelength (low frequency) range. From Eq. (14b) a singularity should appear on the curve of $\cos k_{\mathrm{s}}a = 1/(1-\eta)$ in the parameter plane $(\eta,\ ka)$. It is indeed shown in Fig. 3 as a jump discontinuity of the phase shift δ_{S} from its minimum of $\delta_{\mathrm{S}} \to -\pi/2$ on the $\cos ka < 1/(1-\eta)$ side to its maximum of $\delta_{\mathrm{S}} \to \pi/2$ on the $\cos ka > 1/(1-\eta)$ side of the $(\eta,\ ka)$ plane.

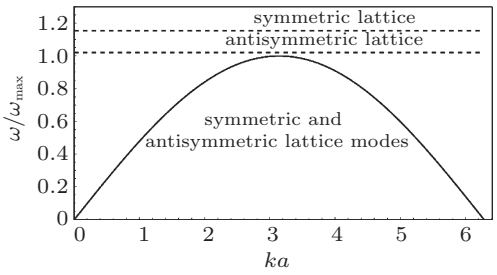


Fig. 2. Dispersion relation of lattice modes and local vibration frequencies with $Q = 2.5q$.

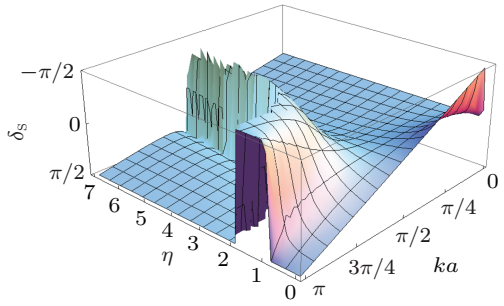


Fig. 3. (colour online) Phase shift of the symmetric lattice mode.

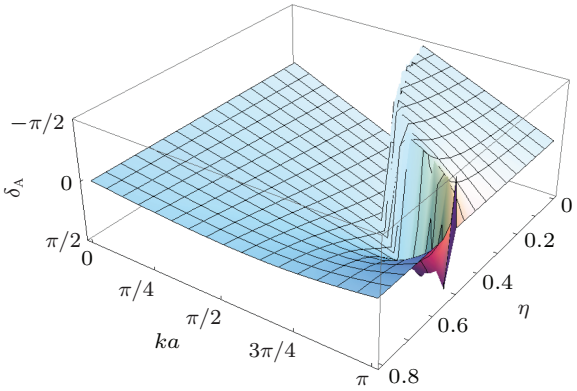


Fig. 4. (colour online) Phase shift of antisymmetric lattice mode.

In addition, from Eq. (16) we can expect a similar singularity to appear on the curve of $\cos k_{\mathrm{s}}a = \eta/(\eta-1)$ in the parameter plane $(\eta,\ ka)$. It is again shown in Fig. 4 when the phase shift δ_{A} jumps from its minimal value of $-\pi/2$ on the $\cos ka > \eta/(\eta-1)$ side to its maximum value of $\pi/2$ on the $\cos ka < \eta/(\eta-1)$ side of the $(\eta,\ ka)$ plane.

In the "negatively dispersive" range of $\pi < ka < 2\pi$, where the direction of the group velocity of the modes is opposite to the phase velocity, the properties of the phase shift is similar to the "positively dispersive" range with respect to the corresponding symmetry.

An interesting point is that at $ka = \pi$, the frequency of both modes reaches its maximum when the group velocity vanishes. Thus, as the frequency further increases to exceed $\omega_{\max}$, both the energy and phase of the mode stop propagating, and only oscillate locally.

3.2. The local modes

For the symmetric local mode, as discussed above, it can only be excited when $0 < \eta < 1$, i.e., $|Q| > |q|$ and $M > m$. For $\eta \to 1$, from Eqs. (18), (22a), and (24) we can obtain $\omega_{\rm LS} = \omega_{\max}$, $\lambda = -1$, and $B/A = 1$. Furthermore, for the symmetric lattice mode with $\eta \to 1$ and $ka \to \pi$, we can obtain $\omega_{\rm S} = \omega_{\max}$, $\delta_{\rm S} = 0$, and $B/A = 1$ from Eqs. (10), (12), and (14b). Thus, for the critical parameter $\eta \to 1$, we have $\omega_{\rm LS} = \omega_{\rm S} = \omega_{\max}$. The perturbation of $\tilde{x}_n = A\lambda^{|n|}\,{\rm e}^{-{\rm i}\omega t} = (-1)^n A\,{\rm e}^{-{\rm i}\omega t}$ for the local mode then becomes the perturbation of $\tilde{x}_n = A\cos(k\,|\bar{x}_n| + \delta_{\rm S})\,{\rm e}^{-{\rm i}\omega t} = (-1)^n A\,{\rm e}^{-{\rm i}\omega t}$ for the lattice mode, i.e., the symmetric local mode and the symmetric lattice mode merge into one branch.

