Chaos suppression of uncertain gyros in a given finite time

Mohammad Pourmahmood Aghababa[a)†] and Hasan Pourmahmood Aghababa[b)c)]

[a)] *Electrical Engineering Department, Urmia University of Technology, Urmia, Iran*

[b)] *Department of Mathematics, University of Tabriz, Tabriz, Iran*

[c)] *Research Center for Industrial Mathematics of University of Tabriz, Tabriz, Iran*

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The gyro is one of the most interesting and everlasting nonlinear dynamical systems, which displays very rich and complex dynamics, such as sub-harmonic and chaotic behaviors. We study the chaos suppression of the chaotic gyros in a given finite time. Considering the effects of model uncertainties, external disturbances, and fully unknown parameters, we design a robust adaptive finite-time controller to suppress the chaotic vibration of the uncertain gyro as quickly as possible. Using the finite-time control technique, we give the exact value of the chaos suppression time. A mathematical theorem is presented to prove the finite-time stability of the proposed scheme. The numerical simulation shows the efficiency and usefulness of the proposed finite-time chaos suppression strategy.

Keywords: chaos suppression, chaotic gyro, finite-time stability, robustness

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1. Introduction

Due to the presence of chaos in different areas of science and engineering, many researchers have been attracted to the control of chaotic behaviors. Chaos possesses some specific features, such as extraordinary sensitivity to initial conditions, broad Fourier transform spectra, fractal properties of the motion in the phase space, and strange attractors. Chaos has become an interesting topic. Irregular and unpredictable vibrations are undesirable in physical devices and suppressing chaos can circumvent damage to physical systems or attenuate the vibratory modes induced by chaos resonance. Since the seminal work of Ott *et al.*,[1] many effective control methodologies have been proposed to suppress chaotic vibrations.[2−8]

The gyro is one of the most interesting and everlasting dynamical systems. Gyros have found various applications in optics, navigation, aeronautics, and space engineering. Recent research has recognized different kinds of gyro systems with linear or nonlinear damping features. Furthermore, it has been shown that these systems display a diverse range of dynamic behaviors including both sub-harmonic and chaotic motions.[9−11]

The chaotic vibration of the gyro was firstly introduced by Leipnik and Newton.[12] Recently, the nonlinear dynamics of symmetric heavy gyros mounted on vibrating platforms was investigated.[9,10,13] Moreover, it has been shown that under the base harmonic excitation and a nonlinear damping force, the gyro exhibits the chaotic vibration.[10,11]

In the past few years, some researchers have paid their attention to the chaos synchronization and/or suppression of chaotic gyros. Lei *et al.*[14] have proposed an active control technique for synchronizing two identical gyros. Hung *et al.*[15] have investigated the problem of the generalized projective synchronization of chaotic gyros in the presence of dead-zone nonlinearity in the control input. In Refs. [16] and [17], fuzzy sliding mode controllers were proposed to synchronize the chaotic gyros. Yan *et al.*[18] have used a variable structure control scheme for chaos elimination in a gyro.

However, the above-mentioned works have been proposed to stabilize or synchronize the chaotic gyros asymptotically. In other words, in those works, the chaotic vibrations of the gyros have been suppressed with ab infinite settling time. Nevertheless, from a practical engineering point of view, it is more valuable to suppress the chaotic vibrations of the gyros in a finite time and as quickly as possible. Therefore, it is

†Corresponding author. E-mail: m.p.aghababa@ee.uut.ac.ir

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important to investigate the finite-time chaotic vibration suppression of the gyros. To achieve faster convergence speeds in control systems, finite-time control methods are effective techniques. Finite-time control means optimality in settling time. Moreover, finite-time control techniques have demonstrated better robustness and disturbance rejection properties.[19]

On the other hand, in real world applications, a gyro's parameters are inevitably disturbed by external inartificial factors, such as environmental temperature, voltage oscillation, and mutual interference among components, and cannot be precisely known in advance. Besides, in practical situations, due to the un-modeled dynamics, modeling errors, structural variations of the systems, and environmental and measurement noise, there are usually some uncertainties and external disturbances in the dynamics of chaotic gyros. Therefore, the effects of unknown parameters, model uncertainties, and external disturbances cannot be neglected in the controller design for the vibration suppression of chaotic gyros. However, to our best knowledge, the finite-time chaos suppression of chaotic gyros with model uncertainties, external disturbances, and unknown parameters has received less attention, and the relevant theoretical advances have seldom been reported in the literature. Therefore, designing a robust controller to suppress the chaotic vibration of uncertain gyros is an important topic to be studied.

