Entanglement concentration for an arbitrary hybrid less-entangled state and W state using quantum dots and a microcavity coupled system*

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We propose a practical entanglement concentration protocol (ECP) for a hybrid entangled state using quantum dots and a microcavity coupled system. A hybrid less-entangled state can be concentrated to a most-entangled state with a certain probability using only one ancillary single photon. Moreover, using this protocol, we can also concentrate an arbitrary three-particle less-entangled W state using two ancillary photons and classical communication. The proposed protocols provide us with a useful method to concentrate less-entangled states, which can be implemented with current technology.

Keywords: entanglement concentration, quantum dot, microcavity, W state

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1. Introduction

Quantum entanglement is a key ingredient of quantum information science and technology. During the past decades, quantum entanglement has provided many useful applications in quantum information processing (QIP), such as quantum key distribution,[1] quantum teleportation,[2] quantum secret sharing,[3] quantum dense coding,[4] and quantum secure direction communication.[5−7] In order to complete QIP, maximally entangled states are necessary. However, the entangled pairs inevitably interact with the environmental noise during the QIP and become less entangled. The participants of QIP can resort to entanglement purification and entanglement concentration to improve the fidelity of entanglement.

Entanglement purification is used to distill a set of mixed entangled states into a set of highly entangled states with local operation and classical communications. Bennett *et al.*[8] proposed the first entanglement purification protocol with CNOT gates and bilateral rotations in 1996. Later, they[9] proposed the first entanglement concentration protocol (ECP) based on the Schmidit projection method, in which entanglement concentration can be used to convert partially entangled pairs into maximally entangled ones. In 2000, Shi *et al.*[10] designed an ECP by exploiting a two-particle collective unitary evaluation. In 2001, Yamamoto *et al.*[11] and Zhao *et al.*[12] proposed two ECPs with polarization beam splitters based on the simplified Schmidt projection method independently. In 2008, Sheng *et al.*[13] proposed an efficient ECP using cross-Kerr nonlinearity. They also designed an ECP for single-photon entangled systems.[14] In 2009, Sheng *et al.*[15] proposed an ECP based on electrons. In 2011, Wang *et al.*[16] proposed a practical ECP using the quantum dot (QD) and microcavity system, in which electron-spin entangled states can be distilled efficiently.

The solid-state system based on the QD–microcavity coupled system is a promising candidate for solid quantum optics and QIP. If the spin coherence time of electrons in the QD is as long as several milliseconds, and if the initialization, manipulation, and measurement of the spin state are easy to carry out with current technology, the QD–microcavity coupled system provides us with a novel way of QIP. In

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2006, Waks *et al.*[17] discussed the interaction of a cavity with a dipole and found that the cavity can be switched perfectly even in the bad cavity limit. The potential application of this system can be further generalized to the realization of a quantum logic gate,[18] photon entangler,[19] entanglement beam splitter,[20] entanglement swapping,[21] and so on. Recently, a single-electron-charged quantum dot in a resonator has been reported, which exhibits a good interaction between a photon and an electron spin.[22] Exploiting this regime, hybrid entanglement between photon and electron spin can be efficiently generated. The hybrid entangled state shows entanglement in different degrees of freedom and provides a novel way of QIP. For example, in Ref. [23], Bonato *et al.* proposed an interface between the spin of a photon and the spin of an electron confined in a quantum dot embedded in a microcavity operating in the weakly-coupled regime. This interface can be used to construct a CNOT gate, multi-photon entangler, and Bell-state analyzer. The protocol for entanglement generators and quantum repeaters based on hybrid entangled states have been reported and experimentally realized.[24−26]

In our previous work, we proposed a hybrid ECP using a single-charged self-assembled GaAs/InAs quantum dot in a microcavity system. We first proposed a parity-check gate (PCG) for electron spins based on the QD–microcavity coupled system, and further generalized the system to complete the ECP for a hybrid entangled state. In the current ECPs, we need at least two copies of the less-entangled pairs and rely on the classical communication to check the measurement results between Alice and Bob. After the concentrating process, one pair of maximally entangled state can be obtained at most. In this paper, we propose a new hybrid entanglement concentration protocol, which is also based on the QD–microcavity coupled system. In our newly proposed ECP, we need only one pair of less-entangled states and one ancillary photon, and the spatial modes of the output photons couldless-entangled w state used to distill the maximally entangled state. By performing the photon-number resolving detection, the two remote parties can distill one pair of maximally entangled photon–electron states with a certain probability. The proposed method may be more flexible and economical compared with the previous work. Furthermore, it can be exploited and generalized to concentrate an arbitrary three-particle less-entangled W state.

This paper is organized as follows. In Section 2, we describe the new ECP for a hybrid entangled state with a QD–microcavity coupled system. In Section 3, we extend the protocol to concentrate on an arbitrary W state. By means of this protocol, an arbitrary less-entangled W state can be concentrated to a maximally entangled W state with a relative high probability, with only two ancillary photons and classical communication. The last section presents the discussion and summary.

2. Photon–electron hybrid entanglement concentration using the QD-microcavity coupled system

Here we consider a single-electron-charged self-assembled GaAs/InAs quantum dot embedded in a micropillar microcavity system. The micropillar microcavity with two GaAs/AlGaAs distributed Bragg reflectors (DBRs) and the transverse index guiding provide the three-dimensional confinement of light. The two DBRs are made to be partially reflective (double-side), symmetric, and low-loss in order to achieve high on-resonance transmission. The cross section of the micropillar is circular so that the cavity mode is degenerate for circular polarization light. According to Pauli's exclusion principle, when a photon passes through the cavity and interacts with QDs, an exciton with negative charges can be created by the optical excitation.[18] The charged exciton consists of two electrons bound in a hole. As shown in Ref. [20], the QD–microcavity coupled system behaves like a beam splitter in the limit of a weak income field. The principle of the system is illustrated in Fig. 1.

