Phase-sensitive nonclassical properties of photon-added-then-subtracted coherent squeezed states*

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We investigate the nonclassical properties of the photon-added-then-subtracted coherent squeezed state (PASCSS) via the sub-Poissonian statistics, the photon-number distribution, and the negativity of the Wigner function. It is found that the PASCSS is a superposition state of $D(\beta)S(\zeta)|0\rangle$, $D(\beta)S(\zeta)|1\rangle$, and $D(\beta)S(\zeta)|2\rangle$. We find that the Mandel Q parameter, the photon-number distribution, and the negative volume of the Wigner function of the PASCSS are all periodic functions of the compound $\phi-\theta/2$ with a period π involved with squeezing and displacement parameters.

Keywords: photon-number distribution, non-Gaussian state, quantum-state engineering

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1. Introduction

Non-Gaussian states with highly nonclassical properties may constitute useful resources in quantum information with continuous variables.[1] Performing photon subtraction (a non-Gaussian operation) on a given Gaussian state may be the simplest way to generate a non-Gaussian state. The photon addition is known to create a nonclassical state from any classical one.[2] Both photon subtraction and photon addition have been realized in experiments.[3−5] For example, the photon-subtracted squeezed states[5] and the photon-subtracted-then-added thermal states have been generated in experiments.[6] Therefore, theoretical and experimental efforts in quantum optics and quantum information have been focused on the engineering of highly nonclassical non-Gaussian states of the radiation field by photon subtraction and photon addition.[7−16] For a review of the quantum-state engineering by photon subtraction and photon addition, refer to Ref. [17] and references therein.

As both single-photon subtraction and addition have been successfully demonstrated in experiment, it is interesting to investigate whether the two non-Gaussian operators can be combined to engineer some useful non-Gaussian states. For example, a coherent superposition of photon subtraction and photon addition was proposed, that is $ta+ra^\dagger$ for quantum state engineering.[18] The coherent operation may create a nonclassical state with observable effects (squeezing and sub-Poissonian statistics) out of a classical state and can also be used to generate an arbitrary superposition of states in the number-state basis.[18] On the other hand, it was suggested that the entanglement distillation can be achieved by some combinations of addition and subtraction, such as photon addition then subtraction $aa^\dagger|\psi\rangle$ and subtraction then addition $a^\dagger a|\psi\rangle$,[19] or by the coherent combinations of two sequences of photon subtraction and photon addition.[20] In particular, the latter scheme was motivated by the recent proposal of Kim *et al.* to prove the bosonic commutation relation,[21] where the coherent superposition of two product operations, $aa^\dagger$ and $a^\dagger a$, was suggested, which was experimentally demonstrated in Ref. [6].

In this paper, we pay our attention to the photon-added-then-subtracted coherent squeezed state (PASCSS), and will investigate its nonclassical properties according to multiple criteria, i.e., the Mandel Q parameter, photon-number distributions, and the Wigner function (WF). In particular, we will present how the compound phase $\phi-\theta/2$ coming from the squeezing parameter $\zeta=r\,\mathrm{e}^{\mathrm{i}\theta}$ and the displacement parameter $\beta=|\beta|\,\mathrm{e}^{\mathrm{i}\phi}$ affects those nonclassical properties. This paper is organized as follows. In Section 2, we introduce the PASCSS. In Section 3, based

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on the normalization constant of the PASCSS, we investigate the sub-Poissonian statistics of the PASCSS and its photon-number distributions. We find that both Q parameter and photon-number distributions of the PASCSS are periodic functions of $\phi - \theta/2$ with a period π. In Section 4, we study how the phases ϕ and θ affect the negativity of the Wigner function of the PASCSS. We find that the negative volume of the Wigner function is also a periodic function of $\phi - \theta/2$ with a period π. However, these quantities are all phase independent at $\beta = 0$. Our main results are summarized in Section 5.

