Leader–following consensus criteria for multi-agent systems with time-varying delays and switching interconnection topologies*

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We consider multi-agent systems with time-varying delays and switching interconnection topologies. By constructing a suitable Lyapunov–Krasovskii functional and using the reciprocally convex approach, new delay-dependent consensus criteria for the systems are established in terms of linear matrix inequalities (LMIs), which can be easily solved by using various effective optimization algorithms. Two numerical examples are given to illustrate the effectiveness of the proposed methods.

Keywords: consensus, multi-agent systems, time delay, Lyapunov method

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1. Introduction

Multi-agent systems (MASs) have received considerable attention due to their extensive applications in many fields, such as biology, physics, robotics, and control engineering.[1−6] In Ref. [4], the consensus problem of high-order MASs with time delays and switching topologies was addressed by using the Lyapunov function and the properties of the Metzler matrix with zero row sums. In Ref. [5], the constrained consensus problem was investigated for a group of agents in disconnected topologies owing to the existence of packet drops, limited communication channels, external disturbances, and other influencing factors. In Ref. [6], the H_∞ consensus control condition of networks of second-order MASs with external disturbances was proposed based on new positive semidefinite Lyapunov functions. A prime concern in those systems is the agreement among the group of agents on their states of leader by the interaction. Namely, this problem is a consensus problem. Specially, the consensus problem with a leader is called the leader–following consensus problem or the consensus regulation. Recently, this problem has been applied in various fields, such as vehicle systems,[7] intelligent decision support systems for power grid dispatching,[8] and networked control systems.[9]

During the last few years, the MASs have been put to use in the consensus problem for systems with time delays. The delays are due to the finite speed of information processing in the implementation of the system. It is well known that the time delay often causes undesirable dynamic behaviors, such as oscillation, performance degradation, and instability of the network. It should be pointed out that analyzing the consensus problem of the MASs with time delays can be regarded as investigating the asymptotical stability of the MASs. The stability criteria for time-delay systems can be classified into two types: delay-dependent ones and delay-independent ones. The former are generally less conservative than the latter, because the delay-dependent stability criteria include the infor-

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mation on the size of delay.[10] Since the consensus issue is a prerequisite to the applications of MASs, various approaches to finding consensus criteria for the MASs with time delays have been investigated in the literature.[11−16] By using the Lyapunov-based approach and the related space decomposition technique, a coordination problem was addressed for the MASs with jointly connected interconnection topologies.[11] Liu *et al.*[12] obtained the consensus conditions for the second-order MASs systems with time-varying communication delays and static interconnection topologies. In addition, the delay-dependent consensus condition was also derived for the system with time-invariant communication delay and switching topology. Xiao *et al.*[13] have studied a consensus problem for discrete-time MASs with changing communication topologies and bounded time-varying communication delays. In Ref. [14], based on the Lyapunov–Razumikhin methods, the leader–following consensus problem for the second-order MASs with identical communication delays was considered. Tian *et al.*[15] studied the consensus problem for the MASs with both communication and input delays. By constructing a Lyapunov–Krasovskii functional with the idea of delay partitioning, Qin *et al.*[16] derived the consensus condition for the directed networks of agents with switching topology and time delay. However, to the best of our knowledge, the delay-dependent consensus conditions for the MASs with time-varying delays and switching interconnection topologies have not been investigated yet.

Motivated by the above discussion, in this paper, we propose new delay-dependent consensus criteria for the MASs with time-varying delays and switching interconnection topologies. By constructing a suitable Lyapunov–Krasovskii functional and using the reciprocally convex approach,[17] the criteria are derived in terms of linear matrix inequalities (LMIs), which can be solved efficiently by the use of standard convex optimization algorithms, such as the interior-point methods.[18] Inspired by the works in Refs. [19] and [20], we use a triple integral form of the Lyapunov–Krasovskii functional. Two numerical examples are included to show the effectiveness of the proposed methods.

Notation The $\mathbb{R}^n$ is the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ denotes the set of $m\times n$ real matrix. For symmetric matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$, $\boldsymbol{X} > \boldsymbol{Y}$ (respectively, $\boldsymbol{X} \geq \boldsymbol{Y}$) means that matrix $\boldsymbol{X} - \boldsymbol{Y}$ is positive definite (nonnegative). The $\boldsymbol{X}^{\perp}$ denotes a basis for the null space of $\boldsymbol{X}$. The $\boldsymbol{I}_n$, $\boldsymbol{0}_n$, and $\boldsymbol{0}_{m\times n}$ denote $n\times n$ identity matrix, $n\times n$ and $m\times n$ zero matrices, respectively. The $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. And diag$\{\cdots\}$ denotes the block diagonal matrix.

2. Problem statement

The interaction topology of a network of agents is represented by a directed graph (digraph) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with nodes $\mathcal{V} = \{1, 2, \ldots, n\}$ and edges $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\} \subset \mathcal{V} \times \mathcal{V}$. A adjacency matrix $\boldsymbol{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ of digraph $\mathcal{G}$ is the matrix with nonnegative elements satisfying $a_{ii} = 0$ and $a_{ij} \geq 0$. If there is an edge between i and j, then the corresponding element in matrix $\boldsymbol{A}$ has $a_{ij} > 0 \Leftrightarrow (i, j) \in \mathcal{E}$. The digraph $\mathcal{G}$ is said to be undirected if $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$. A set of neighbors of agent i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$. The degree of node i is denoted by $\deg(i) = \sum_{j\in\mathcal{N}_i} a_{ij}$. The degree matrix of digraph $\mathcal{G}$ is a diagonal matrix defined as $\boldsymbol{D} = \text{diag}\{\deg(1), \ldots, \deg(n)\}$. More details can be found in Ref. [21].

Consider the following multi-agent system with the second-order agent dynamics:

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t), \quad i \in \mathcal{V} = \{1, \ldots, n\}, \end{aligned} \tag{1}$$

where n is the number of agents, $x_i(t) \in \mathbb{R}$, $v_i(t) \in \mathbb{R}$, and $u_i(t) \in \mathbb{R}$ are the position, the velocity, and the control input of agent i, respectively.

According to Refs. [11] and [12], a consensus algorithm with the leader interconnection topology can be described as

$$\begin{aligned} u_i(t) = &-k_i v_i(t) - \sum_{j\in\mathcal{N}_i} a_{ij}(x_i(t) - x_j(t)) \\ &- b_i(x_i(t) - x_0), \end{aligned} \tag{2}$$

where $x_0 \in \mathbb{R}$ is the position of the leader, $k_i > 0$ are the control parameters, a_{ij} and b_i are the interconnection weights defined as

$$\begin{cases} a_{ij} > 0, & \text{if agent } i \text{ is connected to agent } j, \\ a_{ij} = 0, & \text{otherwise}, \end{cases}$$

$$\begin{cases} b_i > 0, & \text{if agent } i \text{ is connected to the leader}, \\ b_i = 0, & \text{otherwise}. \end{cases}$$

With the communication delay, a consensus algorithm can be given as

$$\begin{aligned} u_i(t) &= -k_i v_i(t) - \sum_{j\in\mathcal{N}_i} a_{ij}(x_i(t) - x_j(t-h(t))) \\ &\quad - b_i(x_i(t) - x_0), \end{aligned} \tag{3}$$

where $h(t)$ denotes a time-varying delay satisfying

$$0 \le h(t) \le h_{\rm M}, \quad \dot{h}(t) \le h_{\rm D}, \tag{4}$$

with $h_{\rm M} > 0$ and $h_{\rm D}$ being known constant scalar values.