Motivated by the aforementioned issues, we aim to investigate the robust vibration suppression of uncertain chaotic gyros in a given finite time via an adaptive finite-time control approach. It is assumed that the chaotic gyro is perturbed by unknown model uncertainties, external disturbances, and fully unknown parameters. To undertake the unknown parameters of the gyro system, appropriate adaptive laws are derived. On the basis of the adaptive laws and the finite-time control technique, adaptive finite-time switching control laws are also proposed to guarantee that the chaotic vibrations of the gyro system converge to zero within a given finite time even when all the parameters of the gyro are fully unknown and the model uncertainties and the external disturbances exist in the system dynamics. It is analytically proved that the closed-loop system is stable in a finite time. An illustrative example is presented to demonstrate the efficiency and applicability of the proposed control scheme.

The rest of this paper is organized as follows. The dynamics of a chaotic gyro and the finite-time control problem formulation are described in Section 2. In Section 3, the design procedure of the proposed robust adaptive finite-time controller is given. Numerical simulations are present in Section 4. In Section 5, the conclusion is given.

2. Gyro dynamics and the chaos suppression problem

In this section, a brief description of the nonlinear chaotic behavior of a chaotic gyro is given. Then, the problem of finite-time robust chaos suppression is formulated.

2.1. Dynamics of a chaotic gyro

The motion of a symmetric gyro with linear-plus-cubic damping mounted on a vibrating base in terms of the rotation angle θ, i.e., the angle between the spin axis of the gyro and the vertical axis, is given by[9]

$$\ddot{\theta} + \alpha^2 \frac{(1-\cos\theta)^2}{\sin^3\theta} - \beta\sin\theta + c_1\dot{\theta} + c_2\dot{\theta}^3 = f\sin(\omega t)\sin\theta, \tag{1}$$

where $f\sin(\omega t)$ is a parametric excitation that models the base excitation, $c_1\dot{\theta}$ and $c_2\dot{\theta}^3$ are the linear and the nonlinear damping terms, respectively, and $\alpha^2(1-\cos\theta)^2/\sin^3\theta - \beta\sin\theta$ is nonlinear resilience.

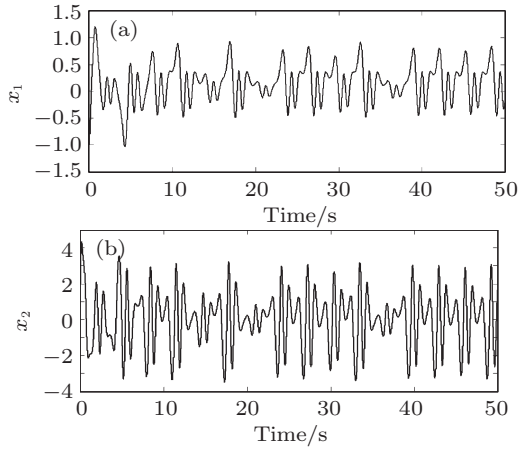


Fig. 1. Chaotic vibrations of the gyro system: (a) x_1, (b) x_2.

The dynamics of this gyro system has been extensively studied for the values of f in the range $32 < f < 36$. In particular, for $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $\omega = 2$, and $f = 35.5$, this gyro exhibits chaotic behavior.[9] The irregular motion and the chaotic vibrations of gyro system (1) with initial

conditions $\theta(0) = -1$ and $\dot{\theta}(0) = 2$ are illustrated in Figs. 1 and 2.

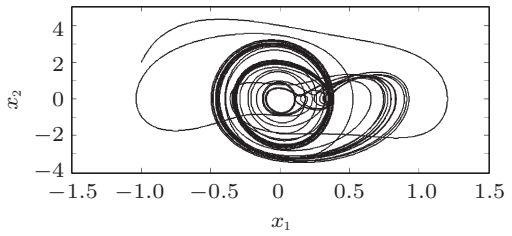


Fig. 2. Phase portrait of the chaotic gyro system.