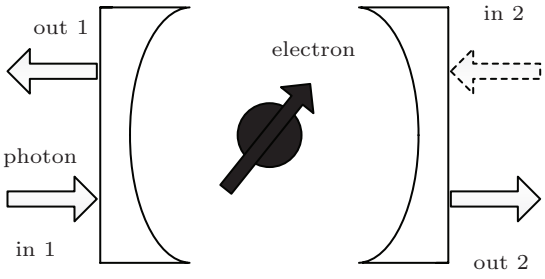


Fig. 1. Schematic diagram of the principle of the QD–microcavity coupled system.

We can define the photon transmission operator and reflection operator of the system as

$$\begin{aligned}&\hat{t}(\omega)\\&= t_0(\omega)\left(\left|R^{\downarrow}\right\rangle\left\langle{}^{\downarrow}R\right|\otimes|\uparrow\rangle\langle\uparrow|+\left|L^{\uparrow}\right\rangle\left\langle{}^{\uparrow}L\right|\otimes|\uparrow\rangle\langle\uparrow|\right.\\&\quad\left.+\left|R^{\uparrow}\right\rangle\left\langle{}^{\uparrow}R\right|\otimes|\downarrow\rangle\langle\downarrow|+\left|L^{\downarrow}\right\rangle\left\langle{}^{\downarrow}L\right|\otimes|\downarrow\rangle\langle\downarrow|\right)\\&\quad+t(\omega)\left(\left|R^{\uparrow}\right\rangle\left\langle{}^{\uparrow}R\right|\otimes|\uparrow\rangle\langle\uparrow|+\left|L^{\downarrow}\right\rangle\left\langle{}^{\downarrow}L\right|\otimes|\uparrow\rangle\langle\uparrow\right.\\&\quad\left.+\left|R^{\downarrow}\right\rangle\left\langle{}^{\downarrow}R\right|\otimes|\downarrow\rangle\langle\downarrow|+\left|L^{\uparrow}\right\rangle\left\langle{}^{\uparrow}L\right|\otimes|\downarrow\rangle\langle\downarrow|\right),\quad(1)\\&\hat{r}(\omega)\\&= r_0(\omega)\left(\left|R^{\downarrow}\right\rangle\left\langle{}^{\downarrow}R\right|\otimes|\uparrow\rangle\langle\uparrow|+\left|L^{\uparrow}\right\rangle\left\langle{}^{\uparrow}L\right|\otimes|\uparrow\rangle\langle\uparrow|\right.\\&\quad\left.+\left|R^{\uparrow}\right\rangle\left\langle{}^{\uparrow}R\right|\otimes|\downarrow\rangle\langle\downarrow|+\left|L^{\downarrow}\right\rangle\left\langle{}^{\downarrow}L\right|\otimes|\downarrow\rangle\langle\downarrow|\right)\\&\quad+r(\omega)\left(\left|R^{\uparrow}\right\rangle\left\langle{}^{\uparrow}R\right|\otimes|\uparrow\rangle\langle\uparrow|+\left|L^{\downarrow}\right\rangle\left\langle{}^{\downarrow}L\right|\otimes|\uparrow\rangle\langle\uparrow|\right.\\&\quad\left.+\left|R^{\downarrow}\right\rangle\left\langle{}^{\downarrow}R\right|\otimes|\downarrow\rangle\langle\downarrow|+\left|L^{\uparrow}\right\rangle\left\langle{}^{\uparrow}L\right|\otimes|\downarrow\rangle\langle\downarrow|\right).\quad(2)\end{aligned}$$

Both operators include the contribution from the uncoupled ($r_0(\omega)$, $t_0(\omega)$) and coupled ($r(\omega)$, $t(\omega)$) cavity. Here the $|L\rangle$ and $|R\rangle$ represent the left and right circularly-polarized states of photons, $|\uparrow\rangle$ and $|\downarrow\rangle$ denote the spin states of electrons, and the superscript represents the propagation direction of the input photon along the quantization z axis for angular momentum. As discussed in Ref. [18], when the incident photon passes through the cavity and interacts with the electron in weak excitation approximation, the reflection and transmission coefficients of the cavity can be investigated by solving the Heisenberg equations of motion for the cavity-field operator and the trion dipole operator, which can be described by[19,27]

$$r(\omega)=\frac{\left[\mathrm{i}\left(\omega_{\chi^-}-\omega\right)+\gamma/2\right]\left[\mathrm{i}\left(\omega_{\mathrm{c}}-\omega\right)+\kappa_{\mathrm{s}}/2\right]+g^2}{\left[\mathrm{i}\left(\omega_{\chi^-}-\omega\right)+\gamma/2\right]\left[\mathrm{i}\left(\omega_{\mathrm{c}}-\omega\right)+\kappa+\kappa_{\mathrm{s}}/2\right]+g^2},\quad(3)$$

$$t(\omega)=\frac{-\kappa\left[\mathrm{i}\left(\omega_{\chi^-}-\omega\right)+\gamma/2\right]}{\left[\mathrm{i}\left(\omega_{\chi^-}-\omega\right)+\gamma/2\right]\left[\mathrm{i}\left(\omega_{\mathrm{c}}-\omega\right)+\kappa+\kappa_{\mathrm{s}}/2\right]+g^2}.\quad(4)$$

Here we denote the frequencies of the input photon, cavity mode, and spin-dependent optical transition by ω, ω_{c}, and ω_{χ^-}, respectively. The g is the coupling constant between cavity and QD, $\gamma/2$ is the exciton dipole decay rate, and κ and κ_{s} are the cavity decay rate and the leaky rate, respectively. In the condition that the system is in the resonant interaction with $\omega_{\mathrm{c}}=\omega_{\chi^-}=\omega_0$ and $g=0$, the reflect and transmission coefficients for the uncoupled cavity system can be given by

$$r_0(\omega)=\frac{\mathrm{i}(\omega_0-\omega)+\kappa_{\mathrm{s}}/2}{\mathrm{i}(\omega_0-\omega)+\kappa+\kappa_{\mathrm{s}}/2},\quad(5)$$

$$t_0(\omega)=\frac{-\kappa}{\mathrm{i}(\omega_0-\omega)+\kappa+\kappa_{\mathrm{s}}/2}.\quad(6)$$