2. Photon-added-then-subtracted coherent squeezed state

Let us firstly review the coherent squeezed state (CSS). The CSS can be obtained by performing the squeezing operator $S(\zeta) = \exp\big(\frac{\zeta}{2}a^{\dagger 2} - \frac{\zeta^*}{2}a^2\big)$ on a vacuum state first and followed by the displacement operator $D(\beta) = \exp(\beta a^\dagger - \beta^* a)$, i.e.,

$$|\beta, \zeta\rangle = D(\beta)S(\zeta)|0\rangle, \tag{1}$$

where $\zeta = r\,\mathrm{e}^{\mathrm{i}\theta}$ is a complex squeezing parameter, and $\beta = |\beta|\,\mathrm{e}^{\mathrm{i}\phi}$ is a complex displacement parameter. The normally ordered form of density operator $\rho_{\rm SCS}$ is[22]

$$\begin{aligned}\rho_{\rm SCS} =& \frac{1}{\cosh r} {:}\ \exp\Big[\frac{\tanh r}{2}\,\mathrm{e}^{-\mathrm{i}\theta}(a-\beta)^2 \\ &+\frac{\tanh r}{2}\,\mathrm{e}^{\mathrm{i}\theta}(a^\dagger-\beta^*)^2 - (a^\dagger-\beta^*) \\ &\times(a-\beta)\Big]{:}\,. \end{aligned}\tag{2}$$

The normally ordered form of $\rho_{\rm SCS}$ directly presents the evaluation of matrix elements in the coherent states.

Theoretically, we can introduce the PASCSS by performing $aa^\dagger$ on the CSS, i.e.,

$$|\psi\rangle_{\rm sa} = N^{-1/2}aa^\dagger D(\beta)S(\zeta)|0\rangle, \tag{3}$$

whose density operator is

$$\rho_{\rm sa} = N^{-1}aa^\dagger\rho_{\rm SCS}aa^\dagger, \tag{4}$$

where $N = \mathrm{Tr}(aa^\dagger\rho_{\rm SCS}aa^\dagger)$ is the normalization constant for the PASCSS. The normalization constant of the PASCSS is given by

$$\begin{aligned}N =& M(2) - M(1) \\ =& (|\beta|^2 + \cosh^2 r)^2 + |\,\mathrm{e}^{\mathrm{i}\theta}\beta^*\sinh r \\ &+\beta\cosh r|^2 + 2\cosh^2 r\sinh^2 r, \end{aligned}\tag{5}$$

where we have set

$$M(m) \equiv \mathrm{Tr}(a^{\dagger m}\rho_{\rm SCS}a^m). \tag{6}$$

Actually, $\mathrm{Tr}(a^{\dagger m}\rho_{\rm SCS}a^m)$ is the normalization constant of the photon added coherent squeezed state[23]

$$\begin{aligned}M(m) =& \sum_{k=0}^{m}\Big[\frac{m!}{(m-k)!}\Big]^2\frac{\cosh^{2k} r}{k!}\Big(\frac{\sinh 2r}{4}\Big)^{m-k} \\ &\times\Big|\,\mathrm{H}_{m-k}\Big(\frac{|\beta|\,\mathrm{e}^{\mathrm{i}(\phi-\theta/2)}}{\mathrm{i}\sqrt{\sin 2r}}\Big)\,\Big|^2, \end{aligned}\tag{7}$$

where H is the Hermite polynomial. Due to $\mathrm{H}_n(-z) = (-1)^n\mathrm{H}_n(z)$, as well as $\mathrm{H}_n(-z)\mathrm{H}_n(-z^*) = \mathrm{H}_n(z)\mathrm{H}_n(z^*)$, the modulus square of the Hermite polynomial in Eq. (7) is invariant under the replacement $\phi - \theta/2 \to \pi + \phi - \theta/2$, so $M(m)$ is a periodic function of the compound phase $\phi - \theta/2$ with a period π (except $m = 0$, 1). Thus, the normalization constant of the PASCSS is also a periodic function of the compound phase $\phi - \theta/2$ with a period π. However, $M(m)$ is phase independent at $\beta = 0$ or $r = 0$.

Actually, by using the following transformations $S^{-1}(\zeta)aS(\zeta) = a\cosh r + \mathrm{e}^{\mathrm{i}\theta}a^\dagger\sinh r$ and $S^{-1}(\zeta)a^\dagger S(\zeta) = a^\dagger\cosh r + \mathrm{e}^{-\mathrm{i}\theta}a\sinh r$, as well as $D^{-1}(\beta)aD(\beta) = a + \beta$ and $D^{-1}(\beta)a^\dagger D(\beta) = a^\dagger + \beta^*$, we can reform Eq. (3) as