2.2. Chaos suppression problem formulation

Considering the effects of model uncertainties, external disturbances, unknown parameters, and control inputs, and defining $x_1 = \theta$ and $x_2 = \dot{\theta}$, we can describe the dynamics of the chaotic gyro system (1) by

$$\begin{aligned}\dot{x}_1 &= x_2 + \Delta f_1(\boldsymbol{x}, t) + u_1(t),\\ \dot{x}_2 &= -\alpha^2 \frac{(1-\cos x_1)^2}{\sin^3 x_1} - c_1 x_2 - c_2 x_2^3 + \beta \sin x_1\\ &\quad + f \sin(\omega t) \sin x_1 + \Delta f_2(\boldsymbol{x}, t) + u_2(t), \end{aligned} \tag{2}$$

where $\boldsymbol{x} = [x_1(t), x_2(t)]^{\mathrm{T}}$ is the state vector of the system, $\Delta f_i(\boldsymbol{x}, t)$, $i = 1, 2$ are unknown model uncertainties and external disturbances of the system, and $\boldsymbol{u}(t) = [u_1(t), u_2(t)]^{\mathrm{T}}$ is the control input to be designed later.

Assumption 1 We assume that the uncertainties $\Delta f_i(\boldsymbol{x}, t)$, $i = 1, 2$ are bounded by

$$|\Delta f_i(\boldsymbol{x}, t)| \leq a_i, \ i = 1, 2, \tag{3}$$

where $a_i, \ i = 1, 2$ are given positive constants.

Let $\boldsymbol{\psi} = [\alpha^2, c_1, c_2, |\beta| + |f|]^{\mathrm{T}} = [\psi_1, \psi_2, \psi_3, \psi_4]^{\mathrm{T}}$ be the unknown vector parameter of the gyro system. Then, the following assumption is made.

Assumption 2 It is assumed that all the parameters of the gyro system are unknown in advance and the unknown vector parameter $\boldsymbol{\psi}$ is norm bounded, i.e.,

$$\|\boldsymbol{\psi}\| \leq \Psi, \tag{4}$$

where $\|\cdot\|$ denotes the Euclidean norm, and Ψ is a given positive constant.

The finite-time vibration suppression of the chaotic gyro system means that the chaotic vibrations of the gyro system converge to zero within a finite time. The precise definition of the finite-time vibration suppression is given as follows.

Definition 1 Consider the nonlinear chaotic gyro system described by Eq. (2). If there exists a constant $T = T(\boldsymbol{x}(0)) > 0$, such that

$$\mathrm{Lim}_{t \to T} \|\boldsymbol{x}(t)\| = 0, \tag{5}$$

and $\|\boldsymbol{x}(t)\| \equiv 0$ for $t \geq T$, then the vibration suppression of chaotic gyro system (2) is achieved in a finite time, and the gyro trajectories will not be chaotic after $t \geq T$.

3. Finite-time controller design for chaos suppression

In this section, an adaptive robust finite-time controller is designed to stabilize the chaotic gyro in a finite time (i.e., to suppress the chaotic vibration of the chaotic gyro) with model uncertainties, external disturbances, and completely unknown parameters.

Lemma 1[19] Assume that a continuous positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -cV^{\xi}(t), \ \ \forall t \geq t_0, \ \ V(t_0) \geq 0, \tag{6}$$

where $c > 0$, $0 < \xi < 1$ are two constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$\begin{aligned} V^{1-\xi}(t) \leq V^{1-\xi}(t_0) - c(1-\xi)(t-t_0),\\ t_0 \leq t \leq t_1, \end{aligned} \tag{7}$$

and $V(t) \equiv 0$, $\forall t \geq t_1$ with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\xi}(t_0)}{c(1-\xi)}. \tag{8}$$

In order to guarantee the finite-time stabilization of uncertain chaotic gyro system (2), suitable control laws are derived. The control laws should have the following features.

i) The control inputs should be robust against the system uncertainties. On the other hand, since the system uncertainties are unknown in practical applications, we use only the norm bounds of the uncertainties to construct a robust and reliable controller. Therefore, motivated by Assumption 1, we assume that the control inputs contain the bounds of the system uncertainties.

ii) Another feature of the control laws is that they should be able to undertake the system's unknown parameters. This problem can be solved using the adaptive control theory. Consequently, proper adaptive parameters are introduced in the control laws to adaptively estimate the system's unknown parameters.

iii) Finally, the control laws should be designed in a form ensuring the finite-time stability of the closed-loop system. According to the finite time control theory (Lemma 1), the control inputs should include some terms to satisfy the results of Lemma 1. As a result, we find that a constant gain with a state-normalized term multiplied by the sum of the bounds of the unknown parameters and the adaptive parameters should be employed.