When $\omega=\omega_0$, for the case of strong coupling and $\kappa_{\mathrm{s}}\ll\kappa$, the transmission coefficient $|t(\omega)|$ and reflection coefficient $|r_0(\omega)|$ approach 0. Thus the transmission and reflection operators can be rewritten as

$$\begin{aligned}&\hat{t}(\omega)\\&= t_0(\omega)\left(\left|R^{\downarrow}\right\rangle\left\langle{}^{\downarrow}R\right|\otimes|\uparrow\rangle\langle\uparrow|+\left|L^{\uparrow}\right\rangle\left\langle{}^{\uparrow}L\right|\otimes|\uparrow\rangle\langle\uparrow|\right.\\&\quad\left.+\left|R^{\uparrow}\right\rangle\left\langle{}^{\uparrow}R\right|\otimes|\downarrow\rangle\langle\downarrow|+\left|L^{\downarrow}\right\rangle\left\langle{}^{\downarrow}L\right|\otimes|\downarrow\rangle\langle\downarrow|\right),\quad(7)\\&\hat{r}(\omega)\\&= r(\omega)\left(\left|R^{\uparrow}\right\rangle\left\langle{}^{\uparrow}R\right|\otimes|\uparrow\rangle\langle\uparrow|+\left|L^{\downarrow}\right\rangle\left\langle{}^{\downarrow}L\right|\otimes|\uparrow\rangle\langle\uparrow|\right.\\&\quad\left.+\left|R^{\downarrow}\right\rangle\left\langle{}^{\downarrow}R\right|\otimes|\downarrow\rangle\langle\downarrow|+\left|L^{\uparrow}\right\rangle\left\langle{}^{\uparrow}L\right|\otimes|\downarrow\rangle\langle\downarrow|\right).\quad(8)\end{aligned}$$

If an incident photon is in the state $\left|L^{\downarrow}\right\rangle$ or $\left|R^{\uparrow}\right\rangle$ ($s_z=1$) and the electron spin is in the state $|\uparrow\rangle$, there is a dipole interaction between them, and the photon is reflected by the cavity. Upon reflection, both the photon polarization and propagation direction are reversed, and the input states are transformed to the states $\left|R^{\uparrow}\right\rangle$ and $\left|L^{\downarrow}\right\rangle$. However, if the electron spin is in the state $|\downarrow\rangle$, the photon states will pass through the cavity and gain a π phase shift relative to the reflected photon state. Similarly, the states $\left|L^{\uparrow}\right\rangle$ and $\left|R^{\downarrow}\right\rangle$ ($s_z=-1$) are reflected if the electron spin state is $|\downarrow\rangle$, while are transmitted through the cavity when the electron spin is $|\uparrow\rangle$. The interaction between the incident photon and the cavity can be described as follows:[16,23]

$$\begin{aligned}&\left|R^{\uparrow},\uparrow\right\rangle\Rightarrow\left|L^{\downarrow},\uparrow\right\rangle,\left|L^{\uparrow},\uparrow\right\rangle\Rightarrow-\left|L^{\uparrow},\uparrow\right\rangle\\&\left|R^{\downarrow},\uparrow\right\rangle\Rightarrow-\left|R^{\downarrow},\uparrow\right\rangle,\left|L^{\downarrow},\uparrow\right\rangle\Rightarrow\left|R^{\uparrow},\uparrow\right\rangle\\&\left|R^{\uparrow},\downarrow\right\rangle\Rightarrow-\left|R^{\uparrow},\downarrow\right\rangle,\left|L^{\uparrow},\downarrow\right\rangle\Rightarrow\left|R^{\downarrow},\downarrow\right\rangle\\&\left|R^{\downarrow},\downarrow\right\rangle\Rightarrow\left|L^{\uparrow},\downarrow\right\rangle,\left|L^{\downarrow},\downarrow\right\rangle\Rightarrow-\left|L^{\downarrow},\downarrow\right\rangle.\end{aligned}\quad(9)$$

The basic principles of entanglement generation, distribution, and concentration process are illustrated in Fig. 2. A and B denote the same QD–microcavity coupled system setups as shown in Fig. 1. The incident photon 2 is along the opposite direction of the z axis on Alice's side, and then is sent to Bob. The S denotes the single photon source. Electron 4 is in the state $|\uparrow\rangle$. The photon-number detector can distinguish the photon number 1 from 0 and 2.

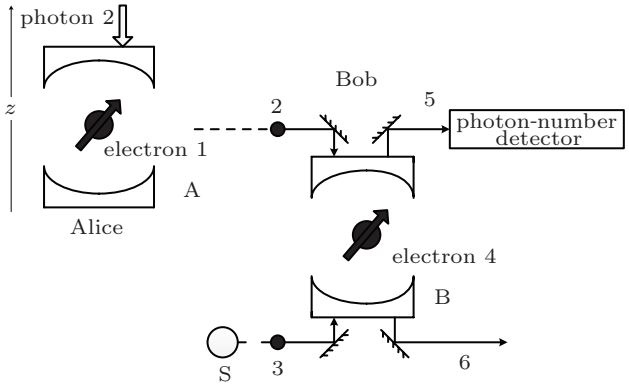


Fig. 2. Schematic diagram of the principles of hybrid entanglement generation, distribution, and concentration process based on remote QDs.