$$\begin{aligned}|\psi\rangle_{\rm sa} =& N^{-1}D(\beta)S(\zeta)[(\cosh^2 r + |\beta|^2)|0\rangle. \\ &(\,\mathrm{e}^{\mathrm{i}\theta}\beta^*\sinh r + \beta\cosh r)|1\rangle. \\ &+\sqrt{2}\,\mathrm{e}^{\mathrm{i}\theta}\sinh r\cosh r|2\rangle], \end{aligned}\tag{8}$$

which indicates that the PASSCS is a superposition state of $D(\beta)S(\zeta)|0\rangle$, $D(\beta)S(\zeta)|1\rangle$, and $D(\beta)S(\zeta)|2\rangle$. According to Eq. (7), from $\mathrm{Tr}(|\psi\rangle_{\rm sa\ sa}\langle\psi|) = 1$, we can easily obtain the normalization constant of the PASCSS expressed in Eq. (5).

3. Mandel Q parameter and photon-number distributions

3.1. Sub-Poissonian statistics

In order to investigate the photon statistical properties of the PASCSS, we calculate the Mandel Q parameter. The nonclassicality of the PASCSS can be analyzed by studying its sub-Poissonian statistics characterized by the Mandel Q parameter, which is defined as

$$Q = \frac{\langle a^{\dagger 2}a^2\rangle}{\langle a^\dagger a\rangle} - \langle a^\dagger a\rangle. \tag{9}$$

The negativity of the Mandel Q parameter refers to the sub-Poissonian statistics. But a state may be nonclassical even though the Q parameter is positive, as pointed out in Ref. [2]. In another words, $Q < 0$ is only a sufficient condition for the state being nonclassical, not a necessary one. All Fock states result in

$Q=-1$, and all coherent states have $Q=0$. For the CSS, its Q parameter is

$$Q_0=\frac{M(2)-4M(1)+2}{M(1)-1}-M(1)+1. \tag{10}$$

From Eqs. (4) and (7), we have

$$\begin{aligned} N\langle a^{\dagger}a\rangle &= \mathrm{Tr}(a^{\dagger 3}\rho_{\mathrm{SCS}}a^3)-4\mathrm{Tr}(a^{\dagger 2}\rho_{\mathrm{SCS}}a^2) \\ &\quad +2\mathrm{Tr}(a^{\dagger}\rho_{\mathrm{SCS}}a), \end{aligned} \tag{11}$$

$$\begin{aligned} N\langle a^{\dagger 2}a^2\rangle &= \mathrm{Tr}(a^{\dagger 4}\rho_{\mathrm{SCS}}a^4)-9\mathrm{Tr}(a^{\dagger 3}\rho_{\mathrm{SCS}}a^3) \\ &\quad +18\mathrm{Tr}(a^{\dagger 2}\rho_{\mathrm{SCS}}a^2)-6\mathrm{Tr}(a^{\dagger}\rho_{\mathrm{SCS}}a), \end{aligned} \tag{12}$$

thus

$$\begin{aligned} Q &= \frac{M(4)-9M(3)+18M(2)-6M(1)}{M(3)-4M(2)+2M(1)} \\ &\quad -\frac{M(3)-4M(2)+2M(1)}{M(2)-M(1)}. \end{aligned} \tag{13}$$

As mentioned above, $M(m)$ is a periodic function of $\phi-\theta/2$, thus the Q parameter of the PASCSS is also a periodic function of $\phi-\theta/2$ with a period π.

For different values of the displacement amplitude $|\beta|$, the Q changing with r is plotted in Fig. 1(a). From Fig. 1(a), we can see that the Q parameter can be negative when the squeezing parameter r is less than a certain threshold value. The threshold value increases as $|\beta|$ increases. In Fig. 1(b), we draw the changes of Q with $\phi-\theta/2$ in the range $[0,2\pi]$ for different $|\beta|$.

