Non-completely elastic interactions in a (2+1)-dimensional dispersive long wave equation*

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With the help of a modified mapping method, we obtain two kinds of variable separation solutions with two arbitrary functions for the (2+1)-dimensional dispersive long wave equation. When selecting appropriate multi-valued functions in the variable separation solution, we investigate the interactions among special multi-dromions, dromion-like multi-peakons, and dromion-like multi-semifoldons, which all demonstrate non-completely elastic properties.

Keywords: modified mapping method, dispersive long wave equation, variable separation solution, exotic interaction between special solitons

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1. Introduction

In the recent decades, there has been a surge of interest in obtaining exact analytical solutions of the nonlinear evolution equations (NLEEs). As one of the important methods in the nonlinear domain, the variable separation approach (VSA) has been extensively investigated. Among those methods, the VSA based on the mapping method has been established and extensively applied to solve various NLEEs.[1−5] Moreover, many direct methods based on different mapping equations, including the improved projective approach,[6−9] the q-deformed hyperbolic function method,[10] and the projective Ricatti equation method (PREM),[11] have been constructed to realize the variable separation for NLEEs.

In (2+1)-dimensional NLEEs, the solitons exhibit colorful localized structures. Many single-valued localized structures (dromions, peakons, compactons, *etc.*) have been extensively investigated.[1−9] Multi-valued localized structures, such as foldons, are related to complicated folded phenomena, including folded proteins,[12] folded brain and skin surfaces, and many other kinds of folded biologic systems.[13] Moreover, semifolded structures can also be realized. That is, ocean waves may fold in one direction, say x, and localize in a usual single-valued way in another direction, say y.

It is well-known that the interactions between solitons may be elastic or non-elastic. The interaction behaviors between solitons in (2+1)-dimensional nonlinear models are usually considered to be completely elastic, which means that the amplitude, the velocity, and the shape of a soliton do not undergo any change after the nonlinear interaction. Otherwise, the interaction between solitons is non-elastic (not completely elastic or completely non-elastic). Like the collisions between two classical particles, a collision in which the solitons stick together is called completely non-elastic. Many researchers have discussed these interaction behaviors. For example, the completely elastic interactions between dromions[2] and between dromion-solitoffs[6,7] have been reported. The non-completely elastic interactions between peakons and semifoldons[11] have been investigated. However, the interactions among some multi-soliton structures of dromions, peakons, and foldons are hardly discussed.

Now some important and interesting issues arise. Whether other mapping equations can be used to obtain variable separation solutions of some (2+1)-dimensional nonlinear physics systems? Based on these variable separation solutions, can we discuss some new dynamical behaviors among some multi-

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soliton structures of dromions, peakons, and foldons? Motivated by these issues, we consider the following (2+1)-dimensional dispersive long wave equation (DLWE):

$$\begin{aligned} &u_{ty} + v_{xx} + u_x u_y + u u_{xy} = 0, \\ &v_t + (uv)_x + u_{xxy} = 0, \end{aligned} \tag{1}$$

which was introduced by Boiti[14] as a compatibility for a weak lax pair. The (1+1)-dimensional DLWE ($y = x$ of Eq. (1)) is called the classical Boussinesq equation. There exist a large number of papers discussing the possible applications and the exact solutions of the (1+1)-dimensional DLWE.[15] In Ref. [16], Paquin and Winternitz showed that the symmetry algebra of Eq. (1) is infinite-dimensional and has the Kac–Moody–Virasoro structure. The variable separation solutions of Eq. (1) have been obtained by the general projective Riccati equation method.[17]

