Tunneling-induced ultraslow bright and dark solitons in quantum wells*

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We show the formation of tunneling-induced ultraslow bright and dark solitons in an asymmetric double-quantum-well structure based on the tunneling induced transparency. In this semiconductor structure, the pump field is replaced by the electron-tunneling coupling, which can be modulated by a static electric field. With appropriate conditions, we demonstrate by modulating the intensity of the static electric field that the interplay between the group velocity dispersion and the self-Kerr nonlinearity results in the generation of dark and bright solitons with ultraslow group velocity.

Keywords: optical solitons, quantum wells, tunneling induced transparency

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1. Introduction

Optical solitons, which are formed as the result of interplay between nonlinearity and dispersion properties of media under excitation, have received a great deal of attention in many fields,[1−13] especially in optical fiber and Bose–Einstein condensates.[14,15] It is very important to find new physical systems producing optical solitons at low light levels with good controllability not only for investigating the nonlinear dynamics quantum solitons but also for inventing new optical devices for quantum information processing. In particular, optical solitons with ultraslow group velocity may lead to important applications, such as high fidelity optical buffers and transmission lines.[16] By means of the quantum interference effect induced by a coupling optical field, the absorption of a probe field could be modified through the effects of electromagnetically induced transparency (EIT). The propagation of a weak probe optical field in such a medium displays many new features, including large suppression of optical absorption, significant reduction of group velocity, and enhancement of the Kerr nonlinearity.[17] Based on these characteristics, Wu and Deng generalized an N-type scheme by including the self-phase modulation for the generation of ultraslow optical solitons.[3] Subsequently, the ultraslow[4,5,7] and the superluminal[8−11] optical solitons in EIT cold atom systems have been studied in detail. Recently, Wu extended their work[3] to a crystal of molecular magnets, and showed that such a solid medium can support the propagation of bright and dark microwave solitons.[12]

Semiconductor nanostructures, such as quantum wells (QWs) and quantum dots (QDs), which confine carriers in one or all three spatial directions, have discrete atomic-like spectra. The atomic-like quantum interference effects include the EIT,[18] the tunneling induced transparency (TIT),[19] slow light,[20] ultrafast all-optical switching,[21] tunneling-induced large cross-phase modulation,[22] resonant four-wave mixing,[23] entangled light,[24−26] and so on.[27,28] The propagation of optical pulses[29−33] has also attracted much attention due to the potentially important applications in optoelectronics and solid state quantum information science. It is desirable to achieve ultraslow optical solitons with controllability in semiconductor nanostructures for understanding the propagation properties in such structures and for the possible designation of optical devices via semiconductor nano-materials. In addition, due to their large dipole moments up to the order of 10^{-17} esu·cm, the interaction between semiconductor QWs and optical light fields is strongly enhanced in comparison with that in

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the atomic systems, making them good candidates to study the nonlinear optical propagation effects. Moreover, the interband energies and the electron function symmetries can be engineered according to requirements. These advantages create the opportunities of building opto-electron devices.

In this paper, we consider an asymmetric double-QW structure and present a study addressing the propagation of a weak probe optical field based on the TIT. Different from Ref. [4], the strong control field is replaced by the electron-tunneling coupling, which can be easily controlled by a static electric field. Our results show that in the QW structure under consideration, the probe pulse spreading and attenuation due to the group velocity dispersion can be balanced by the self-Kerr nonlinearity, leading to the generation of dark and bright optical solitons with ultraslow group velocity.

2. Structure and equations

The asymmetric double-QW structure consists of two wells (the left wide one and the right narrow one) with different band structures. As shown in Fig. 1(a), a weak near resonant optical pulse with center frequency $\nu_{\rm p}$ is applied to excite one electron from the valence band to the conduction band in the left well, thus the slowly varying envelope approximation (SVEA) is satisfied very well.[30] The electron levels in the left wide well and in the right narrow well can be energetically aligned with each other by applying a static electric field, while the corresponding hole levels are never aligned for this polarity of the field (see Fig. 1(b)). By neglecting the hole tunneling coupling and the biexciton state, the energy-level diagram of this structure can be described as Fig. 1(c). Different from the atomic scheme in Ref. [4], the pump field is replaced by the electron-tunneling coupling, which can be tuned by the electric field.[34] Our structure is therefore very simple and requires minimal physical resources, which benefits the experimental investigation. In the interaction picture, the coupled Schrödinger–Maxwell equations under the rotating-wave approximation read