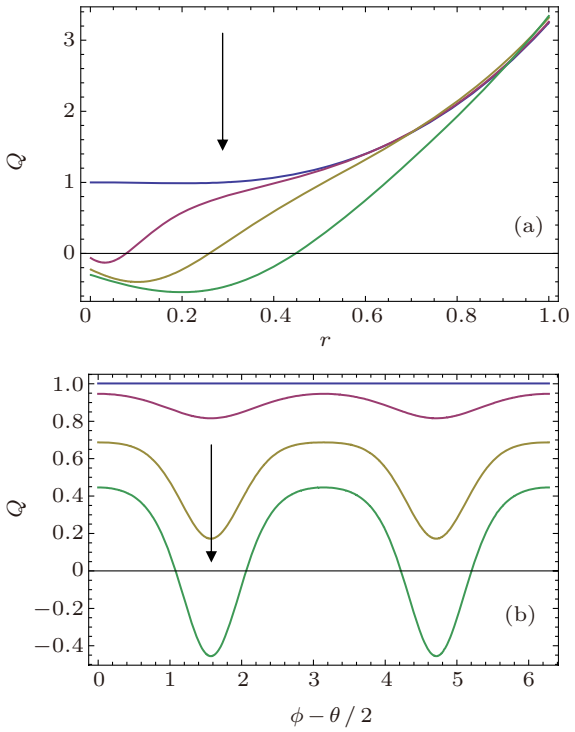


Fig. 1. (colour online) (a) The Q parameter as a function of the squeezing parameter r with $\phi-\theta/2=\pi/2$ for different displacement amplitude $|\beta|$. (b) The Q parameter as a function of the compound phase $\phi-\theta/2$ with $r=0.3$ for different $|\beta|$. From the top line to the bottom one, $|\beta|=0.0,0.2,0.5,1.0$.

3.2. Photon-number distributions of the field

The photon-number distribution is a key characteristic of every optical field. Several indicators of nonclassicality are based directly on the measurement of these statistics, including the negativity of the Mandel Q parameter and the negativity of the Glauber–Sudarshan P function.[24] The photon-number distribution, i.e., the probability of finding n photons in a quantum state described by the density operator ρ, is defined by

$$\mathcal{P}(n)=\mathrm{Tr}(\rho|n\rangle\langle n|)=\langle n|\rho|n\rangle, \tag{14}$$

where $|n\rangle$ is the eigenstate of the number operator $a^{\dagger}a$. Obviously, $\mathcal{P}(n)$ is nothing but the diagonal matrix element ρ_{nn} of ρ in the Fock basis. Considering the normally ordered form of the vacuum projector $|0\rangle\langle 0|=:\exp[-a^{\dagger}a]:$,[25] and splitting the exponential, from Eq. (2), we obtain the explicit form of the CSS

$$\begin{aligned} D(\beta)S(\zeta)|0\rangle &= \frac{1}{\sqrt{\cosh r}}\exp\Big[-\frac{|\beta|^2}{2} \\ &\quad +\frac{\tanh r}{2}\,\mathrm{e}^{\mathrm{i}\theta}(\beta^{*2}+a^{\dagger 2}) \\ &\quad +(\beta-\mathrm{e}^{\mathrm{i}\theta}\beta^{*}\tanh r)a^{\dagger}\Big]|0\rangle. \end{aligned} \tag{15}$$

Noticing the generating function of Hermite polynomial $\mathrm{H}_m(x)$[26,27]

$$\sum_{n=0}^{\infty}\frac{t^n}{n!}\mathrm{H}_n(x)=\mathrm{e}^{2tx-t^2}, \tag{16}$$

we have

$$\begin{aligned} &D(\beta)S(\zeta)|0\rangle \\ &=\frac{1}{\sqrt{\cosh r}}\exp\Big[-\frac{|\beta|^2}{2}+\frac{\tanh r}{2}\,\mathrm{e}^{\mathrm{i}\theta}\beta^{*2}\Big] \\ &\quad\times\sum_{n=0}^{\infty}\frac{1}{\sqrt{n!}}\Big(\mathrm{i}\sqrt{\frac{\tanh r}{2}}\,\mathrm{e}^{\mathrm{i}\theta}\Big)^n\mathrm{H}_n(\Delta)|n\rangle, \end{aligned} \tag{17}$$

where $\Delta\equiv-\mathrm{i}(\beta\,\mathrm{e}^{-\mathrm{i}\theta/2}\cosh r-\mathrm{e}^{\mathrm{i}\theta/2}\beta^{*}\sinh r)/\sqrt{\sin 2r}$. Then the expansion of the PASCSS in the Fock states is

$$\begin{aligned} &|\psi\rangle_{\mathrm{sa}} \\ &=\frac{N^{-1/2}}{\sqrt{\cosh r}}\exp\Big[-\frac{|\beta|^2}{2}+\frac{\tanh r}{2}\,\mathrm{e}^{\mathrm{i}\theta}\beta^{*2}\Big] \\ &\quad\times\sum_{n=0}^{\infty}\frac{(n+1)}{\sqrt{n!}}\Big(\mathrm{i}\sqrt{\frac{\tanh r}{2}}\,\mathrm{e}^{\mathrm{i}\theta}\Big)^n\mathrm{H}_n(\Delta)|n\rangle. \end{aligned} \tag{18}$$