Based on the optical selection rule discussed above, we assume that, on Alice's side, incident photon 2 is in the state $\left|L^{\downarrow}\right\rangle$, and electron 1 is prepared in the state $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. The state $\left(\left|R^{\uparrow},\uparrow\right\rangle - \left|L^{\downarrow},\downarrow\right\rangle\right)/\sqrt{2}$ can be obtained after the interaction in the microcavity between electron 1 and photon 2. Then photon 2 is sent to Bob to set up an entanglement channel between Alice and Bob. However, in the entanglement distribution of the particle, the maximally entangled hybrid state between electron 1 and photon 2 may be affected by environmental noise and become less entangled. Here we consider the non-maximally entangled photon–electron hybrid state

$$|\Phi_{1,2}\rangle = \alpha\,|\uparrow_1\rangle\,|R_2\rangle + \beta\,|\downarrow_1\rangle\,|L_2\rangle \tag{10}$$

as the initial state, where $|\alpha|^2 + |\beta|^2 = 1$, and electron 1 belongs to Alice and photon 2 belongs to Bob.

Alice and Bob utilize the device illustrated in Fig. 2 for the realization of hybrid entanglement concentration. In the first step, another single photon source S on Bob's side emits one single photon in the following form:

$$|\Phi_3\rangle = \alpha\,|R_3\rangle + \beta\,|L_3\rangle\,. \tag{11}$$

Here, the initial state of the photon and photon–spin composite system can be written as

$$\begin{aligned}|\Phi_{\rm in}\rangle &= |\Phi_{1,2}\rangle \otimes |\Phi_3\rangle \\ &= \alpha^2\,|\uparrow_1\rangle\,|R_2\rangle\,|R_3\rangle + \beta^2\,|\downarrow_1\rangle\,|L_2\rangle\,|L_3\rangle \\ &\quad + \alpha\beta\,(|\uparrow_1\rangle\,|R_2\rangle\,|L_3\rangle + |\downarrow_1\rangle\,|L_2\rangle\,|R_3\rangle)\,,\end{aligned} \tag{12}$$

then photons 2 and 3 are sent to the microcavity B in sequence. The electron spin was initially in the spin eigenstate, for example in the state $|\uparrow\rangle$. The time difference between photons 2 and 3 should be less than the spin eigenstate coherence time $T_4^{\rm e}$. After the interaction of photons 2 and 3 with the QD cavity, the final state of the composite system can be described by

$$\begin{aligned}&|\Phi_{\rm out}\rangle \\ &= -\alpha^2\left|R^{\downarrow}\right\rangle\left|L^{\downarrow}\right\rangle|\uparrow_1\rangle - \beta^2\left|R^{\uparrow}\right\rangle\left|L^{\uparrow}\right\rangle|\downarrow_1\rangle \\ &\quad + \alpha\beta\left(\left|R^{\downarrow}\right\rangle\left|L^{\uparrow}\right\rangle|\uparrow_1\rangle + \left|R^{\uparrow}\right\rangle\left|L^{\downarrow}\right\rangle|\downarrow_1\rangle\right).\end{aligned} \tag{13}$$

It is obvious that the states $\left|R^{\downarrow}\right\rangle\left|L^{\uparrow}\right\rangle|\uparrow\rangle_1$ and $\left|R^{\uparrow}\right\rangle\left|L^{\downarrow}\right\rangle|\downarrow\rangle_2$ make both the upper mode 5 and lower mode 6 contain only one photon. However, the state $\left|R^{\uparrow}\right\rangle\left|L^{\uparrow}\right\rangle|\downarrow\rangle_1$ leads to two photons in upper mode 5, and state $\left|R^{\downarrow}\right\rangle\left|L^{\downarrow}\right\rangle|\uparrow\rangle_1$ leads to two photons in lower mode 6. Therefore, by choosing the case when the upper mode 5 contains only one photon, the initial state is projected into a maximally entangled photon–spin hybrid state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\,(|R_6\rangle\,|\uparrow_1\rangle + |L_6\rangle\,|\downarrow_1\rangle)\,. \tag{14}$$

The success probability P of obtaining the maximally entangled state is shown in Fig. 3. We can see that it changes with increasing α and approaches the maximal value when $\alpha = 1/\sqrt{2}$.

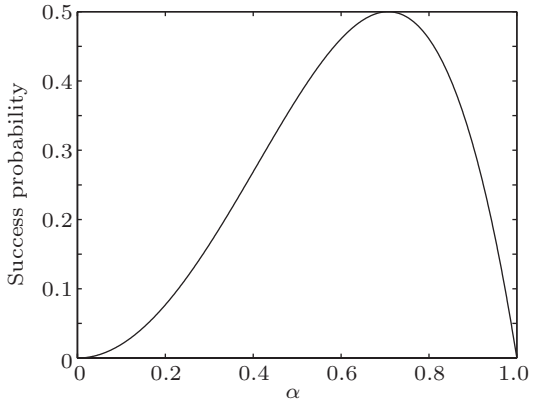


Fig. 3. The success probability P of obtaining the maximally entangled state as a function of the coefficient α in the ECP.

In our proposed ECP, only Bob needs to perform the parity check operation and decides whether to retain or discard their entangled pairs, so Alice and Bob do not need to check their measurement results. In contrast, in our previous work, both Alice and Bob should perform the measurement operations and check their measurement results. Thus, the classical communication is greatly simplified. This new protocol

seems more feasible and economical. Moreover, the QD–microcavity coupled system could be widely used to deterministically create photon–photon entanglement of Bell states and N-photon GHZ states nonlocally.