$$\frac{\partial A_0}{\partial t} = {\rm i}\Omega_{\rm p}^* A_1, \tag{1}$$

$$\frac{\partial A_1}{\partial t} = {\rm i}(\Delta_1 + {\rm i}\gamma_1)A_1 + {\rm i}\Omega_{\rm p}A_0 - {\rm i}T_{\rm e}A_2, \tag{2}$$

$$\frac{\partial A_2}{\partial t} = {\rm i}(\Delta_2 + {\rm i}\gamma_2)A_2 - {\rm i}T_{\rm e}A_1, \tag{3}$$

$$\frac{\partial \Omega_{\rm p}}{\partial z} + \frac{1}{c}\frac{\partial \Omega_{\rm p}}{\partial t} = {\rm i}\kappa_{01}A_1A_0^*, \tag{4}$$

where Δ_1 and Δ_2 are the detunings defined by $\Delta_1 = \nu_{\rm p} - \omega_{10}$ and $\Delta_2 = \Delta_1 + \omega_{12}$ with $\omega_{ij} = \omega_i - \omega_j$ being the corresponding energy level difference between states $|i\rangle$ and $|j\rangle$, $T_{\rm e}$ denotes the electron tunneling matrix element, and $\kappa_{01} = 2N\nu_{\rm p}|\mu_{01}|^2/(\hbar\varepsilon_0 c)$, with N being the electron volume density. Half of the probe Rabi frequency is defined by $\Omega_{\rm p} = \boldsymbol{\mu}_{01}\cdot\boldsymbol{E}_{\rm p}(t)/2\hbar$, where the electric dipole moment $\boldsymbol{\mu}_{01}$ describes the coupling to the radiation field of the excitonic transition, and $E_{\rm p}(t)$ is the pulse amplitude. Here $\gamma_1 = \gamma_1^{\rm ph} + \gamma_1^{\rm deph}$ is the sum of the electron decay $\gamma_1^{\rm ph}$ and the dephasing rate $\gamma_1^{\rm deph}$. The electron decay in the QW structure is largely affected by temperature, well width, intensity of applied DC field, and so on. The $\gamma_1^{\rm deph}$ is introduced to account for not only the intra-subband phonon scattering and the electron–electron scattering but also the inhomogeneous broadening due to the scattering caused by the interface roughness. The direct decay from state $|2\rangle$ to ground state $|0\rangle$ (i.e., the electron goes directly from the conductive band of the right well to the valence band of the left well) is negligibly small because of the high potential barrier, and thus $\gamma_2 \approx \gamma_2^{\rm deph}$. In Eqs. (2) and (3), γ_1 and γ_2 are added in a phenomenological way.

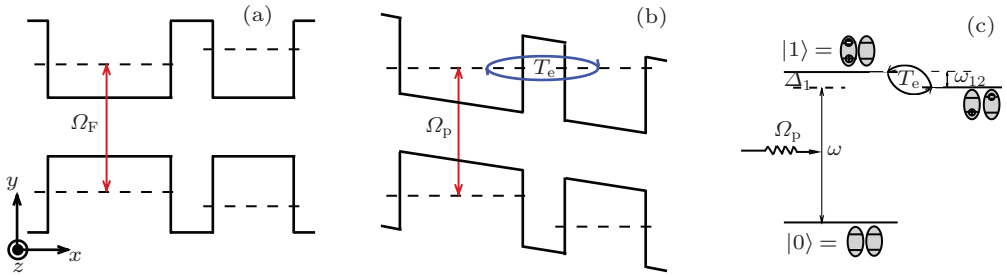


Fig. 1. (colour online) Schematic band diagrams of asymmetric QW structures (a) without and (b) with resonant tunneling. By applying an appropriate static electric field, the electrons of the left conduction band can tunnel into the right conduction band, and then delocalize over both wells. While the holes remain localized. (c) Schematic representation of the model structure, where $\Omega_{\rm p}$ is the Rabi frequency of the optical field, and $T_{\rm e}$ denotes the electron tunneling matrix element.