Consider a model of MASs with the consensus algorithm (3)

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= -k_i v_i(t) - \sum_{j\in\mathcal{N}_i} a_{ij}(x_i(t) - x_j(t-h(t))) \\ &\quad - b_i(x_i(t) - x_0). \end{aligned} \tag{5}$$

Then, system (5) can be rewritten in the matrix form

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{v}(t), \\ \dot{\boldsymbol{v}}(t) &= -\boldsymbol{K}\boldsymbol{v}(t) - (\boldsymbol{D}+\boldsymbol{B})(\boldsymbol{x}(t) - x_0\mathbf{1}_n) \\ &\quad + \boldsymbol{A}(\boldsymbol{x}(t-h(t)) - x_0\mathbf{1}_n) + (\boldsymbol{A}-\boldsymbol{D})x_0\mathbf{1}_n, \end{aligned} \tag{6}$$

where $\boldsymbol{x}(t) = [x_1(t), \ldots, x_n(t)]^{\rm T} \in \mathbb{R}^n$ is the position vector of agents, $\boldsymbol{v}(t) = [v_1(t), \ldots, v_n(t)]^{\rm T} \in \mathbb{R}^n$ is the velocity vector of agents, $\boldsymbol{K} = {\rm diag}\{k_1, \ldots, k_n\}$, $\boldsymbol{A}$ is the adjacency matrix of $\mathcal{G}$, $\boldsymbol{B} = {\rm diag}\{b_1, \ldots, b_n\}$, $\boldsymbol{D}$ is the degree matrix of $\mathcal{G}$, and $\mathbf{1}_n = [1, 1, \ldots, 1]^{\rm T} \in \mathbb{R}^n$. Since the following equality with $x_0\mathbf{1}_n$ holds

$$\begin{aligned} &\boldsymbol{A}x_0\mathbf{1}_n \\ &= \begin{bmatrix} 0 & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & a_{n-1n} \\ a_{n1} & a_{n2} & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_0 \\ \vdots \\ x_0 \end{bmatrix} \\ &= \begin{bmatrix} \sum_{j\in\mathcal{N}_i} a_{1j}x_0 \\ \sum_{j\in\mathcal{N}_i} a_{2j}x_0 \\ \vdots \\ \sum_{j\in\mathcal{N}_i} a_{nj}x_0 \end{bmatrix} \\ &= \begin{bmatrix} \sum_{j\in\mathcal{N}_i} a_{1j} & 0 & \cdots & 0 \\ 0 & \sum_{j\in\mathcal{N}_i} a_{2j} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \sum_{j\in\mathcal{N}_i} a_{nj} \end{bmatrix} \begin{bmatrix} x_0 \\ x_0 \\ \vdots \\ x_0 \end{bmatrix} \\ &= \boldsymbol{D}x_0\mathbf{1}_n, \end{aligned}$$

system (6) can be rewritten as

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{v}(t), \\ \dot{\boldsymbol{v}}(t) &= -\boldsymbol{K}\boldsymbol{v}(t) - (\boldsymbol{D}+\boldsymbol{B})(\boldsymbol{x}(t) - x_0\mathbf{1}_n) \\ &\quad + \boldsymbol{A}(\boldsymbol{x}(t-h(t)) - x_0\mathbf{1}_n). \end{aligned} \tag{7}$$

In this paper, we consider the consensus for MASs with time-varying delays and switching interconnection topologies. It should be noted that $\hat{\mathcal{G}}$ denotes the topology composed of the agents and the leader, and $\Gamma = \{\hat{\mathcal{G}}_m, m = 1, \ldots, N\}$ is defined as the union of the topologies. Here, N is the number of topologies.

Let us define

$$\hat{\boldsymbol{x}}(t) = \boldsymbol{x}(t) - x_0\mathbf{1}_n, \quad \boldsymbol{\eta}(t) = [\hat{\boldsymbol{x}}^{\rm T}(t), \boldsymbol{v}^{\rm T}(t)]^{\rm T}.$$

Then, system (7) with switching interconnection topology m can be rewritten as

$$\begin{aligned} &\dot{\boldsymbol{\eta}}(t) = \boldsymbol{E}_m\boldsymbol{\eta}(t) + \boldsymbol{F}_m\boldsymbol{\eta}(t-h(t)), \\ &m = s(t), \quad \hat{\mathcal{G}}_m \in \Gamma, \end{aligned} \tag{8}$$

where $I_\Gamma = \{1, \ldots, N\}$ is the index set associated with the elements of Γ, $s(t) : \mathbb{R}^+ \to I_\Gamma$ is a switching signal, and

$$\boldsymbol{E}_m = \begin{bmatrix} \mathbf{0}_n & \boldsymbol{I}_n \\ -(\boldsymbol{D}_m + \boldsymbol{B}_m) & -\boldsymbol{K} \end{bmatrix},$$

$$\boldsymbol{F}_m = \begin{bmatrix} \mathbf{0}_n & \mathbf{0}_n \\ \boldsymbol{A}_m & \mathbf{0}_n \end{bmatrix}.$$

Remark 1 The model (8) can be extend to the case without leader as follows:

$$\begin{aligned} &\bar{\dot{\boldsymbol{\eta}}}(t) = \bar{\boldsymbol{E}}_m\bar{\boldsymbol{\eta}}(t) + \boldsymbol{F}_m\bar{\boldsymbol{\eta}}(t-h(t)), \\ &m = s(t), \quad \hat{\mathcal{G}}_m \in \Gamma, \end{aligned} \tag{9}$$

where

$$\bar{\boldsymbol{\eta}}(t) = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{v}(t) \end{bmatrix}, \quad \bar{\boldsymbol{E}}_m = \begin{bmatrix} \mathbf{0}_n & \boldsymbol{I}_n \\ -\boldsymbol{D}_m & -\boldsymbol{K} \end{bmatrix},$$

$$\boldsymbol{F}_m = \begin{bmatrix} \mathbf{0}_n & \mathbf{0}_n \\ \boldsymbol{A}_m & \mathbf{0}_n \end{bmatrix}.$$

In model (8) with a leader, take the group of environment monitoring robot fishes in Fig. 1 as an example, the interconnection between the leader and one particular agent can provide the information of both the route of agent group and the consensus of each agent. However, model (9) without leader provides only the information of the consensus of each agent. That is, to perform the task of agent group, we should consider the interconnection between the leader and one

particular agent, as shown Fig. 1. For the above mentioned reason, model (9) without leader has less practical meaning than model (8) with a leader.