2. Variable separation solutions

In order to solve Eq. (1), first, let us make a transformation for Eq. (1): $v = u_y$. Substituting the transformation into Eq. (1) yields

$$u_{ty} + (u_x u)_y + u_{xxy} = 0. \tag{2}$$

Next we suppose that Eq. (2) has the following formal solution:

$$u = a_0 + a_1\phi(R) + a_2\sqrt{[A\phi(R) - a][B\phi(R) - b]}, \tag{3}$$

where a_0, a_1, a_2, and R are functions of $\{x, y, t\}$ to be determined. The ϕ satisfies a new project equation[18]

$$\mathrm{d}\phi/\mathrm{d}R = (A\phi - a)(B\phi - b), \tag{4}$$

with the general solution

$$\phi = \frac{b\exp[(aB - Ab)R] - a\exp[C_1(Ab - aB)]}{B\exp[(aB - Ab)R] - A\exp[C_1(Ab - aB)]}, \tag{5}$$

where C_1 is an integration constant, and A, B, a, and b are arbitrary constants. Inserting Eqs. (3) and (4) into Eq. (2), selecting the variable separation ansatz

$$R = p(x,t) + q(y), \tag{6}$$

and eliminating all the coefficients of the powers of ϕ^i and $\sqrt{(A\phi - a)(B\phi - b)}$, we obtain a set of partial differential equations, from which we derive two special solutions, namely,

$$a_0 = [(Ab + Ba)p_x^2 - p_t - p_{xx}]/p_x,$$

$$a_1 = -2ABp_x, \quad a_2 = 0, \tag{7}$$

and

$$\begin{aligned} &a_0 = -(p_t + p_{xx})/p_x, \quad a_1 = -ABp_x, \\ &a_2 = \pm\sqrt{AB}p_x, \quad Ab + Ba = 0, \end{aligned} \tag{8}$$

where $p \equiv p(x,t)$ and $q \equiv q(y)$.

Therefore, from Eqs. (3) and (5)–(8), we can derive the variable separation solutions of the (2+1)-dimensional DLWE. According to Eqs. (3), (5), (6), and (8), the physical quantity v reads

$$\begin{aligned} v &= u_y \\ &= \left\{ -\frac{p_t + p_{xx}}{p_x} \right. \\ &\quad \left. + abp_x \left[\frac{\Gamma_+}{\Gamma_-} \pm \sqrt{\left(\frac{\Gamma_+}{\Gamma_-} + 1\right)\left(\frac{\Gamma_+}{\Gamma_-} - 1\right)} \right] \right\}_y \\ &= -2A^2b^2Bp_xq_y \exp[-2Ab(p+q)] \\ &\quad \times \left[\frac{1}{\Gamma_-} + \frac{\Gamma_+}{\Gamma_-^2} - \frac{4A\exp(2AbC_1)\Gamma_+}{\Gamma_-^3\sqrt{\left(\frac{\Gamma_+}{\Gamma_-} + 1\right)\left(\frac{\Gamma_+}{\Gamma_-} - 1\right)}} \right], \end{aligned} \tag{9}$$

where $\Gamma_\pm = A\exp(2AbC_1) \pm B\exp[-2Ab(p+q)]$, $p \equiv p(x,t)$, and $q \equiv q(y)$.

3. The non-completely elastic interactions

In this section, we will focus on the interaction between special multi-solitons for the physical quantity v expressed by Eq. (9). Firstly, we discuss three special multi-soliton structures, i.e., special multi-dromions, dromion-like multi-peakons, and dromion-like multi-semifoldons, by introducing the multi-valued function

$$\begin{aligned} &p_x = 0.5\mathrm{sech}^2(\zeta - 0.5t), \\ &x = \zeta - C\tanh(\zeta - 0.5t), \end{aligned} \tag{10}$$

with

$$q = \sum_{j=-N}^{N} b_j \tanh(l_j y + y_{0j}), \tag{11}$$

where C is a characteristic parameter that determines the localized structure. From Eq. (10), we can know that ζ is a multi-valued function in some suitable regions of x, and function p_x is a multi-valued function of x in these areas, though it is a single-valued function of ζ. From Eq. (11), we can construct $(2N + 1)$ dromions. Here we choose $N = 1, b_j = 1, l_j = 0.5$, and $y_{0j} = 4j$.