3. Entanglement concentration for arbitrary three-particle less-entangled W states

The ECPs described above all deal with the two-particle state. In the three-particle system, the Greenberger–Horne–Zeilinger (GHZ) state and the W state are two kinds of tripartite-entangled states which cannot be converted into each other with local operation and classical communications. Our ECP for hybrid entangled concentration is also suitable for the case of a multipartite GHZ state, but cannot deal with the case of a less-entangled W state directly. In 2003, Cao and Yang[28] proposed an ECP for the W state using joint unitary transformation. In 2007, Zhang *et al.*[29] proposed an ECP based on Bell-state measurement. However, both joint unitary transformation and Bell-state measurement are not easy to realize in current experiment conditions. In 2010, Wang *et al.*[30] proposed an ECP for a W state with linear optics and conventional photon detectors. In their schemes, three distant parties, Alice, Bob, and Charlie, could obtain one three-photon maximally entangled W state from two identical partially-entangled W states in the special form $\alpha\left|\mathrm{HHV}\right\rangle+\beta\left(\left|\mathrm{HVH}\right\rangle+\left|\mathrm{VHH}\right\rangle\right)$. In 2012, Sheng *et al.*[31] proposed the concentration protocol for arbitrary W states.

In this section, we extend our ECP to concentrate an arbitrary three-photon W state $\alpha\left|LRR\right\rangle+\beta\left|RLR\right\rangle+\gamma\left|RRL\right\rangle$ to a standard maximally entangled W state $\left(\left|LRR\right\rangle+\left|RLR\right\rangle+\left|RRL\right\rangle\right)/\sqrt{3}$ with only local operation and classical communications. Our protocol is quite different from Wang's because we focus on an arbitrary three-particle entangled W state. We do not need two identical arbitrary three-photon W states, but only need one pair of arbitrary three-photon less-entangled W states and two ancillary photons.

Figure 4 shows the basic principle of the ECP. Suppose a pair of less-entangled W state photons is sent to Alice, Bob, and Charlie, respectively. The photon in the spatial mode a1 is sent to Alice, the photon in the special mode b1 is sent to Bob, and the photon in the special mode c1 is sent to Charlie. The photons are initially in the less-entangled state

$$\begin{aligned}\left|\Phi_{\mathrm{a1b1c1}}\right\rangle=&\,\alpha\left|L_{\mathrm{a1}}\right\rangle\left|R_{\mathrm{b1}}\right\rangle\left|R_{\mathrm{c1}}\right\rangle+\beta\left|R_{\mathrm{a1}}\right\rangle\left|L_{\mathrm{b1}}\right\rangle\left|R_{\mathrm{c1}}\right\rangle\\&+\gamma\left|R_{\mathrm{a1}}\right\rangle\left|R_{\mathrm{b1}}\right\rangle\left|L_{\mathrm{c1}}\right\rangle,\end{aligned}\tag{15}$$

where $\left|\alpha\right|^2+\left|\beta\right|^2+\left|\gamma\right|^2=1$.

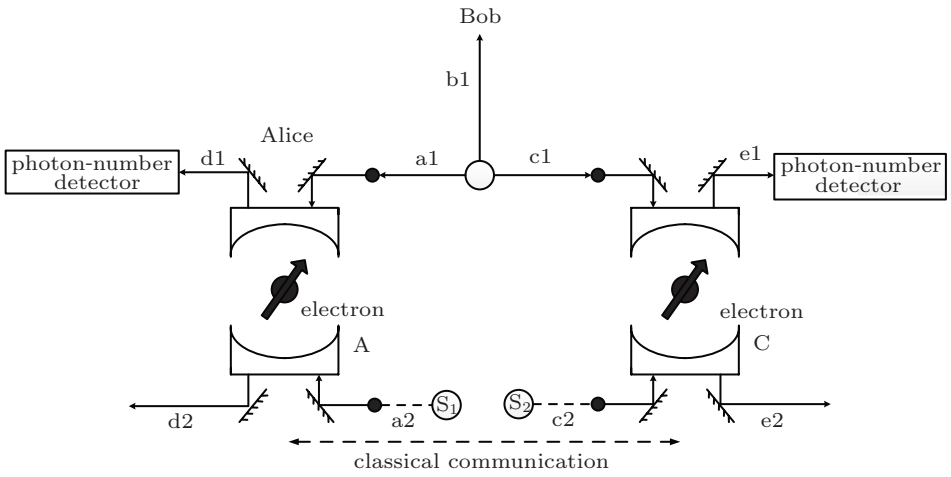


Fig. 4. Schematic diagram of the principle of the ECP. S: less-entangled W state source; S_1 and S_2: single photon source; A and C: the same setup of QD–microcavity coupled system as that described in Fig. 1. The photon-number detector can distinguish the photon number 1 from 0 and 2.

In order to explain this protocol clearly, we divide the ECP into two steps. First, a single photon in the spatial mode a2 emitted from a single-photon source S_1 is sent to Alice, which is described by

$$|\Phi_{a2}\rangle = \frac{1}{\sqrt{\alpha^2+\beta^2}}\left(\alpha|L_{a2}\rangle + \beta|R_{a2}\rangle\right). \quad (16)$$

Thus, the whole four-photon system can be described as

$$\begin{aligned}|\Phi\rangle = & \frac{\alpha^2}{\sqrt{\alpha^2+\beta^2}}|L_{a1}\rangle|L_{a2}\rangle|R_{b1}\rangle|R_{c1}\rangle \\ & + \frac{\alpha\beta}{\sqrt{\alpha^2+\beta^2}}|R_{a1}\rangle|L_{a2}\rangle|L_{b1}\rangle|R_{c1}\rangle \\ & + \frac{\alpha\gamma}{\sqrt{\alpha^2+\beta^2}}|R_{a1}\rangle|L_{a2}\rangle|R_{b1}\rangle|L_{c1}\rangle \\ & + \frac{\alpha\beta}{\sqrt{\alpha^2+\beta^2}}|L_{a1}\rangle|R_{a2}\rangle|R_{b1}\rangle|R_{c1}\rangle \\ & + \frac{\beta^2}{\sqrt{\alpha^2+\beta^2}}|R_{a1}\rangle|R_{a2}\rangle|L_{b1}\rangle|R_{c1}\rangle \\ & + \frac{\beta\gamma}{\sqrt{\alpha^2+\beta^2}}|R_{a1}\rangle|R_{a2}\rangle|R_{b1}\rangle|L_{c1}\rangle. \quad (17)\end{aligned}$$