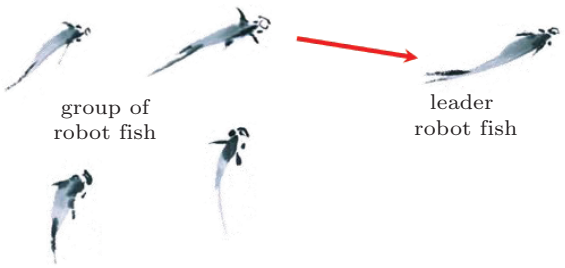


Fig. 1. (colour online) Environment monitoring robot fishes.

The aim of this paper is to investigate the delay-dependent consensus (in other word, stability analysis) of system (8) with time-varying delays. To do this, we introduce the following lemmas.

Lemma 1 For any constant matrix $\boldsymbol{M} \in \mathbb{R}^{n\times n}$, $\boldsymbol{M} = \boldsymbol{M}^{\mathrm{T}} > 0$, and scalars β, α satisfying $\beta < \alpha$, vector function $\boldsymbol{x} : [\beta, \alpha] \to \mathbb{R}^n$ such that the integrations concerned are well defined, then

$$\begin{aligned}
&(\alpha-\beta)\int_\beta^\alpha \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{M}\boldsymbol{x}(s)\mathrm{d}s \\
&\geq \left(\int_\beta^\alpha \boldsymbol{x}(s)\mathrm{d}s\right)^{\mathrm{T}} \boldsymbol{M}\left(\int_\beta^\alpha \boldsymbol{x}(s)\mathrm{d}s\right), \qquad (10)\\
&\frac{(\alpha-\beta)^2}{2}\int_\beta^\alpha\int_s^\alpha \boldsymbol{x}^{\mathrm{T}}(u)\boldsymbol{M}\boldsymbol{x}(u)\mathrm{d}u\mathrm{d}s \\
&\geq \left(\int_\beta^\alpha\int_s^\alpha \boldsymbol{x}(u)\mathrm{d}u\mathrm{d}s\right)^{\mathrm{T}} \\
&\quad \times \boldsymbol{M}\left(\int_\beta^\alpha\int_s^\alpha \boldsymbol{x}(u)\mathrm{d}u\mathrm{d}s\right). \qquad (11)
\end{aligned}$$

Proof According to Jensen's inequality in Ref. [22], we can obtain inequality (10). Moreover, the following inequality holds:

$$\begin{aligned}
&(\alpha-s)\int_s^\alpha \boldsymbol{x}^{\mathrm{T}}(u)\boldsymbol{M}\boldsymbol{x}(u)\mathrm{d}u \\
&\geq \left(\int_s^\alpha \boldsymbol{x}(u)\mathrm{d}u\right)^{\mathrm{T}} \boldsymbol{M}\left(\int_s^\alpha \boldsymbol{x}(u)\mathrm{d}u\right). \qquad (12)
\end{aligned}$$

According to the Schur Complements,[18] equation (12) is equivalent to

$$\begin{bmatrix} \int_s^\alpha \boldsymbol{x}^{\mathrm{T}}(u)\boldsymbol{M}\boldsymbol{x}(u)\mathrm{d}u & \int_s^\alpha \boldsymbol{x}^{\mathrm{T}}(u)\mathrm{d}u \\ \int_s^\alpha \boldsymbol{x}(u)\mathrm{d}u & (\alpha-s)\boldsymbol{M}^{-1} \end{bmatrix} \geq \boldsymbol{0}. \qquad (13)$$

Integrating inequality (13) from β to α yields

$$\begin{bmatrix} \int_\beta^\alpha\int_s^\alpha \boldsymbol{x}^{\mathrm{T}}(u)\boldsymbol{M}\boldsymbol{x}(u)\mathrm{d}u\mathrm{d}s & \int_\beta^\alpha\int_s^\alpha \boldsymbol{x}^{\mathrm{T}}(u)\mathrm{d}u\mathrm{d}s \\ \int_\beta^\alpha\int_s^\alpha \boldsymbol{x}(u)\mathrm{d}u\mathrm{d}s & \int_\beta^\alpha(\alpha-s)\boldsymbol{M}^{-1}\mathrm{d}s \end{bmatrix} \geq \boldsymbol{0}. \qquad (14)$$

Therefore, inequality (14) is equivalent to inequality (11) according to the Schur Complements. This complete the proof.

Lemma 2 (Finsler's lemma[23]) Let $\boldsymbol{\zeta} \in \mathbb{R}^n$, $\boldsymbol{\Phi} = \boldsymbol{\Phi}^{\mathrm{T}} \in \mathbb{R}^{n\times n}$, and $\boldsymbol{B} \in \mathbb{R}^{m\times n}$ such that $\mathrm{rank}(\boldsymbol{B}) < n$. The following statements are equivalent:

(i) $\boldsymbol{\zeta}^{\mathrm{T}}\boldsymbol{\Phi}\boldsymbol{\zeta} < \boldsymbol{0}$, $\forall \boldsymbol{B}\boldsymbol{\zeta} = 0$, $\boldsymbol{\zeta} \neq \boldsymbol{0}$,

(ii) ${\boldsymbol{B}^{\perp}}^{\mathrm{T}}\boldsymbol{\Phi}\boldsymbol{B}^{\perp} < \boldsymbol{0}$,

(iii) $\exists \boldsymbol{X} \in \mathbb{R}^{n\times m} : \boldsymbol{\Phi} + \boldsymbol{X}\boldsymbol{B} + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{X}^{\mathrm{T}} < \boldsymbol{0}$.

3. Main results

In this section, we propose new stability criteria for network (8). For the simplicity of matrix representation, $\boldsymbol{e}_i \in \mathbb{R}^{14n\times 2n}$ and $\boldsymbol{e}_{i\pm j} \in \mathbb{R}^{14n\times 2n}$ $(i, j = 1, \ldots, 7)$ are defined as block entry matrices; e.g., $\boldsymbol{e}_2 = [\boldsymbol{0}_{2n}, \boldsymbol{I}_{2n}, \boldsymbol{0}_{2n}, \ldots, \boldsymbol{0}_{2n}]^{\mathrm{T}}$ and $\boldsymbol{e}_{1\pm 2} = \boldsymbol{e}_1 \pm \boldsymbol{e}_2$. Several matrices are defined as

$$\boldsymbol{\zeta}(t) = \left[\boldsymbol{\eta}^{\mathrm{T}}(t), \boldsymbol{\eta}^{\mathrm{T}}(t-h(t)), \boldsymbol{\eta}^{\mathrm{T}}(t-h_{\mathrm{M}}), \dot{\boldsymbol{\eta}}^{\mathrm{T}}(t), \dot{\boldsymbol{\eta}}^{\mathrm{T}}(t-h_{\mathrm{M}}), \int_{t-h(t)}^{t} \boldsymbol{\eta}^{\mathrm{T}}(s)\mathrm{d}s, \int_{t-h_{\mathrm{M}}}^{t-h(t)} \boldsymbol{\eta}^{\mathrm{T}}(s)\mathrm{d}s\right]^{\mathrm{T}},$$