Figure 1 describes these special multi-soliton structures, i.e., special multi-dromions, dromion-like multi-peakons, and dromion-like multi-semifoldons with $C = 0.5, 0.95, 1.5$, respectively. They localize as three dromions with three up peaks in the y direction and bell-like soliton, peakon, and loop soliton in the x direction, respectively.

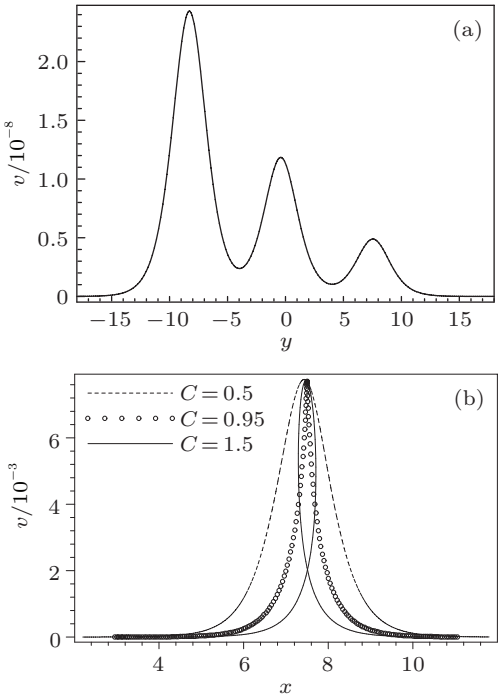


Fig. 1. Sectional views of special multi-solitons at (a) $x = 0$ and (b) $y = 0$ with parameters $A = 2, B = 1, C_1 = 1, a = 0.5, b = -0.25$ at time $t = 15$.

Next, let us study the interaction among these special multi-solitons by setting

$$p_x = 0.5\mathrm{sech}^2(\zeta - 0.5t) + 0.9\mathrm{sech}^2(\zeta + 0.5t),$$
$$x = \zeta - C\tanh(\zeta - 0.5t) - D\tanh(\zeta + 0.5t), \quad (12)$$

with q in Eq. (11), where C and D are characteristic parameters, which determine the types of interaction. From the expression of v and Eqs. (11) and (12), we can obtain two multi-solitons, one is moving along the positive x direction, and the other is moving along the negative x direction.

When choosing the specific values $C = 0.2$ and $D = 1.5$ in Eq. (12), we can successfully construct the interaction between the special multi-dromions and dromion-like multi-semifoldons for the physical quantity v depicted in Fig. 2. From Fig. 2, we can find that the interaction may exhibit a non-completely elastic behavior since solitons' shapes and amplitudes are not completely maintained after the interaction. Prior to the interaction, the velocities of the special multi-dromions and dromion-like multi-semifoldons are set to be $\{v_{01x} = 0.5, \}$ and $\{v_{02x} = -0.5, \}$, respectively. The special multi-dromions are moving along the positive x direction, and the dromion-like multi-semifoldons are moving along the negative x direction, then the head-on collision happens. After the interaction, the final velocities v_{1x} and v_{2x} of the special multi-dromions and dromion-like multi-semifoldons completely maintain their initial velocities $\{v_{1x} = v_{01x} = 0.5\}$ and $\{v_{2x} = v_{02x} = -0.5\}$.