The electron in cavity A is initialized in the spin-up state $|\uparrow\rangle$. After the two photons in spatial modes a1 and a2 pass through the cavity A, the state becomes

$$\begin{aligned}|\Phi_1\rangle = & -\frac{\alpha^2}{\sqrt{\alpha^2+\beta^2}}\left|R^{\uparrow}\right\rangle\left|L^{\uparrow}\right\rangle|R_{b1}\rangle|R_{c1}\rangle \\ & + \frac{\alpha\beta}{\sqrt{\alpha^2+\beta^2}}\left|R^{\downarrow}\right\rangle\left|L^{\uparrow}\right\rangle|L_{b1}\rangle|R_{c1}\rangle \\ & + \frac{\alpha\gamma}{\sqrt{\alpha^2+\beta^2}}\left|R^{\downarrow}\right\rangle\left|L^{\uparrow}\right\rangle|R_{b1}\rangle|L_{c1}\rangle \\ & + \frac{\alpha\beta}{\sqrt{\alpha^2+\beta^2}}\left|R^{\uparrow}\right\rangle\left|L^{\downarrow}\right\rangle|R_{b1}\rangle|R_{c1}\rangle \\ & - \frac{\beta^2}{\sqrt{\alpha^2+\beta^2}}\left|R^{\downarrow}\right\rangle\left|L^{\downarrow}\right\rangle|L_{b1}\rangle|R_{c1}\rangle \\ & - \frac{\beta\gamma}{\sqrt{\alpha^2+\beta^2}}\left|R^{\downarrow}\right\rangle\left|L^{\downarrow}\right\rangle|R_{b1}\rangle|L_{c1}\rangle, \quad (18)\end{aligned}$$

where three terms

$$\begin{aligned}&\left|R^{\uparrow}\right\rangle\left|L^{\uparrow}\right\rangle|R_{b1}\rangle|R_{c1}\rangle, \\ &\left|R^{\downarrow}\right\rangle\left|L^{\downarrow}\right\rangle|L_{b1}\rangle|R_{c1}\rangle, \\ &\left|R^{\downarrow}\right\rangle\left|L^{\downarrow}\right\rangle|R_{b1}\rangle|L_{c1}\rangle,\end{aligned}$$

lead to the two input photons in spatial modes a1 and a2 in the same output mode (d1 or d2). However, the other three terms $\left|R^{\downarrow}\right\rangle\left|L^{\uparrow}\right\rangle|L_{b1}\rangle|R_{c1}\rangle$, $\left|R^{\downarrow}\right\rangle\left|L^{\uparrow}\right\rangle|R_{b1}\rangle|L_{c1}\rangle$, and $\left|R^{\uparrow}\right\rangle\left|L^{\downarrow}\right\rangle|R_{b1}\rangle|R_{c1}\rangle$ lead to the two output modes of the cavity A (d1 and d2) which contain only one photon exactly. Therefore, if we choose the case that the spatial modes d1, d2, b1, and c2 all contain exactly one photon with a photon-number detection in mode d1, then the initial state collapses to the following state:

$$\begin{aligned}|\Phi_2\rangle = & \frac{\beta}{\sqrt{\gamma^2+2\beta^2}}(|R_{d2}\rangle|L_{b1}\rangle|R_{c1}\rangle \\ & + |L_{d2}\rangle|R_{b1}\rangle|R_{c1}\rangle) \\ & + \frac{\gamma}{\sqrt{\gamma^2+2\beta^2}}|R_{d2}\rangle|R_{b1}\rangle|L_{c1}\rangle, \quad (19)\end{aligned}$$

with a success probability of

$$P_1 = \frac{\alpha^2\left(\alpha^2+2\beta^2\right)}{\alpha^2+\beta^2}.$$

The first step of our protocol acts as a post-selection principle.

The second step is similar to the first one. It is performed by Charlie, as shown in Fig. 4. After we obtain the state $|\Phi_2\rangle$, another single photon in the state $|\Phi_{c2}\rangle$ emitted from a single-photon source S_2 is sent to Charlie. The state is described by

$$|\Phi_{c2}\rangle = \frac{1}{\sqrt{\gamma^2+\beta^2}}\left(\gamma|L_{c2}\rangle + \beta|R_{c2}\rangle\right). \quad (20)$$

Then the combination of four-photon states can be written as

$$\begin{aligned}&|\Psi\rangle \\ = & \frac{\beta^2}{\sqrt{\gamma^2+2\beta^2}\sqrt{\gamma^2+\beta^2}}|R_{d2}\rangle|L_{b1}\rangle|R_{c1}\rangle|R_{c2}\rangle \\ & + \frac{\beta^2}{\sqrt{\gamma^2+2\beta^2}\sqrt{\gamma^2+\beta^2}}|L_{d2}\rangle|R_{b1}\rangle|R_{c1}\rangle|R_{c2}\rangle \\ & + \frac{\gamma^2}{\sqrt{\gamma^2+2\beta^2}\sqrt{\gamma^2+\beta^2}}|R_{d2}\rangle|R_{b1}\rangle|L_{c1}\rangle|L_{c2}\rangle \\ & + \frac{\beta\gamma}{\sqrt{\gamma^2+2\beta^2}\sqrt{\gamma^2+\beta^2}}|R_{d2}\rangle|L_{b1}\rangle|R_{c1}\rangle|L_{c2}\rangle \\ & + \frac{\beta\gamma}{\sqrt{\gamma^2+2\beta^2}\sqrt{\gamma^2+\beta^2}}|L_{d2}\rangle|R_{b1}\rangle|R_{c1}\rangle|L_{c2}\rangle \\ & + \frac{\beta\gamma}{\sqrt{\gamma^2+2\beta^2}\sqrt{\gamma^2+\beta^2}}|R_{d2}\rangle|R_{b1}\rangle|L_{c1}\rangle|R_{c2}\rangle. \quad (21)\end{aligned}$$