$$\begin{aligned}
\boldsymbol{\Upsilon}_m &= [\boldsymbol{E}_m, \boldsymbol{F}_m, \boldsymbol{0}_{2n}, -\boldsymbol{I}_{2n}, \boldsymbol{0}_{2n}, \boldsymbol{0}_{2n}, \boldsymbol{0}_{2n}],\\
\boldsymbol{\Xi}_1 &= [\boldsymbol{e}_1, \boldsymbol{e}_3, \boldsymbol{e}_{6+7}]\boldsymbol{\mathcal{P}}[\boldsymbol{e}_4, \boldsymbol{e}_5, \boldsymbol{e}_{1-3}]^{\mathrm{T}}\\
&\quad + [\boldsymbol{e}_4, \boldsymbol{e}_5, \boldsymbol{e}_{1-3}]\boldsymbol{\mathcal{P}}[\boldsymbol{e}_1, \boldsymbol{e}_3, \boldsymbol{e}_{6+7}]^{\mathrm{T}},\\
\boldsymbol{\Xi}_2 &= \boldsymbol{e}_1\boldsymbol{Q}_1\boldsymbol{e}_1^{\mathrm{T}} - \boldsymbol{e}_2((1-h_{\mathrm{D}})\boldsymbol{Q}_1)\boldsymbol{e}_2^{\mathrm{T}}\\
&\quad + [\boldsymbol{e}_1, \boldsymbol{e}_4]\boldsymbol{\mathcal{Q}}_2[\boldsymbol{e}_1, \boldsymbol{e}_4]^{\mathrm{T}} - [\boldsymbol{e}_3, \boldsymbol{e}_5]\boldsymbol{\mathcal{Q}}_2[\boldsymbol{e}_3, \boldsymbol{e}_5]^{\mathrm{T}},\\
\boldsymbol{\Xi}_3 &= [\boldsymbol{e}_1, \boldsymbol{e}_4](h_{\mathrm{M}}^2\boldsymbol{\mathcal{R}}_1)[\boldsymbol{e}_1, \boldsymbol{e}_4]^{\mathrm{T}} - [\boldsymbol{e}_6, \boldsymbol{e}_{1-2}]\boldsymbol{\mathcal{R}}_1[\boldsymbol{e}_6, \boldsymbol{e}_{1-2}]^{\mathrm{T}}\\
&\quad - [\boldsymbol{e}_7, \boldsymbol{e}_{2-3}]\boldsymbol{\mathcal{R}}_1[\boldsymbol{e}_7, \boldsymbol{e}_{2-3}]^{\mathrm{T}}\\
&\quad - [\boldsymbol{e}_6, \boldsymbol{e}_{1-2}]\boldsymbol{\mathcal{S}}[\boldsymbol{e}_7, \boldsymbol{e}_{2-3}]^{\mathrm{T}}\\
&\quad - [\boldsymbol{e}_7, \boldsymbol{e}_{2-3}]\boldsymbol{\mathcal{S}}^{\mathrm{T}}[\boldsymbol{e}_6, \boldsymbol{e}_{1-2}]^{\mathrm{T}},\\
\boldsymbol{\Xi}_4 &= \boldsymbol{e}_4\left(\frac{h_{\mathrm{M}}^2}{2}\right)^2\boldsymbol{R}_2\boldsymbol{e}_4^{\mathrm{T}} - [\boldsymbol{e}_1 - \boldsymbol{e}_{6+7}]\boldsymbol{R}_2[\boldsymbol{e}_1 - \boldsymbol{e}_{6+7}]^{\mathrm{T}},\\
\boldsymbol{\Phi} &= \sum_{i=1}^{4}\boldsymbol{\Xi}_i. \qquad (15)
\end{aligned}$$

Then, we have the following theorem.

Theorem 1 For given scalars $0 < h_{\mathrm{M}}$ and h_{D},

the agents in system (8) converge to the state of the leader for switching signal $s(t)$ asymptotically, if there exist positive definite matrices $\boldsymbol{\mathcal{P}} \in \mathbb{R}^{6n\times 6n}$, $\boldsymbol{Q}_1 \in \mathbb{R}^{2n\times 2n}$, $\boldsymbol{\mathcal{Q}}_2 \in \mathbb{R}^{4n\times 4n}$, $\boldsymbol{\mathcal{R}}_1 =\in \mathbb{R}^{4n\times 4n}$, $\boldsymbol{R}_2 \in \mathbb{R}^{2n\times 2n}$, and any matrix $\boldsymbol{\mathcal{S}} \in \mathbb{R}^{4n\times 4n}$ satisfying the following LMIs:

$$(\boldsymbol{\Upsilon}_m^{\perp})^{\mathrm{T}}\boldsymbol{\Phi}(\boldsymbol{\Upsilon}_m^{\perp}) < \boldsymbol{0}, \tag{16}$$

$$\begin{bmatrix} \boldsymbol{\mathcal{R}}_1 & \boldsymbol{\mathcal{S}} \\ \boldsymbol{\mathcal{S}}^{\mathrm{T}} & \boldsymbol{\mathcal{R}}_1 \end{bmatrix} \geq \boldsymbol{0}, \tag{17}$$

where $\boldsymbol{\Phi}$ and $\boldsymbol{\Upsilon}_m$ are defined in Eq. (15).

Proof Let us consider the following Lyapunov–Krasovskii functional candidate:

$$V = V_1 + V_2 + V_3 + V_4, \tag{18}$$

where

$$\begin{aligned}
V_1 &= \begin{bmatrix} \boldsymbol{\eta}(t) \\ \boldsymbol{\eta}(t-h_{\mathrm{M}}) \\ \int_{t-h_{\mathrm{M}}}^{t}\boldsymbol{\eta}(s)\mathrm{d}s \end{bmatrix}^{\mathrm{T}} \boldsymbol{\mathcal{P}} \begin{bmatrix} \boldsymbol{\eta}(t) \\ \boldsymbol{\eta}(t-h_{\mathrm{M}}) \\ \int_{t-h_{\mathrm{M}}}^{t}\boldsymbol{\eta}(s)\mathrm{d}s \end{bmatrix}, \\
V_2 &= \int_{t-h(t)}^{t}\boldsymbol{\eta}^{\mathrm{T}}(s)\boldsymbol{Q}_1\boldsymbol{\eta}(s)\mathrm{d}s \\
&\quad + \int_{t-h_{\mathrm{M}}}^{t}\begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix}^{\mathrm{T}} \boldsymbol{\mathcal{Q}}_2 \begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix}\mathrm{d}s, \\
V_3 &= h_{\mathrm{M}}\int_{t-h_{\mathrm{M}}}^{t}\int_{s}^{t}\begin{bmatrix} \boldsymbol{\eta}(u) \\ \dot{\boldsymbol{\eta}}(u) \end{bmatrix}^{\mathrm{T}} \boldsymbol{\mathcal{R}}_1 \begin{bmatrix} \boldsymbol{\eta}(u) \\ \dot{\boldsymbol{\eta}}(u) \end{bmatrix}\mathrm{d}u\mathrm{d}s, \\
V_4 &= \frac{h_{\mathrm{M}}^2}{2}\int_{t-h_{\mathrm{M}}}^{t}\int_{s}^{t}\int_{u}^{t}\dot{\boldsymbol{\eta}}^{\mathrm{T}}(v)\boldsymbol{R}_2\dot{\boldsymbol{\eta}}(v)\mathrm{d}v\mathrm{d}u\mathrm{d}s.
\end{aligned}$$