3. Results and discussion

It is well known that the significant probe field spread and attenuation due to the group velocity dispersion can be balanced by the self-Kerr nonlinearity effect, leading to the formation of ultraslow optical solitons. In order to obtain a better insight into the interplay between the group velocity dispersion and the self-Kerr nonlinearity, following the standard process,[3,4] we take the time Fourier transform of Eqs. (2)–(4) and keep terms up to the first order of $\Omega_{\rm p}$, yielding the linear dispersion relation as

$$\begin{aligned} K(\omega) &= \frac{\omega}{c} + \frac{\kappa_{01}(\omega+\Delta_2+{\rm i}\gamma_2)}{|T_{\rm e}|^2-(\omega+\Delta_1+{\rm i}\gamma_1)(\omega+\Delta_2+{\rm i}\gamma_2)} \\ &\simeq K_0 + K_1\omega + K_2\omega^2 + \mathcal{O}(\omega^3), \end{aligned} \tag{5}$$

where $K_0 = \phi + {\rm i}\alpha/2$ describes the phase shift ϕ per unit length and the absorption coefficient α of the pulsed laser field, ${\rm Re}[K_1] = 1/v_{\rm g}$ is related to the group velocity $v_{\rm g}$, ${\rm Im}[K_1]$ always contributes to the gain of the probe field, and K_2 represents the group velocity dispersion, which contributes both to the probe pulse shape change and the additional loss of probe field intensity.

We now examine the nonlinear effect of the structure. In the moving frame ($\xi \equiv z$, $\eta \equiv t - z/v_{\rm g}$), we can obtain the nonlinear wave equation of the slowly varying envelope $\Omega_{\rm p}(z,t)$

$${\rm i}\frac{\partial}{\partial\xi}\Omega_{\rm p} - K_2\frac{\partial^2}{\partial\eta^2}\Omega_{\rm p} = W\exp(-\alpha\xi)|\Omega_{\rm p}|^2\Omega_{\rm p}, \tag{6}$$

where the coefficient of self-Kerr nonlinearity W is given by

$$W = \frac{\kappa_{01}(\Delta_2+{\rm i}\gamma_2)(\Delta_2^2+\gamma_2^2+T_{\rm e}^2)}{\mathcal{Z}|\mathcal{Z}|^2}, \tag{7}$$

with $\mathcal{Z} = |T_{\rm e}|^2-(\Delta_1+{\rm i}\gamma_1)(\Delta_2+{\rm i}\gamma_2)$. These results in our structure are similar to those in Ref. [4], however, we do not need to introduce the pump field. If the QW structure has no absorption for the probe pulse, i.e., $\exp(-\alpha L) \simeq 1$ and the imaginary parts of K_2 and W are much smaller than their corresponding real parts, then, Eq. (6) can be reduced to the standard nonlinear Schrödinger equation

$${\rm i}\frac{\partial}{\partial\xi}\Omega_{\rm p} - K_{2\rm r}\frac{\partial^2}{\partial\eta^2}\Omega_{\rm p} = W_{\rm r}|\Omega_{\rm p}|^2\Omega_{\rm p}, \tag{8}$$

where $K_{2\rm r}$ and $W_{\rm r}$ are the real parts of K_2 and W, respectively. Note that $W = W_{\rm r} + {\rm i}W_{\rm i} \simeq W_{\rm r}$ is required by the integrability of Eq. (6). Equation (8) suggests that the QW structure supports bright and dark solitons solutions, including the N soliton for dark and bright solitons. The fundamental bright soliton solution with $K_{2\rm r}W_{\rm r} > 0$ is

$$\Omega_{\rm p} = \Omega_{\rm p0}\,{\rm sech}(\eta/\tau)\,{\rm e}^{-\alpha\xi}\exp(-{\rm i}\xi W_{\rm r}|\Omega_{\rm p0}|^2/2), \tag{9}$$

where ${\rm sech}(\eta/\tau)$ is the hyperbolic secant function. While for $K_{2\rm r}W_{\rm r} < 0$, the dark soliton solution is

$$\Omega_{\rm p} = \Omega_{\rm p0}\tanh(\eta/\tau)\,{\rm e}^{-\alpha\xi}\exp(-{\rm i}\xi W_{\rm r}|\Omega_{\rm p0}|^2), \tag{10}$$