It is evident to find that after photons in special modes c1 and c2 pass through the cavity C with the electron spin in the spin-up state $|\uparrow\rangle$, the whole state becomes

$$\begin{aligned}&|\Psi_1\rangle \\ = & -\frac{\beta^2}{\sqrt{\gamma^2+2\beta^2}\sqrt{\gamma^2+\beta^2}}|R_{d2}\rangle|L_{b1}\rangle\left|R^{\downarrow}\right\rangle\left|L^{\downarrow}\right\rangle \\ & - \frac{\beta^2}{\sqrt{\gamma^2+2\beta^2}\sqrt{\gamma^2+\beta^2}}|L_{d2}\rangle|R_{b1}\rangle\left|R^{\downarrow}\right\rangle\left|L^{\downarrow}\right\rangle \\ & - \frac{\gamma^2}{\sqrt{\gamma^2+2\beta^2}\sqrt{\gamma^2+\beta^2}}|R_{d2}\rangle|R_{b1}\rangle\left|R^{\uparrow}\right\rangle\left|L^{\uparrow}\right\rangle \\ & + \frac{\beta\gamma}{\sqrt{\gamma^2+2\beta^2}\sqrt{\gamma^2+\beta^2}}|R_{d2}\rangle|L_{b1}\rangle\left|R^{\downarrow}\right\rangle\left|L^{\uparrow}\right\rangle \\ & + \frac{\beta\gamma}{\sqrt{\gamma^2+2\beta^2}\sqrt{\gamma^2+\beta^2}}|L_{d2}\rangle|R_{b1}\rangle\left|R^{\downarrow}\right\rangle\left|L^{\uparrow}\right\rangle \\ & + \frac{\beta\gamma}{\sqrt{\gamma^2+2\beta^2}\sqrt{\gamma^2+\beta^2}}|R_{d2}\rangle|R_{b1}\rangle\left|R^{\uparrow}\right\rangle\left|L^{\downarrow}\right\rangle, \quad (22)\end{aligned}$$

where the three terms $|R_{d2}\rangle|L_{b1}\rangle|R^{\downarrow}\rangle|L^{\downarrow}\rangle$, $|L_{d2}\rangle|R_{b1}\rangle|R^{\downarrow}\rangle|L^{\downarrow}\rangle$, and $|R_{d2}\rangle|R_{b1}\rangle|R^{\uparrow}\rangle|L^{\uparrow}\rangle$ lead to two photons on Charlie's side in the same output mode e1 or e2, and the other three terms $|R_{d2}\rangle|L_{b1}\rangle|L^{\uparrow}\rangle|R^{\downarrow}\rangle$, $|L_{d2}\rangle|R_{b1}\rangle|L^{\uparrow}\rangle|R^{\downarrow}\rangle$, and $|R_{d2}\rangle|R_{b1}\rangle|R^{\uparrow}\rangle|L^{\downarrow}\rangle$ lead to two photons in different output modes e1 and e2. Therefore, similar to the first step, Charlie chooses the case that two output modes of cavity C both contain only one photon with photon-number detection in mode e1. The initial state finally becomes the maximally entangled W state

$$|\Psi_2\rangle = \frac{1}{\sqrt{3}}(|R_{d2}\rangle|L_{b1}\rangle|R_{e2}\rangle + |L_{d2}\rangle|R_{b1}\rangle|R_{e2}\rangle + |R_{d2}\rangle|R_{b1}\rangle|L_{e2}\rangle) \quad (23)$$

with a success probability

$$P_2 = \frac{3\beta^2\gamma^2}{(\gamma^2+\beta^2)(\gamma^2+2\beta^2)}.$$

Bob only needs to retain or discard his photon according to the measurement results from Alice and Charlie by classical communication. The total success probability P^* for obtaining a maximally entangled state is

$$P^* = P_1P_2 = \frac{3\alpha^2\beta^2\gamma^2}{(\alpha^2+\beta^2)(\beta^2+\gamma^2)}. \quad (24)$$

At this point, we have fully explained our ECP for an arbitrary three-particle less-entangled W states. We calculate the total success probability P^* of obtaining a maximally entangled W state when $\gamma = 1/\sqrt{2}, 1/\sqrt{3}$, and $1/2$. Numerical simulation results are shown in Fig. 5 and P^* changes with increasing initial coefficient α in the ECP and approaches the maximal value as $\alpha \approx 0.55$, 0.66, and 0.71, respectively.

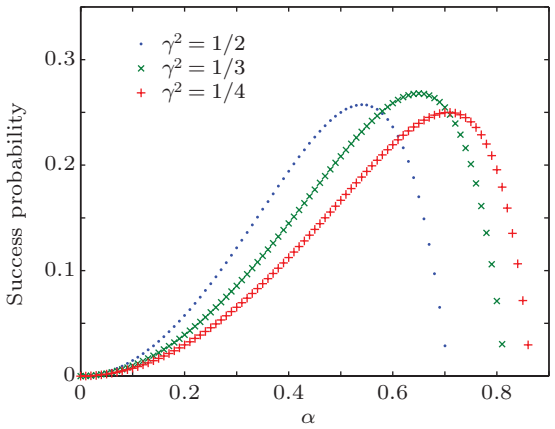


Fig. 5. (colour online) Success probability P^* of obtaining the maximally entangled W state as a function of the initial coefficient α when $\gamma = 1/\sqrt{2}$, $1/\sqrt{3}$, and $1/2$.