The time derivative of V_1 is calculated as

$$\begin{aligned}
\dot{V}_1 &= 2\begin{bmatrix} \boldsymbol{\eta}(t) \\ \boldsymbol{\eta}(t-h_{\mathrm{M}}) \\ \int_{t-h_{\mathrm{M}}}^{t}\boldsymbol{\eta}(s)\mathrm{d}s \end{bmatrix}^{\mathrm{T}} \boldsymbol{\mathcal{P}} \begin{bmatrix} \dot{\boldsymbol{\eta}}(t) \\ \dot{\boldsymbol{\eta}}(t-h_{\mathrm{M}}) \\ \boldsymbol{\eta}(t)-\boldsymbol{\eta}(t-h_{\mathrm{M}}) \end{bmatrix} \\
&= 2\begin{bmatrix} \boldsymbol{\eta}(t) \\ \boldsymbol{\eta}(t-h_{\mathrm{M}}) \\ \int_{t-h(t)}^{t}\boldsymbol{\eta}(s)\mathrm{d}s + \int_{t-h_{\mathrm{M}}}^{t-h(t)}\boldsymbol{\eta}(s)\mathrm{d}s \end{bmatrix}^{\mathrm{T}} \\
&\quad \times \boldsymbol{\mathcal{P}} \begin{bmatrix} \dot{\boldsymbol{\eta}}(t) \\ \dot{\boldsymbol{\eta}}(t-h_{\mathrm{M}}) \\ \boldsymbol{\eta}(t)-\boldsymbol{\eta}(t-h_{\mathrm{M}}) \end{bmatrix} \\
&= \boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\Xi}_1\boldsymbol{\zeta}(t).
\end{aligned} \tag{19}$$

By calculation of the time-derivative of V_2, we have

$$\begin{aligned}
\dot{V}_2 &\leq \boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{\eta}(t) - (1-h_{\mathrm{D}})\boldsymbol{\eta}^{\mathrm{T}}(t-h(t)) \\
&\quad \times \boldsymbol{Q}_1\boldsymbol{\eta}(t-h(t)) \\
&\quad + \begin{bmatrix} \boldsymbol{\eta}(t) \\ \dot{\boldsymbol{\eta}}(t) \end{bmatrix}^{\mathrm{T}} \boldsymbol{\mathcal{Q}}_2 \begin{bmatrix} \boldsymbol{\eta}(t) \\ \dot{\boldsymbol{\eta}}(t) \end{bmatrix} \\
&\quad - \begin{bmatrix} \boldsymbol{\eta}(t-h_{\mathrm{M}}) \\ \dot{\boldsymbol{\eta}}(t-h_{\mathrm{M}}) \end{bmatrix}^{\mathrm{T}} \boldsymbol{\mathcal{Q}}_2 \begin{bmatrix} \boldsymbol{\eta}(t-h_{\mathrm{M}}) \\ \dot{\boldsymbol{\eta}}(t-h_{\mathrm{M}}) \end{bmatrix} \\
&= \boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\Xi}_2\boldsymbol{\zeta}(t).
\end{aligned} \tag{20}$$

By utilizing Eq. (10) of Lemma 1, an upper bound for the time derivative of V_3 can be estimated as

$$\begin{aligned}
\dot{V}_3 &= \begin{bmatrix} \boldsymbol{\eta}(t) \\ \dot{\boldsymbol{\eta}}(t) \end{bmatrix}^{\mathrm{T}} (h_{\mathrm{M}}^2\boldsymbol{\mathcal{R}}_1) \begin{bmatrix} \boldsymbol{\eta}(t) \\ \dot{\boldsymbol{\eta}}(t) \end{bmatrix} \\
&\quad - h_{\mathrm{M}}\int_{t-h(t)}^{t}\begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix}^{\mathrm{T}} \boldsymbol{\mathcal{R}}_1 \begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix}\mathrm{d}s \\
&\quad - h_{\mathrm{M}}\int_{t-h_{\mathrm{M}}}^{t-h(t)}\begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix}^{\mathrm{T}} \boldsymbol{\mathcal{R}}_1 \begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix}\mathrm{d}s \\
&\leq \begin{bmatrix} \boldsymbol{\eta}(t) \\ \dot{\boldsymbol{\eta}}(t) \end{bmatrix}^{\mathrm{T}} \left(h_{\mathrm{M}}^2\boldsymbol{\mathcal{R}}_1\right) \begin{bmatrix} \boldsymbol{\eta}(t) \\ \dot{\boldsymbol{\eta}}(t) \end{bmatrix} \\
&\quad - \begin{bmatrix} \int_{t-h(t)}^{t}\begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix}\mathrm{d}s \\ \int_{t-h_{\mathrm{M}}}^{t-h(t)}\begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix}\mathrm{d}s \end{bmatrix}^{\mathrm{T}} \\
&\quad \times \begin{bmatrix} \frac{1}{\alpha(t)}\boldsymbol{\mathcal{R}}_1 & \boldsymbol{0}_{4n} \\ \boldsymbol{0}_{4n} & \frac{1}{1-\alpha(t)}\boldsymbol{\mathcal{R}}_1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} \int_{t-h(t)}^{t}\begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix}\mathrm{d}s \\ \int_{t-h_{\mathrm{M}}}^{t-h(t)}\begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix}\mathrm{d}s \end{bmatrix},
\end{aligned} \tag{21}$$

where $\alpha(t) = h(t)h_{\mathrm{M}}^{-1}$.

By using the reciprocally convex approach,[17] we find that the following inequality for any matrix $\boldsymbol{\mathcal{S}}$ and $0 < \alpha(t) < 1$ holds:

$$\begin{bmatrix} \sqrt{\frac{1-\alpha(t)}{\alpha(t)}}\boldsymbol{I}_{4n} & \boldsymbol{0}_{4n} \\ \boldsymbol{0}_{4n} & -\sqrt{\frac{\alpha(t)}{1-\alpha(t)}}\boldsymbol{I}_{4n} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mathcal{R}}_1 & \boldsymbol{\mathcal{S}} \\ \boldsymbol{\mathcal{S}}^{\mathrm{T}} & \boldsymbol{\mathcal{R}}_1 \end{bmatrix}$$

$$\times \begin{bmatrix} \sqrt{\dfrac{1-\alpha(t)}{\alpha(t)}}\boldsymbol{I}_{4n} & \boldsymbol{0}_{4n} \\ \boldsymbol{0}_{4n} & -\sqrt{\dfrac{\alpha(t)}{1-\alpha(t)}}\boldsymbol{I}_{4n} \end{bmatrix} \geq \boldsymbol{0}, \quad (22)$$

which implies

$$\begin{bmatrix} \dfrac{1}{\alpha(t)}\boldsymbol{\mathcal{R}}_1 & \boldsymbol{0}_{4n} \\ \boldsymbol{0}_{4n} & \dfrac{1}{1-\alpha(t)}\boldsymbol{\mathcal{R}}_1 \end{bmatrix} \geq \begin{bmatrix} \boldsymbol{\mathcal{R}}_1 & \boldsymbol{\mathcal{S}} \\ \boldsymbol{\mathcal{S}}^{\mathrm{T}} & \boldsymbol{\mathcal{R}}_1 \end{bmatrix}. \quad (23)$$