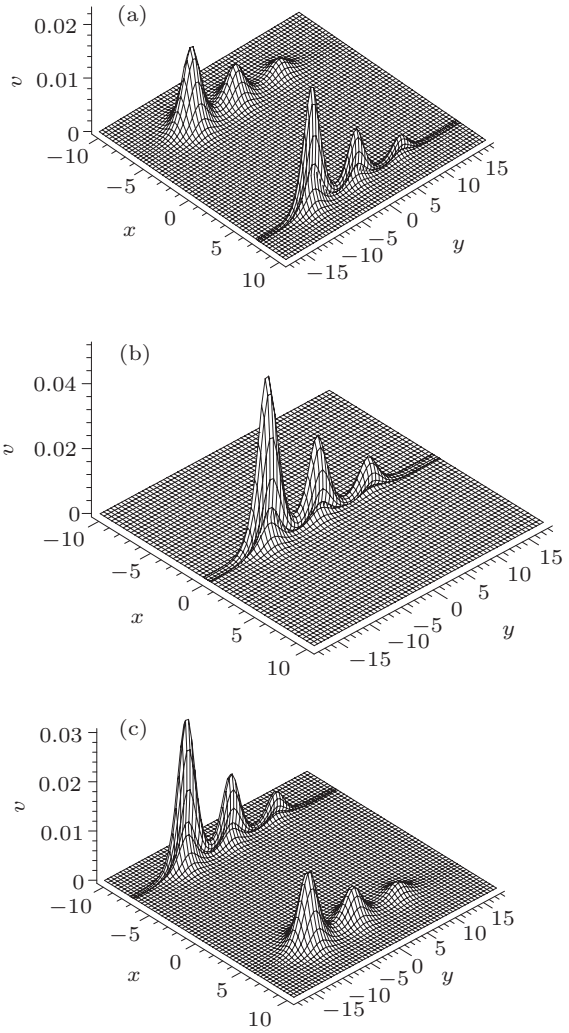


Fig. 2. The interaction between special multi-dromions and dromion-like multi-semifoldons at (a) $t = -15$, (b) $t = 0.1$, and (c) $t = 15$. The parameters are chosen to be $A = -1, B = 0.2, C_1 = 10, a = -2.5, b = -0.5, C = 0.2, D = E = 1.5$.

If we choose the specific values $C = 0.95$ and $D = 1.5$ in Eq. (12), then we can successfully obtain the interaction between dromion-like multi-peakons and dromion-like multi-semifoldons. When we set the specific values $C = D = 1.5$ in Eq. (12),

we can discuss the interaction between dromion-like multi-semifoldons. These interactions are also non-completely elastic, since solitons' shapes and amplitudes are not completely maintained after the interaction (Figs. 3 and 4). Moreover, the final velocities of the special multi-solitons also maintain their initial velocities.

Similarly, we can discuss the interaction between special multi-dromions and dromion-like multi-peakons by setting the specific values $C = 0.2$ and $D = 0.95$ in Eq. (12). This case also shows the non-completely elastic interaction. Due to the limit of length, we omit the detailed discussion about it.