where $\tanh(\eta/\tau)$ is the hyperbolic tangent function, and amplitude $\Omega_{\rm p0}$ and width τ are arbitrary constants with $|\Omega_{\rm p0}\tau|^2 = 2|K_{2\rm r}/W_{\rm r}|$. We have assumed $L_{\rm D} = L_{\rm NL}$, i.e., the balance of the dispersion and the nonlinearity, in order to favor the formation of soliton, where $L_{\rm D}$ and $L_{\rm NL}$ denote the dispersion and the nonlinear distances, respectively. It should be noted that in deriving Eqs. (5) and (7), we assume $|\Omega_{\rm p0}\tau|^2 \ll |T_{\rm e}\tau|^2$. Therefore, width τ should be chosen to satisfy $2|K_{2\rm r}/W_{\rm r}| \ll |T_{\rm e}\tau|^2$.

In the absence of the tunneling $T_{\rm e}$, owing to the strong normal absorption of the excitonic transition,[20] the optical solitons cannot be obtained. When the tunneling is turned on, the structure becomes transparent due to the destructive interference, i.e., the TIT. However, in the ideal EIT case, the self-Kerr nonlinearity vanishes at the transparent point $\Delta_2 = 0$, (see Eq. (7) above), which leads to significant probe field attenuation and distortion. Therefore, we must take a slightly different probe detuning such that the group velocity dispersion can be balanced by the self-Kerr nonlinearity. Simultaneously, the excitonic absorption is very weak and can be neglected. This QW structure therefore, in principle, supports the propagation of ultraslow optical solitons, it is induced by the electric field-controlled resonant tunneling.

We now present numerical examples to demonstrate that this asymmetric QW structure supports the existence of bright and dark solitons with ultraslow group velocity. Consider the asymmetric double QWs with $\kappa_{01} = 9.4\times10^5$ m$^{-1}\cdot$meV,[30] the other parameters are taken as $\hbar\gamma_1 = 1.0$ meV, $\gamma_2 = 10^{-2}\gamma_1$, $T_{\rm e} = 10.0\gamma_1$, $\Delta_1 = -10.0\gamma_1$ s^{-1}, and $\omega_1 - \omega_2 = 10.001\gamma_1$. The numerical findings show that $v_{\rm g}/c \simeq 3.65\times10^{-5}$, and $\alpha \simeq 0.0188(1)$ cm^{-1}. With this set of parameters, the standard nonlinear Schrödinger equation (8) with $K_{2\rm r}W_{\rm r} < 0$ is well characterized, and hence the formation of dark solitons traveling

with ultraslow group velocity occurs in this asymmetric double QW structure. If we modulate ω_{12} to $9.999\gamma_1$ by tuning the intensity of the static electric field, we have $K_{2\mathrm{r}}W_{\mathrm{r}} > 0$ with $v_{\mathrm{g}}/c \simeq 3.36\times10^{-5}$ and $\alpha \simeq 0.0188(5)$ cm^{-1}, which suggest that this asymmetric QW structure can also support the ultraslow bright soliton.

In Figs. 2(a) and 2(c), we have plotted respectively the bright soliton solution of Eq. (8) and the numerical solution of Eq. (6) as the functions of η/τ and ξ/l for $\xi = z = 2.0$ cm and $\tau = 10^{-6}$ s with the above described parameters. As can be seen, these parameters can lead to a negligible probe field attenuation. The significant probe field spread and attenuation due to the group velocity dispersion can be balanced by the self-Kerr nonlinearity effect. This can also be seen from the numerical result of dark soliton of Eq. (6), which is shown in Fig. 3. As a result, the generated bright soliton can maintain its shape invariant propagation in this asymmetric QW structure.