Substituting Eq. (18) into Eq. (14), we can easily obtain the photon-number distribution of the PASCSS

$$\mathcal{P}(n)=\langle n'|\psi\rangle_{\mathrm{sa\,sa}}\langle\psi|n'\rangle=\frac{(n+1)^2}{N}\mathcal{P}_0(n), \tag{19}$$

where

$$\mathcal{P}_0(n) = \frac{\exp[-|\beta|^2 + \frac{\tanh r}{2}(\mathrm{e}^{-\mathrm{i}\theta}\beta^2 + \mathrm{e}^{\mathrm{i}\theta}\beta^{*2})]}{n!\cosh r} \times \left(\frac{\tanh r}{2}\right)^n |\mathrm{H}_n(\Delta)|^2 \quad (20)$$

is the photon-number distribution of the CSS. From Eq. (20), we can see that $\mathcal{P}_0(n)$ is a periodic function of $\phi - \theta/2$ with a period π. Equation (19) indicates that the probability of finding n photons in the PASCSS is modulated by a factor that is a monotonically increasing function of n, which is different from the photon-number distribution of the CSS. And the photon addition-then-subtraction operation essentially shifts the photon-number distribution, which is also presented in Fig. 2.

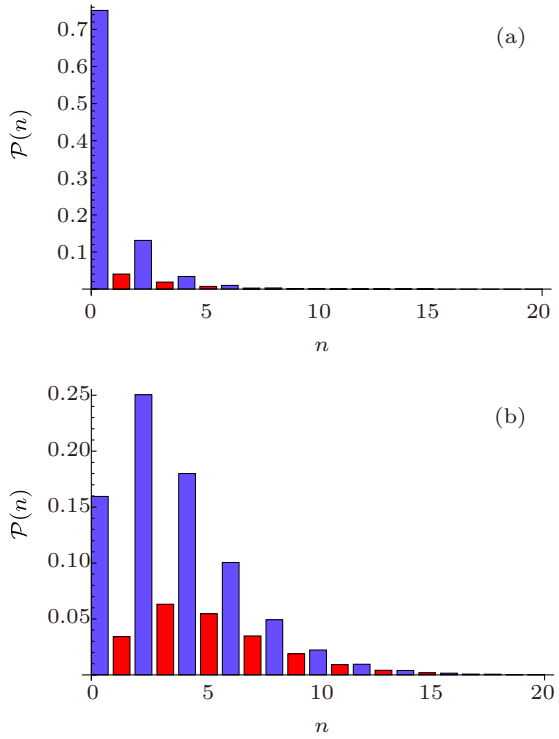


Fig. 2. (colour online) Photon-number distributions of (a) SCS and (b) PASSCS with $|\beta| = 0.5$ and $r = 0.6$ for $\phi - \theta/2 = 0$.

4. Wigner function of PASCSS

A quantum state can be well-described in terms of its Wigner function. The Wigner function was first introduced by Wigner,[27,28] which now becomes a very popular tool to study the nonclassical properties of quantum states. The presence of negativity of the Wigner function is a signature of highly nonclassical character of an optical field. For a single-mode pure state, its Wigner function is expressed as

$$W(\alpha) = \mathrm{Tr}(\rho\Delta(\alpha,\alpha^*)) = \langle\Phi|\Delta(\alpha,\alpha^*)|\Phi\rangle, \quad (21)$$

where $\Delta(\alpha,\alpha^*)$ is the Wigner operator

$$\Delta(\alpha,\alpha^*) = \frac{1}{\pi}{:}\ \exp[-2(a^\dagger - \alpha^*)(a-\alpha)]{:}\,. \quad (22)$$