4. Discussion and conclusion

In this section, we will discuss the feasibility of implementing the ECPs with GaAs- or InAs-based QDs in a micropillar microcavity in realistic experimental conditions. In order to calculate the yield for the entanglement concentration, we use the transmission operation $\hat{t}$ and reflection operation $\hat{r}$ to describe the photon–electron interaction in the microcavity. The success probability P of gaining the maximally entangled state is $2|\alpha\beta|^2$, which is defined as the ratio of the number of maximally entangled pairs and the number of original less-entangled pairs. However, in the above analysis, we did not take into account the influences of the coupling strength g between the QD and the microcavity, and the cavity field decay rate κ. Actually, they are playing a significant role in the realization of entanglement concentration, and the success probability in our ECP varies with the coefficient of the coupling strength g over the cavity decay rate κ. Experimentally, the χ^- dipole decay rate γ is about several micro eV, so we take $\gamma = 0.1\kappa$. Then we employed the coupling constant to calculate the yield in our ECP system in Fig. 6. It is obvious that the yield of the maximally entangled hybrid state increases rapidly with increasing coefficient g/κ smaller than 2. The curve tends to be flat for higher values of g/κ. In Ref. [32], Reithmaier *et al.* reported their observation of strong coupling of a single two-level solid-state system with a photon, which was realized by a single quantum dot in a semiconductor microcavity. The coupling strength was reported to be $g = 0.5$ in the microcavity with a diameter of $d = 1.5$ μm, corresponding to $g/(\kappa + \kappa_s) = 2.4$ with the cavity leakage κ_s acceptable. Here the coefficient κ_s describes the cavity side leakage of the transmission process, which may also affect the success probability of our ECP in a practical pillar microcavity. When $\kappa_s \ll \kappa$, we can gain high fidelity entanglement in the coupling regime and κ_s can be safely neglected.

Furthermore, the time difference between incident photons should be less than the spin eigenstate coherence time T_4^e in our ECPs. The QD-spin eigenstate can be prepared by optical spin pumping and cooling.[33,34] The proposed single-shot QND measurement could also be used to prepare the spin state via single-photon measurement[35] or via the quantum Zeno effect.[36,37] In Ref. [38], Press *et al.* demonstrated the complete coherent control over an initialized electron spin state in a quantum dot using picosecond optical pulses. Their experiment has shown

that the coherence time of the GaAs- or InAs-based QDs could be extended to microseconds, which is long enough for our ECP.[39]

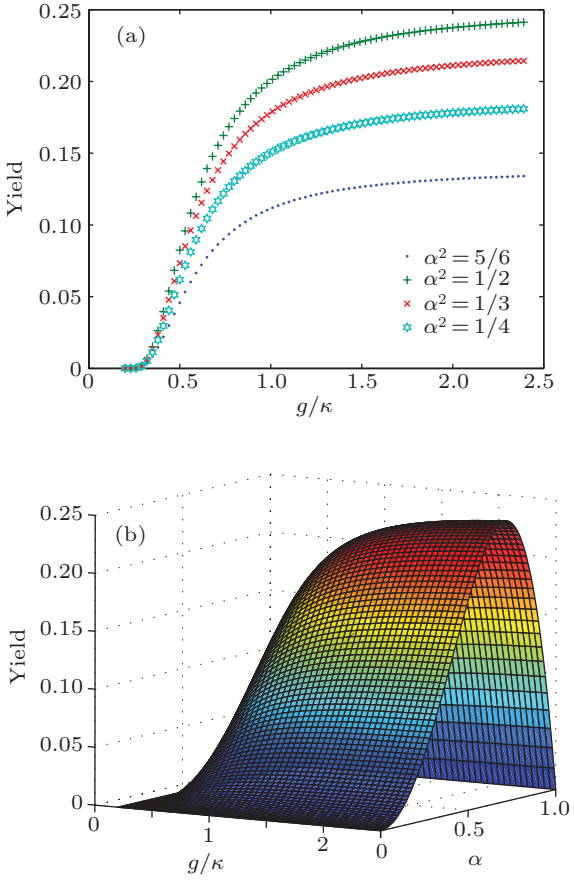


Fig. 6. (colour online) Yield of the maximally hybrid entangled state in our proposed ECP when the cavity field decay rate κ is considered. (a) Yield as a function of the ratio g/κ. (b) Yield as a function of g/κ and α.

For the realistic QD, the optical selection rule may not be perfect. For example, the spin decoherence may reduce the fidelity of entanglement in our ECP by a factor $[1+\exp(-t/T^{\mathrm{e}})]/2$,[19] where t is the cavity photon coherence time. Therefore, the fidelity of entanglement relies on the ratio of the coefficient of the cavity photon lifetime and the trion coherence time. In Ref. [40], Borri *et al.* measured the dephasing time of several hundred picoseconds at low temperature in the ground state transition of strongly confined InGaAs quantum dots, which is ten times larger than the cavity photon lifetime. Furthermore, the hole–spin dephasing and the heavy–light hole mixing may also affect the optical selection rule. Recently, great progress has been made on the optical spin manipulation in QDs,[41−43] which provides us with some useful methods for controlling the interaction between the income photon and electron spin.

In summary, we present a protocol for nonlocal entanglement concentration of a hybrid entangled state. In the hybrid platform, the proposed setup provides a method to realize the entanglement transfer between photons and electron spins. Compared with a previous work, our protocol have several advantages. First of all, we do not need to know the rigorously exact amplitudes α and β of the less-entangled pairs. Second, only one pair of less-entangled states is needed, providing a higher entanglement transformation efficiency. Third, this protocol is more feasible because of the simplification of classical communication for each party. Furthermore, our new protocol can be extended to concentrate the photon–photon less-entangled state and multipartite less-entangled GHZ state. We also discussed a possible application of our protocol for concentrating an arbitrary three-photon W state $\alpha|LRR\rangle+\beta|RLR\rangle+\gamma|RRL\rangle$ to a standard maximally entangled W state $(|LRR\rangle+|RLR\rangle+|RRL\rangle)/\sqrt{3}$ with only local operation and classical communication. Our protocol deals with the arbitrary three-particle entangled W state, and does not need two identical arbitrary three-photon W state, but need only one pair of arbitrary three-photon less-entangled W states and two ancillary photons.

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