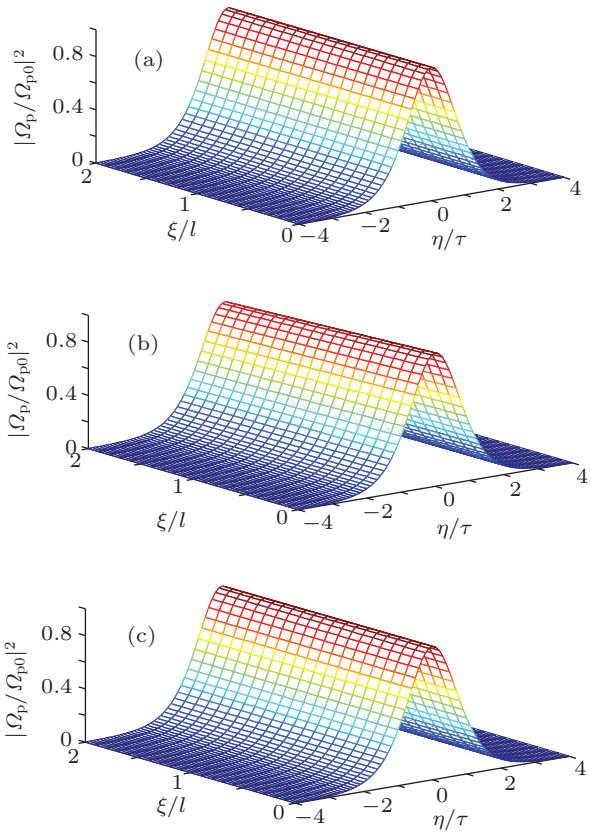


Fig. 2. (colour online) Bright soliton wave shapes $|\Omega_{\mathrm{p}}/\Omega_{\mathrm{p0}}|^2$ versus dimensionless time η/τ and distance ξ/l for $l = 1.0$ cm and $\tau = 10^{-6}$ s. In (a) and (b), $|\Omega_{\mathrm{p}}|^2$ is the numerical solution to Eq. (6) with $T_{\mathrm{e}} = 10.0\gamma_1$ and $T_{\mathrm{e}} = 8.0\gamma_1$, respectively. Panel (c) shows the single bright soliton given in Eq. (8) with $T_{\mathrm{e}} = 10.0\gamma_1$. The other parameters are explained in the text.

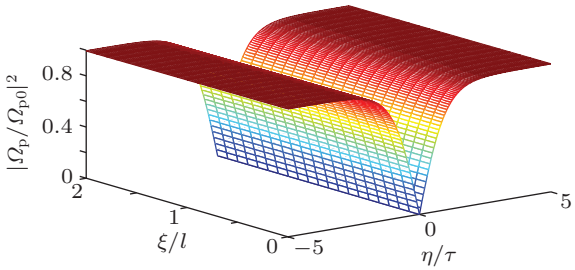


Fig. 3. (colour online) Dark soliton wave shape $|\Omega_{\mathrm{p}}/\Omega_{\mathrm{p0}}|^2$ versus dimensionless time η/τ and distance ξ/l for $l = 1.0$ cm, $\tau = 10^{-6}$ s, in which $|\Omega_{\mathrm{p}}|^2$ is the numerical solution to Eq. (6). The other parameters are explained in the text.

Finally, we demonstrate that a proper choice of electron tunneling T_{e} may further decrease the group velocity of solitons. For example, taking $T_{\mathrm{e}} = 8.0\gamma_1$ with the other parameters unchanged, we obtain $\alpha \simeq 0.0293$ cm^{-1} and $v_{\mathrm{g}}/c \simeq 2.148\times10^{-5}$. In order to see more clearly, we plot the numerical solution of Eq. (6) versus η/τ and ξ/l associated with this case in Fig. 2(b), the other parameters are the same as those in Fig. 2(a). The physics behind is clear, the width of transparency window becomes narrower by decreasing the intensity of the tunneling coupling (this can be seen more clearly in the dressed picture), and therefore resulting in a smaller group velocity. Recalling the transparency condition $|T_{\mathrm{e}}|^2 \gg \gamma_1\gamma_2$, we know that the increased absorption is a natural outcome.

4. Conclusion

In conclusion, we have shown, based on the TIT, that the asymmetric QW structure supports the propagation of ultraslow optical dark and bright solitons with the Schrödinger–Maxwell equations and the pulsed probe field. The semiconductor QW structure can provide all necessary ingredients for generating both bright and dark optical solitons. The self-Kerr nonlinearity can balance the significant probe field spreading and attenuation due to the group velocity dispersion. By adjusting the static electric field appropriately, the interplay between the dispersion and the self-Kerr nonlinearity results in the formation of tunneling-induced ultraslow bright and dark solitons. The present investigation is much more practical than its atomic and quantum dot counterparts due to its flexible design and the controllable electron-tunneling coupling strength. Such ultraslow optical solitons have potential applications in high-fidelity optical delay lines and other information processes based on the semiconductor QW structure.

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