By using the following transformations

$$S^{-1}(\zeta)aS(\zeta) = a\cosh r + \mathrm{e}^{\mathrm{i}\theta}a^\dagger\sinh r$$

and

$$S^{-1}(\zeta)a^\dagger S(\zeta) = a^\dagger\cosh r + \mathrm{e}^{-\mathrm{i}\theta}a\sinh r,$$

as well as

$$D^{-1}(\beta)aD(\beta) = (a+\beta)$$

and

$$D^{-1}(\beta)a^\dagger D(\beta) = (a^\dagger + \beta^*),$$

we can reform Eq. (3) as

$$|\psi\rangle_{\mathrm{sa}} = N^{-1}D(\beta)S(\zeta)[(\cosh^2 r + |\beta|^2)|0\rangle. + (\mathrm{e}^{\mathrm{i}\theta}\beta^*\sinh r + \beta\cosh r)|1\rangle. + \sqrt{2}\,\mathrm{e}^{\mathrm{i}\theta}\sinh r\cosh r|2\rangle], \quad (23)$$

which indicates that the PASCSS is a superposition state of $D(\beta)S(\zeta)|0\rangle$, $D(\beta)S(\zeta)|1\rangle$, and $D(\beta)S(\zeta)|2\rangle$. According to Eq. (23), from $\mathrm{Tr}(|\psi\rangle_{\mathrm{sa\,sa}}\langle\psi|) = 1$, we can easily obtain the normalization constant of the PASCSS expressed in Eq. (5). Form Eq. (23), we can see that the Wigner function of the PASCSS can be decomposed into nine components. Substituting Eq. (23) into Eq. (21), after a length but straightforward calculation, we obtain the Wigner function of the PASCSS as

$$W(\alpha) = \frac{\exp[-2|\lambda|^2]}{N\pi}[W_{00}(\alpha) + W_{11}(\alpha) + W_{22}(\alpha). + 2\,\mathrm{Re}(W_{01}(\alpha)) + 2\,\mathrm{Re}(W_{02}(\alpha)) + 2\,\mathrm{Re}(W_{12}(\alpha))], \quad (24)$$

where

$$\begin{aligned}
W_{00}(\alpha) &= (\cosh^2 r + |\beta|^2)^2,\\
W_{11}(\alpha) &= |(\mathrm{e}^{\mathrm{i}\theta}\beta^*\sinh r + \beta\cosh r)|^2(4|\lambda|^2 - 1),\\
W_{22}(\alpha) &= 2\sinh^2 r\cosh^2 r\mathrm{L}_2(4|\lambda|^2),\\
W_{01}(\alpha) &= W_{10}^*(\alpha) = 2(\cosh^2 r + |\beta|^2)\\
&\quad\times(\mathrm{e}^{\mathrm{i}\theta}\beta^*\sinh r + \beta\cosh r)\lambda^*,\\
W_{02}(\alpha) &= W_{20}^*(\alpha)\\
&= 4\,\mathrm{e}^{\mathrm{i}\theta}\sinh r\cosh r\big(\cosh^2 r + |\beta|^2\big)\lambda^{*2},\\
W_{12}(\alpha) &= W_{21}^*(\alpha)\\
&= -2\big(\beta\cosh r\sinh^2 r + \beta^*\,\mathrm{e}^{\mathrm{i}\theta}\cosh^2 r\sinh r\big)\\
&\quad\times\big[-2\lambda^*\mathrm{L}_1^1\big(4|\lambda|^2\big)\big], \quad (25)\\
\lambda &= (\alpha-\beta)\cosh r - \mathrm{e}^{\mathrm{i}\theta}(\alpha^* - \beta^*)\sinh r. \quad (26)
\end{aligned}$$

Equation (24) indicates that the Wigner function of the PASCSS is a non-Gaussian Wigner function

in the phase space due to the presence of a non-Gaussian item, which arises from the photon-added-then-subtracted operation.

Using Eq. (24), the Wigner functions of the PASCSS are depicted for two different values of displacement amplitude $|\beta|$ in Fig. 3. We can clearly see that there are some negative regions of the Wigner functions, which are the evidence of the nonclassicality of the state. Moreover, the Wigner function of the PASCSS presents one peak in the middle and two negative valleys in both sides (see Fig. 3).