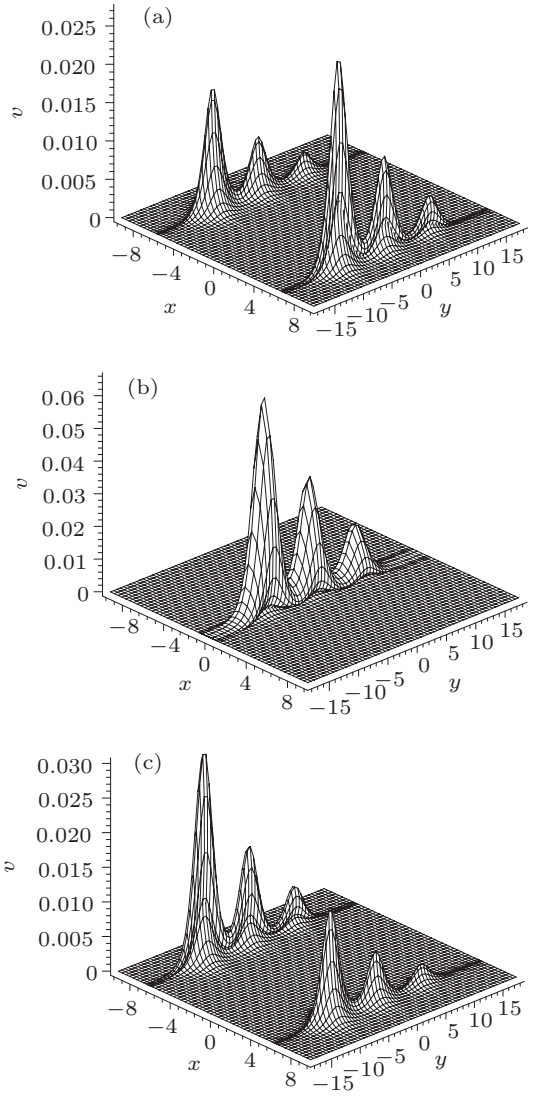


Fig. 3. The interaction between dromion-like multi-peakons and dromion-like multi-semifoldons at (a) $t = -15$, (b) $t = 0.1$, and (c) $t = 15$. The parameters are chosen to be $A = 2, B = 1, C_1 = 1, a = 0.5, b = -0.25, C = 0.95, D = 1.5$.

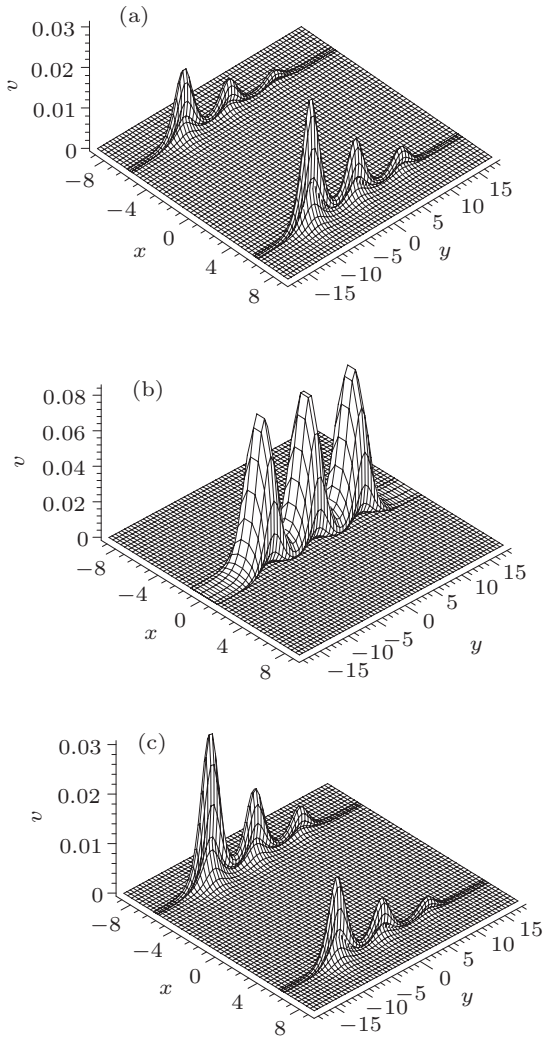


Fig. 4. The interaction between dromion-like multi-semifoldons at (a) $t = -15$, (b) $t = 0$, and (c) $t = 15$. The parameters are chosen to be $A = 2, B = 1, C_1 = 1, a = 0.5, b = -0.25, C = D = 1.5$.

4. Discussion and summary

In summary, with the help of a modified mapping method, we obtain two kinds of variable separation solutions with two arbitrary functions for the (2+1)-dimensional dispersive long wave equation. When selecting appropriate multi-valued functions in the variable separation solution, we investigate the interactions among special multi-dromions, dromion-like multi-peakons, and dromion-like multi-semifoldons, which all demonstrate non-completely elastic properties. Of course, there are some pending issues to be further studied, e.g., how to more suitably quantify the notion of non-complete elasticity? What is the general equation for the distribution of the energy and momentum for these interactions?

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