According to the above comments, the following control laws are introduced:

$$\begin{aligned} u_1(t) =& -x_2 - \mu\left(\Psi + \left\|\hat{\boldsymbol{\psi}}\right\|\right)\left(\frac{x_1}{\|\boldsymbol{x}\|^2}\right) \\ & - (a_1 + k_1)\,\mathrm{sgn}(x_1), \\ u_2(t) =& \hat{\psi}_1 \frac{(1-\cos x_1)^2}{\sin^3 x_1} + \hat{\psi}_2 x_2 + \hat{\psi}_3 x_2^3 \\ & - \hat{\psi}_4 \left|\sin x_1\right| \mathrm{sgn}(x_2) \\ & - \mu\left(\Psi + \left\|\hat{\boldsymbol{\psi}}\right\|\right)\left(\frac{x_2}{\|\boldsymbol{x}\|^2}\right) \\ & - (a_2 + k_2)\,\mathrm{sgn}(x_2), \end{aligned} \tag{9}$$

where $\hat{\psi}_1$, $\hat{\psi}_2$, $\hat{\psi}_3$, and $\hat{\psi}_4$ are estimations for unknown parameters ψ_1, ψ_2, ψ_3, and ψ_4, respectively; $\mu = \min\{k_1, k_2\} > 0$; k_1 and k_2 are positive constant gains; if $\boldsymbol{x} = 0$, then $x_i/\|x\|^2 = 0$, $i = 1, 2$; and $\mathrm{sgn}(\cdot)$ is the sign function.

Subsequently, the following adaptive laws are proposed to estimate the unknown parameters:

$$\begin{aligned} &\dot{\hat{\psi}}_1(t) = \frac{-x_2(1-\cos x_1)^2}{\sin^3 x_1},\ \hat{\psi}_1(0) = \hat{\psi}_{10}, \\ &\dot{\hat{\psi}}_2(t) = -x_2^2,\ \hat{\psi}_2(0) = \hat{\psi}_{20}, \\ &\dot{\hat{\psi}}_3(t) = -x_2^4,\ \hat{\psi}_3(0) = \hat{\psi}_{30}, \\ &\dot{\hat{\psi}}_4(t) = \left|x_2\right|\left|\sin x_1\right|,\ \hat{\psi}_4(0) = \hat{\psi}_{40}, \end{aligned} \tag{10}$$

where $\hat{\psi}_{10}$, $\hat{\psi}_{20}$, $\hat{\psi}_{30}$, and $\hat{\psi}_{40}$ are the initial values of the adaptive parameters $\hat{\psi}_1(t)$, $\hat{\psi}_2(t)$, $\hat{\psi}_3(t)$, and $\hat{\psi}_4(t)$, respectively.

Theorem 1 The chaotic vibrations of gyro system (2) with unknown model uncertainties, external disturbances, and fully unknown parameters are suppressed in a given finite time, if this system is controlled by the control signals in Eq. (9) with the adaptive laws in Eq. (10).

Proof Consider the following positive definite function to be a Lyapunov function candidate of the system:

$$V(t) = \frac{1}{2}\left(\|\boldsymbol{x}\|^2 + \left\|\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}\right\|^2\right). \tag{11}$$

Taking the time derivative of $V(t)$, we have

$$\begin{aligned} \dot{V}(t) =& x_1\dot{x}_1 + x_2\dot{x}_2 + \left(\hat{\psi}_1 - \psi_1\right)\dot{\hat{\psi}}_1 + \left(\hat{\psi}_2 - \psi_2\right)\dot{\hat{\psi}}_2 \\ & + \left(\hat{\psi}_3 - \psi_3\right)\dot{\hat{\psi}}_3 + \left(\hat{\psi}_4 - \psi_4\right)\dot{\hat{\psi}}_4. \end{aligned} \tag{12}$$

Inserting $\dot{x}_1$ and $\dot{x}_2$ from Eq. (2) into the above equation gives

$$\begin{aligned} \dot{V}(t) =& x_1\left(x_2 + \Delta f_1(x,t) + u_1(t)\right) \\ & + x_2\left(\frac{-\alpha^2(1-\cos x_1)^2}{\sin^3 x_1} - c_1 x_2 - c_2 x_2^3 \right. \\ & \left. + \beta\sin x_1 + f\sin(\omega t)\sin x_1 + \Delta f_2(x,t) \right. \\ & \left. + u_2(t)\right) + \left(\hat{\psi}_1 - \psi_1\right)\dot{\hat{\psi}}_1 + \left(\hat{\psi}_2 - \psi_2\right)\dot{\hat{\psi}}_2 \\ & + \left(\hat{\psi}_3 - \psi_3\right)\dot{\hat{\psi}}_3 + \left(\hat{\psi}_4 - \psi_4\right)\dot{\hat{\psi}}_4. \end{aligned} \tag{13}$$