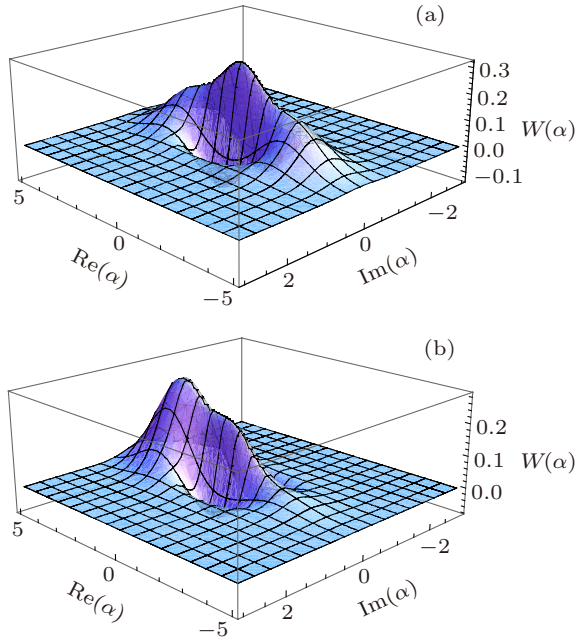


Fig. 3. Wigner functions of PASCSS with $\phi = \theta = 0$ and $r = 0.6$ for (a) $|\beta| = 0$ and (b) $|\beta| = 0.5$.

In order to evaluate how the compound phase $\phi - \theta/2$ affects the Wigner function of the PASCSS, we consider the negative volume of the Wigner function defined by[29]

$$V = \frac{1}{2}\Big[\int \mathrm{d}^2\alpha |W(\alpha, \alpha^*)| - 1\Big]. \tag{27}$$

Our numerical investigation shows that the negative volume of the Wigner function of the PASCSS is also a periodic function of $\phi - \theta/2$ with a period π, as shown in Fig. 4(a). In addition, our numerical analysis also indicates that the negative volume of the Wigner function increases with the increasing squeezing parameter (see Fig. 4(b)). On the other hand, we can prove that the negative volume of the Wigner function decreases with the increase of the displacement amplitude $|\beta|$.

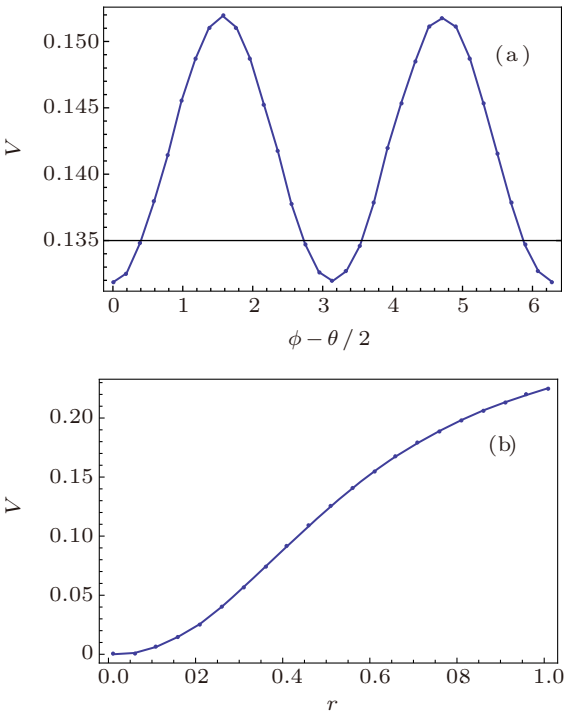


Fig. 4. (colour online) (a) The negative volume of the Wigner function of PASCSS as a function of the compound phase $\phi - \theta/2$ with $r = 0.6$ and $|\beta| = 0.3$. (b) The negative volume of the Wigner function of PASCSS as a function of the squeezing parameter r with $\phi - \theta/2 = \pi/2$ and $|\beta| = 0.3$.

5. Conclusion

In summary, we have investigated the nonclassical properties of the PASCSS based on the sub-Poissonian statistics, photon-number distributions, and the negativity of the Wigner function. We find that the Mandel Q parameter, the photon-number distributions, and the negative volume of the Wigner function are all periodic functions of the compound phase $\phi - \theta/2$ involved in the squeezing and the displacement parameters with a period π. These results indicate that the nonclassical properties of the PASCSS are also dependent on $\phi - \theta/2$. Without considering the displacement parameter ($\beta = 0$), these quantities are all phase independent. Therefore, the displacement parameter plays a key role in the nonclassical properties of the PASCSS.

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