Thus, an upper bound of $\dot{V}_3$ is

$$\begin{aligned} \dot{V}_3 \leq & \begin{bmatrix} \boldsymbol{\eta}(t) \\ \dot{\boldsymbol{\eta}}(t) \end{bmatrix}^{\mathrm{T}} (h_{\mathrm{M}}^2 \boldsymbol{\mathcal{R}}_1) \begin{bmatrix} \boldsymbol{\eta}(t) \\ \dot{\boldsymbol{\eta}}(t) \end{bmatrix} \\ & - \begin{bmatrix} \int_{t-h(t)}^{t} \begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix} \mathrm{d}s \\ \int_{t-h_{\mathrm{M}}}^{t-h(t)} \begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix} \mathrm{d}s \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{\mathcal{R}}_1 & \boldsymbol{\mathcal{S}} \\ \boldsymbol{\mathcal{S}}^{\mathrm{T}} & \boldsymbol{\mathcal{R}}_1 \end{bmatrix} \\ & \times \begin{bmatrix} \int_{t-h(t)}^{t} \begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix} \mathrm{d}s \\ \int_{t-h_{\mathrm{M}}}^{t-h(t)} \begin{bmatrix} \boldsymbol{\eta}(s) \\ \dot{\boldsymbol{\eta}}(s) \end{bmatrix} \mathrm{d}s \end{bmatrix} \\ = & \boldsymbol{\zeta}^{\mathrm{T}}(t) \boldsymbol{\Xi}_3 \boldsymbol{\zeta}(t). \end{aligned} \quad (24)$$

Finally, by utilizing Eq. (11) of Lemma 1, the time-derivative of V_4 can be bounded as

$$\begin{aligned} \dot{V}_4 \leq & \left(\frac{h_{\mathrm{M}}^2}{2}\right)^2 \dot{\boldsymbol{\eta}}^{\mathrm{T}}(t) \boldsymbol{R}_2 \dot{\boldsymbol{\eta}}(t) \\ & - \left(\int_{t-h_{\mathrm{M}}}^{t} \int_{s}^{t} \dot{\boldsymbol{\eta}}(u) \mathrm{d}u \mathrm{d}s\right)^{\mathrm{T}} \\ & \times \boldsymbol{R}_2 \left(\int_{t-h_{\mathrm{M}}}^{t} \int_{s}^{t} \dot{\boldsymbol{\eta}}(u) \mathrm{d}u \mathrm{d}s\right) \\ = & \left(\frac{h_{\mathrm{M}}^2}{2}\right)^2 \dot{\boldsymbol{\eta}}^{\mathrm{T}}(t) \boldsymbol{R}_2 \dot{\boldsymbol{\eta}}(t) - \left(\boldsymbol{\eta}(t) - \int_{t-h(t)}^{t} \boldsymbol{\eta}(s) \mathrm{d}s \right. \\ & \left. - \int_{t-h_{\mathrm{M}}}^{t-h(t)} \boldsymbol{\eta}(s) \mathrm{d}s\right)^{\mathrm{T}} \boldsymbol{R}_2 \\ & \times \left(\boldsymbol{\eta}(t) - \int_{t-h(t)}^{t} \boldsymbol{\eta}(s) \mathrm{d}s - \int_{t-h_{\mathrm{M}}}^{t-h(t)} \boldsymbol{\eta}(s) \mathrm{d}s\right) \\ = & \boldsymbol{\zeta}^{\mathrm{T}}(t) \boldsymbol{\Xi}_4 \boldsymbol{\zeta}(t). \end{aligned} \quad (25)$$

Inequalities (18)–(25) lead to a new upper bound for the time derivative of V

$$\dot{V} \leq \boldsymbol{\zeta}^{\mathrm{T}}(t) \boldsymbol{\Phi} \boldsymbol{\zeta}(t). \quad (26)$$

System (8) with the augmented vector $\boldsymbol{\zeta}(t)$ can be rewritten as

$$\boldsymbol{\Upsilon}_m \boldsymbol{\zeta}(t) = \boldsymbol{0}. \quad (27)$$

Therefore, a stability condition for system (8) is

$$\boldsymbol{\zeta}^{\mathrm{T}}(t) \boldsymbol{\Phi} \boldsymbol{\zeta}(t) < \boldsymbol{0}, \quad \text{subject to } \boldsymbol{\Upsilon}_m \boldsymbol{\zeta}(t) = \boldsymbol{0}. \quad (28)$$

According to Lemma 2, condition (28) is equivalent to the following inequality:

$$[\boldsymbol{\Upsilon}_m^{\perp}]^{\mathrm{T}} \boldsymbol{\Phi} [\boldsymbol{\Upsilon}_m^{\perp}] < \boldsymbol{0}. \quad (29)$$

From inequalities (29), by the use of Lemma 2, if LMI (16) satisfies, then stability condition (28) holds. This complete our proof.

When the information about $\dot{h}(t)$ is unknown, then, by setting $\boldsymbol{Q}_1 = \boldsymbol{0}_{2n}$ in the Lyapunov–Krasovskii functional (18), we can obtain the following corollary.

Corollary 1 For a given scalar $0 < h_{\mathrm{M}}$, the agents in system (8) converge to the state of the leader asymptotically, if there exist positive definite matrices $\boldsymbol{\mathcal{P}} \in \mathbb{R}^{6n \times 6n}$, $\boldsymbol{\mathcal{Q}}_2 \in \mathbb{R}^{4n \times 4n}$, $\boldsymbol{\mathcal{R}}_1 \in \mathbb{R}^{4n \times 4n}$, $\boldsymbol{R}_2 \in \mathbb{R}^{2n \times 2n}$, and any matrix $\boldsymbol{\mathcal{S}} \in \mathbb{R}^{4n \times 4n}$ satisfying the following LMIs:

$$(\boldsymbol{\Upsilon}_m^{\perp})^{\mathrm{T}} (\boldsymbol{\Phi} - \boldsymbol{\Xi}_2) (\boldsymbol{\Upsilon}_m^{\perp}) < \boldsymbol{0}, \quad (30)$$

$$\begin{bmatrix} \boldsymbol{\mathcal{R}}_1 & \boldsymbol{\mathcal{S}} \\ \boldsymbol{\mathcal{S}}^{\mathrm{T}} & \boldsymbol{\mathcal{R}}_1 \end{bmatrix} \geq \boldsymbol{0}, \quad (31)$$

where $\boldsymbol{\Phi}$, $\boldsymbol{\Xi}_2$, and $\boldsymbol{\Upsilon}_m$ are defined in Eq. (15).

Proof The proof of Corollary 1 is very similar to that of Theorem 1, so it is omitted here.

Remark 2 In Refs. [12] and [16], the consensus criteria for the MASs with switching interconnection topologies were considered with the time-invariant delays. However, the time-varying delay handled in this paper is more practical. Furthermore, the presented criteria in this paper are less conservative than the results in Ref. [12]. By numerical examples, the less conservatism will be shown in the next section.