It is obvious that

$$\begin{aligned} \dot{V}(t) \le& x_1\left(x_2 + u_1(t)\right) + \left|x_1\right|\left|\Delta f_1(x,t)\right| \\ & + x_2\left(-\alpha^2\frac{(1-\cos x_1)^2}{\sin^3 x_1} - c_1 x_2 \right. \\ & \left. - c_2 x_2^3 + u_2(t)\right) + \left|x_2\right|\left(|\beta| + |f|\right)\left|\sin x_1\right| \\ & + \left|x_2\right|\left|\Delta f_2(x,t)\right| + \left(\hat{\psi}_1 - \psi_1\right)\dot{\hat{\psi}}_1 \\ & + \left(\hat{\psi}_2 - \psi_2\right)\dot{\hat{\psi}}_2 + \left(\hat{\psi}_3 - \psi_3\right)\dot{\hat{\psi}}_3 \\ & + \left(\hat{\psi}_4 - \psi_4\right)\dot{\hat{\psi}}_4. \end{aligned} \tag{14}$$

Since

$$\begin{aligned} &\psi_1\dot{\hat{\psi}}_1 = -\alpha^2 x_2(1-\cos x_1)^2/\sin^3 x_1, \\ &\psi_2\dot{\hat{\psi}}_2 = -c_1 x_2^2, \quad \psi_3\dot{\hat{\psi}}_3 = -c_3 x_2^4, \end{aligned}$$

and $\psi_4\dot{\hat{\psi}}_4 = \left(|\beta| + |f|\right)\left|x_2\right|\left|\sin x_1\right|$, and by Assumption 1, we have

$$\begin{aligned} \dot{V}(t) \le& x_1\left(x_2 + u_1(t)\right) + a_1\left|x_1\right| + x_2 u_2(t) + a_2\left|x_2\right| \\ & + \hat{\psi}_1\dot{\hat{\psi}}_1 + \hat{\psi}_2\dot{\hat{\psi}}_2 + \hat{\psi}_3\dot{\hat{\psi}}_3 + \hat{\psi}_4\dot{\hat{\psi}}_4. \end{aligned} \tag{15}$$

Introducing $u_1(t)$ and $u_2(t)$ from Eq. (9) into the right-hand side of the above inequality, we have

$$\begin{aligned} \dot{V}(t) \le& x_1\left(x_2 + \left[-x_2 - \mu\left(\Psi + \left\|\hat{\boldsymbol{\psi}}\right\|\right)\left(\frac{x_1}{\|\boldsymbol{x}\|^2}\right) \right.\right. \\ & \left.\left. - (a_1 + k_1)\,\mathrm{sgn}(x_1)\right]\right) + a_1\left|x_1\right| \\ & + x_2\left[\hat{\psi}_1\frac{(1-\cos x_1)^2}{\sin^3 x_1} + \hat{\psi}_2 x_2 + \hat{\psi}_3 x_2^3 \right. \\ & - \hat{\psi}_4\left|\sin x_1\right|\mathrm{sgn}(x_2) - \mu\left(\Psi + \left\|\hat{\boldsymbol{\psi}}\right\|\right) \\ & \left. \times\left(\frac{x_2}{\|\boldsymbol{x}\|^2}\right) - (a_2 + k_2)\,\mathrm{sgn}(x_2)\right] \end{aligned}$$

$$+ a_2 |x_2| + \hat{\psi}_1 \dot{\hat{\psi}}_1 + \hat{\psi}_2 \dot{\hat{\psi}}_2 + \hat{\psi}_3 \dot{\hat{\psi}}_3 + \hat{\psi}_4 \dot{\hat{\psi}}_4. \quad (16)$$