For the antisymmetric local mode, it can only be excited in the parameter range of $0 < \eta < 1/2$, i.e., $|Q| > 2|q|$ and $M > 2m$. For $\eta \to 1/2$, from Eqs. (28) and (29), we can obtain $\omega_{\rm LA} = \omega_{\max}$ and $\lambda = -1$. In addition, for the antisymmetric lattice mode with $\eta \to 1/2$ and $ka \to \pi$, we can obtain $\omega_{\rm A} = \omega_{\max}$ and $\delta_{\rm A} = -\pi/2$ from Eqs. (15) and (16). Then for the critical parameter $\eta \to 1/2$, we have $\omega_{\rm LA} = \omega_{\rm A} = \omega_{\max}$. The perturbation of $\tilde{x}_n = \pm A\lambda^{|n|}\,{\rm e}^{-{\rm i}\omega t} = \pm(-1)^n A\,{\rm e}^{-{\rm i}\omega t}$ for the local mode becomes the same as the perturbation of $\tilde{x}_n = A\sin(k\bar{x}_n \pm \delta_{\rm A})\,{\rm e}^{-{\rm i}\omega t} = \pm(-1)^n A\,{\rm e}^{-{\rm i}\omega t}$ for the lattice mode. In other words, the antisymmetric local mode and the antisymmetric lattice mode merge into one branch.

Thus, at the critical parameter $\eta \to \eta_{\max}$, the local and lattice modes are converted to each other with $\omega = \omega_{\max}$. Then from Eqs. (24) and (29), local mode frequencies $\omega_{\rm LS}$ and $\omega_{\rm LA}$ are always higher than the critical point $\omega_{\max}$so that the "forbidden band" of the lattice modes is filled out, as shown in Fig. 2. They are "long range" modes with a decay length $1/K \to \infty$, as $\omega \to \omega_{\max}$. However, in the higher-frequency regime, the decay length gets shorter, as shown in Fig. 5.

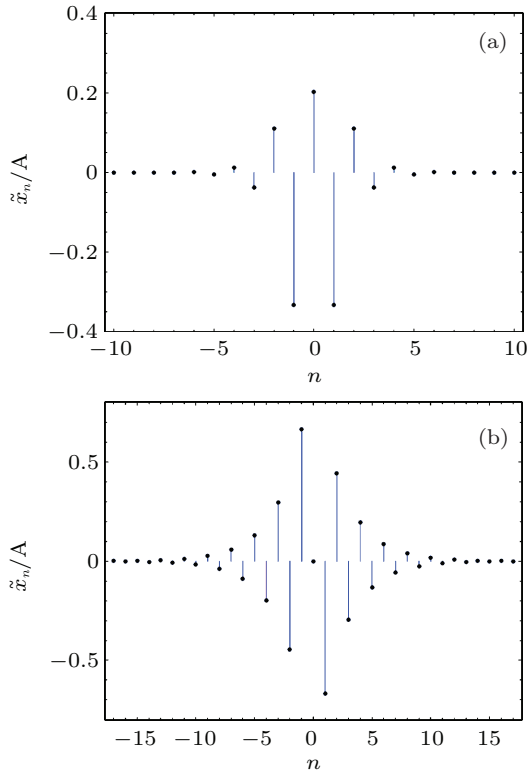


Fig. 5. (a) Symmetric local vibration mode with $Q = 2.5q$. (b) Antisymmetric local vibration mode with $Q = 2.5q$.

4. Summary and conclusion

In this paper we have obtained the lattice wave and local vibration mode solutions in a one-dimensional dusty plasma lattice with an impurity particle. Both symmetric and antisymmetric lattice dispersion relations and their phase shifts are derived. Further discussions reveal a strong dependence of the phase shifts on the wave number.

The local vibration modes can be excited in the frequency band higher than the frequency range of the lattice modes. The critical point is at $\omega = \omega_{\max}$, where the lattice mode becomes a standing wave. At this critical frequency $\omega = \omega_{\max}$, the mode conversion happens, i.e., the local and the lattice modes are converted to each other. Beyond the critical point, the frequency of the local mode gets higher than $\omega_{\max}$ to fill out the "forbidden band" of the lattice modes.

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