4. Numerical examples

In this section, two numerical examples are given to illustrate the effectiveness of the proposed stability criteria.

Example 1 Consider the MAS (8) with the switching interconnection topology described in Fig. 2.

Clearly, the leader is globally reachable. We choose the control gain $\boldsymbol{K} = \mathrm{diag}\{5,5,5,5\}$. From Fig. 2, $\boldsymbol{A}_m$, $\boldsymbol{B}_m$, and $\boldsymbol{D}_m$ at switching interconnection topology m are

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.1 \\ 0 & 0.7 & 0 & 0 \\ 0.3 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{D}_1 = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix},$$

for topology 1, and

$$\boldsymbol{A}_2 = \begin{bmatrix} 0 & 0 & 0.4 & 0 \\ 0.6 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.7 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{D}_2 = \begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix},$$

for topology 2. In Ref. [12], for the above networks, the obtained upper bound on time-invariant delay was 2.491. However, when $h_{\mathrm{D}} = 0$, our obtained result by Theorem 1 is 14.43, which is much larger than that of Ref. [12]. By using Theorem 1 and Corollary 1, we obtain the results of upper bound of time delay with different h_{D}, as listed in Table 1. In order to confirm these results, we simulate positions and velocities of the agents of the network with time-varying delay and switching interconnection topology ($N = 1, 2$) with conditions of time-delay given in Table 2. The switching period of the interconnection topology is 2 s. The simulation results are given in Figs. 3–5. The figures show that each agent converges to the position of the leader for given chosen initial states of the agents.

Table 1. Upper bounds of time-delay for different h_{D} in example 1.

h_{D}	0	0.2	0.5	0.8	unknown
h_{M}	14.43	9.79	8.56	8.37	8.37

Table 2. Conditions of simulation in example 1.

No.	h_{M}	h_{D}	$h(t)$
C1-1	14.43	0	14.43
C1-2	8.56	0.5	$8.56\sin^2(0.05t)$
C1-3	8.37	unknown	$8.37\lvert\sin(t)\rvert$

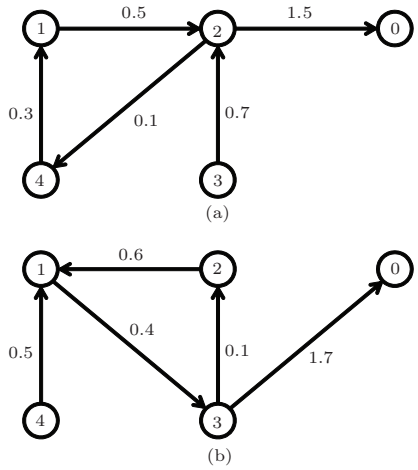


Fig. 2. Topologies (a) 1 and (b) 2 used in example 1.

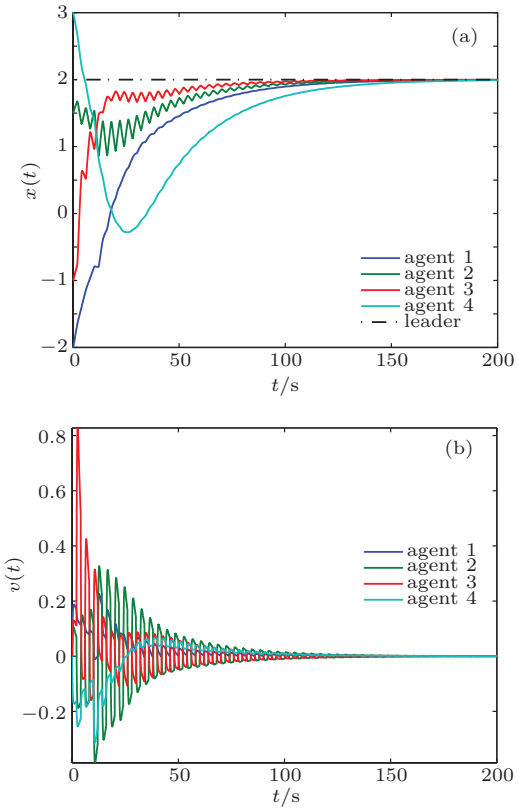


Fig. 3. (colour online) (a) Positions and (b) velocities of the agents with condition C1-1.

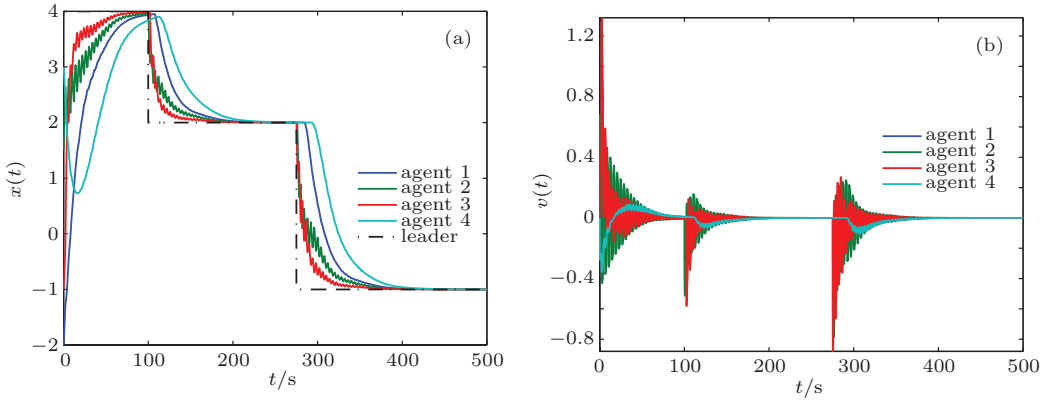


Fig. 4. (colour online) (a) Positions and (b) velocities of the agents with condition C1-2.

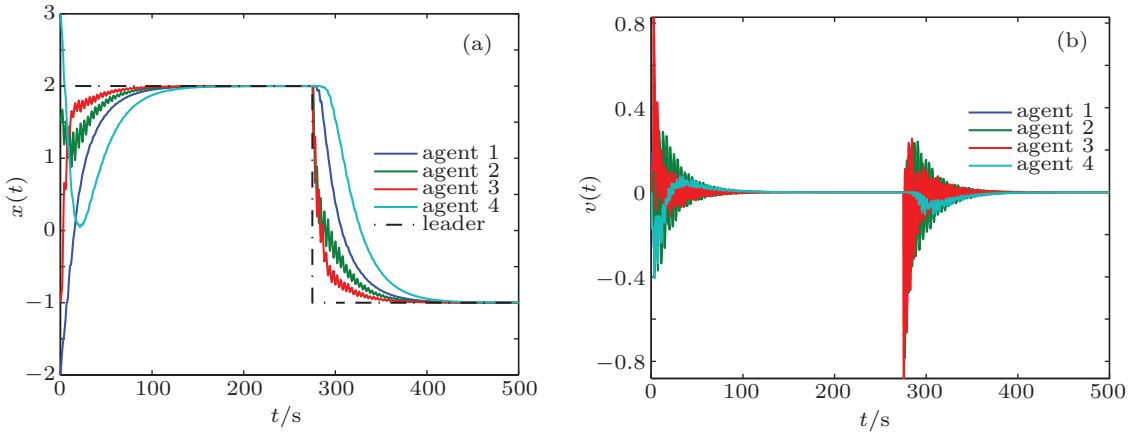


Fig. 5. (colour online) (a) Positions and (b) velocities of the agents with condition C1-3.