Knowing $\hat{\psi}_1 \dot{\hat{\psi}}_1 = -\hat{\psi}_1 x_2 (1 - \cos x_1)^2 / \sin^3 x_1$, $\hat{\psi}_2 \dot{\hat{\psi}}_2 = -\hat{\psi}_2 x_2^2$, $\hat{\psi}_3 \dot{\hat{\psi}}_3 = -\hat{\psi}_3 x_2^4$, $\hat{\psi}_4 \dot{\hat{\psi}}_4 = -\hat{\psi}_4 |\sin x_1| |x_2|$, and $x_1/\|\boldsymbol{x}\|^2 + x_2/\|\boldsymbol{x}\|^2 = 1$ and using $x_i \mathrm{sgn}(x_i) = |x_i|$, we have

$$\dot{V}(t) \leq -k_1 |x_1| - k_2 |x_2| - \mu \left(\Psi + \left\| \hat{\boldsymbol{\psi}} \right\| \right). \quad (17)$$

From a fundamental mathematical fact, we have

$$\dot{V}(t) \leq -\mu \|\boldsymbol{x}\| - \mu \left(\Psi + \left\| \hat{\boldsymbol{\psi}} \right\| \right). \quad (18)$$

Using Assumption 2 and since $\left\| \hat{\boldsymbol{\psi}} - \boldsymbol{\psi} \right\| \leq \left\| \hat{\boldsymbol{\psi}} \right\| + \|\boldsymbol{\psi}\| \leq \left\| \hat{\boldsymbol{\psi}} \right\| + \Psi$ is always satisfied, we have

$$\begin{aligned} \dot{V}(t) &\leq -\mu \left(\|\boldsymbol{x}\| + \left\| \hat{\boldsymbol{\psi}} - \boldsymbol{\psi} \right\| \right) \\ &\leq -\sqrt{2}\mu \left(\frac{1}{2} \left(\|\boldsymbol{x}\|^2 + \left\| \hat{\boldsymbol{\psi}} - \boldsymbol{\psi} \right\|^2 \right) \right)^{1/2} \\ &= -\sqrt{2}\mu V^{1/2}(t). \end{aligned} \quad (19)$$

Hence, from Lemma 1, it can be concluded that gyro system trajectories x_1 and x_2 will converge to zero in the finite time $T = (\sqrt{2}/\mu)((\|x(0)\|^2 + \|\hat{\psi}(0) - \psi\|^2)/2)^{1/2}$. Therefore, the chaotic vibrations of the uncertain gyro system will be suppressed in finite time.

Remark 1 Since the control inputs in Eq. (9) include the terms $\mathrm{sgn}(x_i) = x_i/|x_i|$ and $x_i^2/\|\boldsymbol{x}\|^2$, the undesirable chattering phenomenon may take place. In order to remove the chattering, these terms are modified as $\mathrm{sgn}(x_i) = x_i/(|x_i| + \varepsilon)$ and $x_i^2/(\|\boldsymbol{x}\|^2 + \varepsilon)$, respectively; where $\varepsilon > 0$ is a sufficiently small constant.[20]

Remark 2 If we assume that the parameters of gyro system (1) are known and there are no model uncertainties and external disturbances in the system dynamics, then control laws (9) and (10) are simplified into the following equations:

$$\begin{aligned} u_1(t) &= -x_2 - k_1 \mathrm{sgn}(x_1), \\ u_2(t) &= \alpha^2 \frac{(1 - \cos x_1)^2}{\sin^3 x_1} + c_1 x_2 + c_2 x_2^3 - \beta \sin x_1 \\ &\quad - (|f| |\sin x_1| + k_2) \mathrm{sgn}(x_2). \end{aligned} \quad (20)$$

Choosing a Lyapunov function candidate in the form of $V(t) = \|\boldsymbol{x}\|^2/2$, we can easily prove the finite time stability of the above system.

4. Numerical simulations

In this section, numerical simulations are given to validate the applicability and effectiveness of the proposed finite-time controller in the vibration suppression of the chaotic gyros in spite of model uncertainties, external disturbances, and fully unknown parameters. The simulations are carried out using MATLAB software. The parameters $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $\omega = 2$, and $f = 35.5$ are used in the simulations to guarantee the existence of chaos for the chaotic gyro.[9] Subsequently, the following uncertainties and the external disturbances are considered in the simulation:

$$\begin{aligned} \Delta f_1(\boldsymbol{x}, t) &= 0.15 \cos(4x_1) + 0.2 \sin(2t), \\ \Delta f_2(\boldsymbol{x}, t) &= -0.25 \cos(6x_2) - 0.15 \cos(3t). \end{aligned} \quad (21)$$

As a result, we can obtain that $a_1 = 0.35$ and $a_1 = 0.4$ satisfy Assumption 1. The Ψ is chosen to be 120. The initial values of the gyro are chosen as $x_1(0) = -0.5$ and $x_2(0) = 2.5$. In addition, the initial values of the adaptive parameters $\hat{\psi}_1$, $\hat{\psi}_2$, $\hat{\psi}_3$, and $\hat{\psi}_4$ are all set to 1. Both constant gains k_1 and k_2 are chosen to be 1. And ε is chosen to be 0.001.