Example 2 Consider the MAS (8) with the switching interconnection topology described in Fig. 6. We choose the control gain $\boldsymbol{K} = \mathrm{diag}\{k_1, \ldots, k_8\}$, where $k_i = 5$ $(i = 1, \ldots, 8)$. From Fig. 6, $\boldsymbol{A}_m$, $\boldsymbol{B}_m$, and $\boldsymbol{D}_m$ at switching interconnection topology m are

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 \\ 0 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{B}_1 = \mathrm{diag}\{0, 0, 0, 1.2, 0, 0, 0, 0\},$$

$$\boldsymbol{D}_1 = \mathrm{diag}\{0.2, 0.7, 0.1, 0.3, 1.5, 0.1, 0.5, 0.4\},$$

for topology 1, and

$$\boldsymbol{B}_2 = \mathrm{diag}\{0, 0, 1.9, 0, 0, 0, 0, 0\},$$

$$\boldsymbol{D}_2 = \mathrm{diag}\{0.7, 0.2, 1.1, 0.7, 1.1, 0.1, 0.8, 0.3\},$$

$$\boldsymbol{A}_2 = \begin{bmatrix} 0 & 0.2 & 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0.7 & 0 \\ 0.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

for topology 2. The results of the upper bound on time-delay with different h_{D} are listed in Table 3. It is assumed that the time delays satisfy the conditions listed in Table 4, and the switching period of the interconnection topology is 5 s. Then, the simulation results for positions and velocities of the agents of network are shown in Figs. 7 and 8, which show that all the agents follow the position of the leader.

Table 3. Upper bounds of time-delay for different h_{D} in example 2.

h_{D}	0	0.2	0.5	0.8	unknown
h_{M}	15.24	10.71	8.76	8.57	8.57

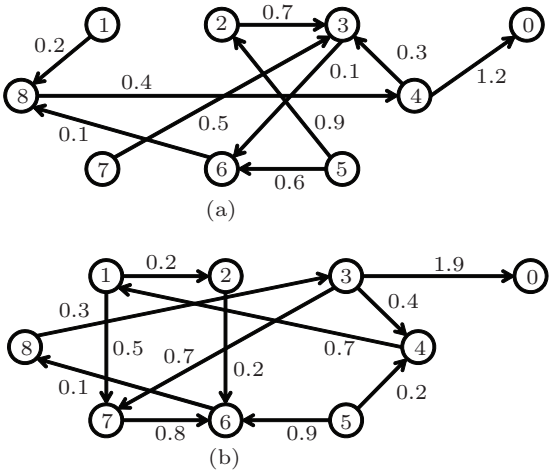


Fig. 6. Topologies (a) 1 and (b) 2 used in example 2.

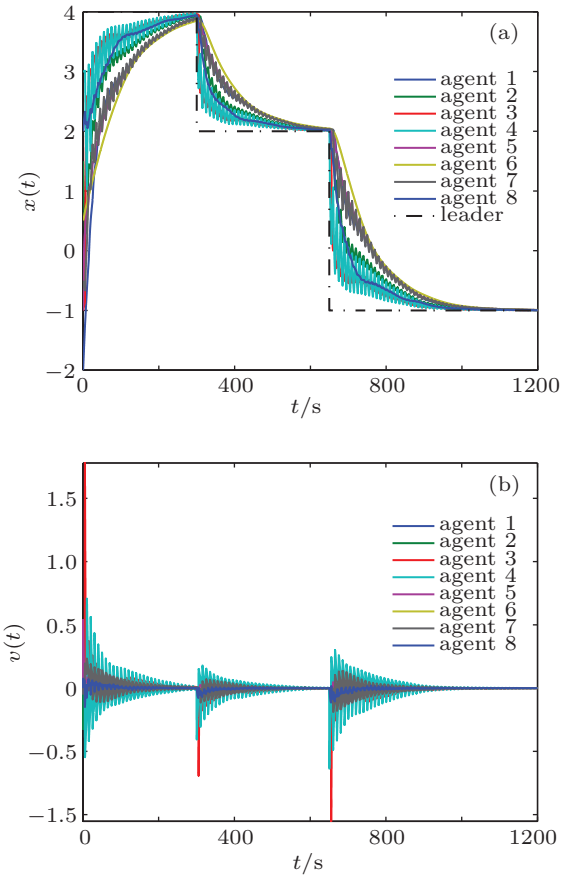


Fig. 7. (colour online) (a) Positions and (b) velocities of the agents with condition C2-1.

Table 4. Conditions of simulation in example 2.

No.	h_{M}	h_{D}	$h(t)$
C2-1	10.71	0.2	$10.71\sin^2(0.01t)$
C2-2	8.57	unknown	$8.57\lvert\sin(t)\rvert$

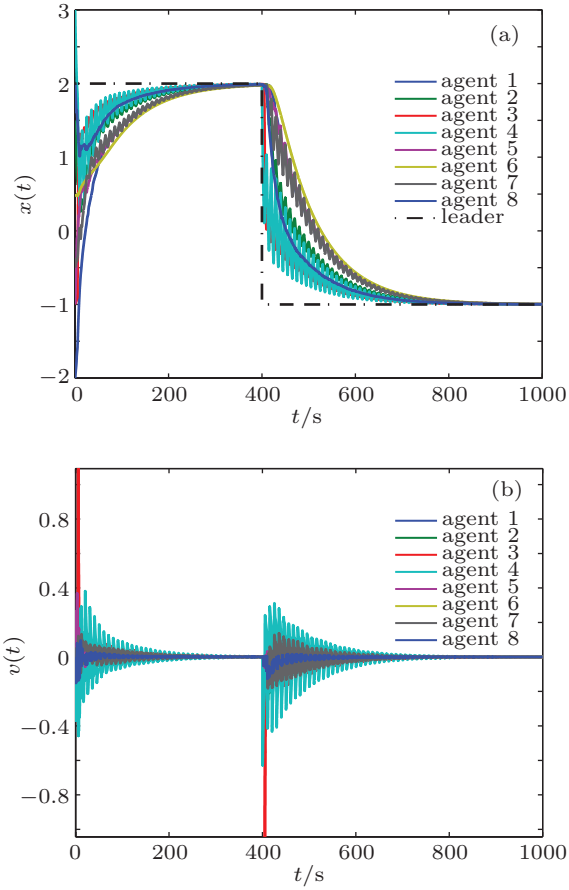


Fig. 8. (colour online) (a) Positions and (b) velocities of the agents with condition C2-2.

5. Conclusion

In this paper, the delay-dependent consensus criteria for the MASs with time-varying delays and switching interconnection topologies are proposed. To do this, a suitable Lyapunov–Krasovskii functional is used to investigate the feasible region of consensus criteria. Two numerical examples have been given to show the effectiveness and usefulness of the presented criteria.

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