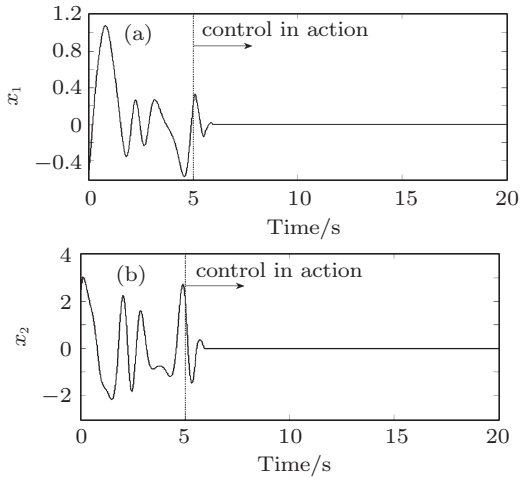


Fig. 3. Vibrations of the gyro system controlled by the finite-time controller: (a) x_1, (b) x_2.

Figure 3 depicts the state trajectories of the uncertain chaotic gyro (2), where the control inputs are turned on at $t = 5$ s. It is observed that when the controller is activated, the chaotic vibrations of the system trajectories converge to zero quickly. This means that with the proposed robust adaptive finite-time controller, the chaotic gyro system with model uncertainties, external disturbances, and fully unknown parameters has been stabilized within a finite time,

and the closed-loop system no longer has chaotic vibration. The time histories of the adaptive parameters $\hat{\psi}_1$, $\hat{\psi}_2$, $\hat{\psi}_3$, and $\hat{\psi}_4$ are illustrated in Fig. 4. We can see that all the adaptive parameters converge to some constants. The used control inputs are shown in Fig. 5. It is seen that the control inputs have no chattering and are feasible to implement in practice.

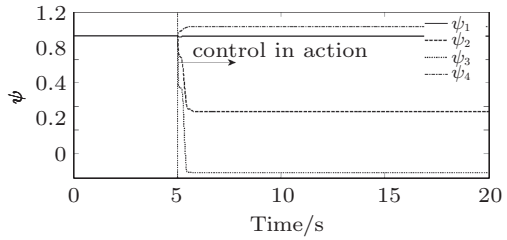


Fig. 4. Time responses of the adaptive vector parameter $\hat{\boldsymbol{\psi}}$.

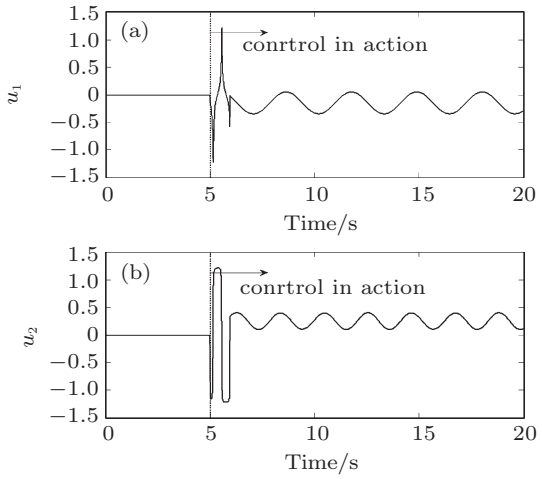


Fig. 5. The control inputs used to suppress the chaotic vibrations of the uncertain gyro: (a) u_1, (b) u_2.

5. Conclusion

In this paper, the finite-time chaos suppression of uncertain gyros is investigated. It is supposed that the parameters of the gyro system are fully unknown in advance and the system is perturbed by unknown model uncertainties and external disturbances. By means of the finite-time control theory and some adaptive laws, a finite-time robust adaptive controller is designed. Numerical simulations indicate that the proposed controller can suppress the chaos of the gyro in a finite time, even when the system parameters are fully unknown and the model uncertainties and the external disturbances are present in the system dynamics.

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