# Comparison and control of the robustness between quantum entanglement and quantum correlation in an open quantum system*

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We investigate the influence of environmental decoherence on the dynamics of a coupled qubit system and quantum correlation. We analyse the relationship between concurrence and the degree of initial entanglement or the purity of initial quantum state, and also their relationship with quantum discord. The results show that the decrease of the purity of an initial quantum state can induce the attenuation of concurrence or quantum discord, but the attenuation of quantum discord is obviously slower than the concurrence's, correspondingly the survival time of quantum discord is longer. Further investigation reveals that the robustness of quantum discord and concurrence relies on the entanglement degree of the initial quantum state. The higher the degree of entanglement, the more robust the quantum discord is than concurrence. And the reverse is equally true. Birth and death happen to quantum discord periodically and a newborn quantum discord comes into being under a certain condition, so does the concurrence.

**Keywords:** coupled qubits, non-Markovian process, entanglement, quantum discord

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## 1. Introduction

Quantum entanglement is the kernel resource for quantum information and quantum calculation. It is also the fundamental reason why this field has exhibited tremendous advantages and application prospects. We can implement some tasks that cannot be realized by classical means, by utilizing quantum entanglement, such as quantum teleportation, quantum dense encryption, and quantum crytography based on quantum entanglement.[1−5] Every system inevitably interacts with an external environmental thermal bath and this local decoherence ultimately destroys the entanglemet of the whole quantum system, thereby restricting the applications of entanglement in practical quantum information and quantum calculation. The thermal bath, however, provides an indirect interaction, which possibly induces entanglement. Aiming at the influence of the environment on the dynamics of entanglement, Almeida *et al.*[6] suggested entanglement death and observed the phenomenon in a succedent experiment. Based on the interaction model between a multimode cavity and an atom, Lopez *et al.*[7] advanced the concepts of entanglement sudden death (ESD) and birth. For an arbitrary 2×2 hierarchy, a state form in which the entanglement death is avoided has been provided.[8−11] Meantime, in Ref. [12], it was found that the ESD is largely dependent on the form of the quantum state no matter whether it occurs or not. Thus, we change the form of the state during the entanglement evolution so as to retard the ESD process.

Recently, Datta *et al.*[13] gave a model which was independent of quantum entanglement but could realize a quantum acceleration algorithm. In this model, no entanglement resource was used and it maintained a separating state at all times. We cannot help questioning what on Earth acts as acceleration algorithm. Gradually, we are aware that quantum entanglement is not the indispensable resource for certain quantum computations. Further investigations find that quantum correlation can better measure the quantum characteristics of one state and it brings hope to actually realizing upgraded quantum computation.[14−16] We can make the experiment less difficult and reduce the device requirement to a large extent if we can imple-

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ment quantum computation without the help of entanglement since the maintenance of quantum entanglement is the most difficult factor in experiments. Quantum correlation differs from quantum entanglement. For a complete separating state, if we describe it with entanglement, we will regard the degree of entanglement as being zero, which is traditionally believed to be of no use to quantum computation. Factually, however, the complete separating state may still possess quantum characteristics. How to measure it has become a research direction and hotspot at the moment. Presently, people usually adopt the quantum discord advanced by Ollivier and Zurek[17] to measure quantum correlation. While there is much dispute on the action of quantum correlation in quantum computation, particularly, the magnitudes of action in different quantum algorithms are still unknown.

In this article, we make a comparison between the quantum entanglement and quantum correlation of interacted coupled qubits in the same thermal bath in a non-Markovian process. We find that under some conditions, there is no entanglement between the coupled qubits. However, we can find a quantum correlation under other conditions. Inversely, the quantum discord is zero but the quantum entanglement is nonzero. On the one hand, it indicates that we should specifically select the quantum discord or quantum entanglement as an information resource. On the other hand, it may imply that it is not fully appropriate to measure quantum correlation with quantum discord and in this case we need to reconsider the measure of quantum correlation from a new point view and evaluate the effect of quantum correlation on quantum computation and quantum information.

## 2. Model

The model consists of two interacting coupled qubits in thermal equilibrium. Assuming that the two qubits independently interact with the environment, the Hamiltonian ($k_{\rm B} = \hbar = 1$) of the model in interaction picture is[18,19]

$$H_{\rm I}(t) \equiv H_{\rm I}^{(\rm s)}(t) + H_{\rm I}^{(\rm d)}(t), \tag{1}$$

where

$$H_{\rm I}^{(\rm s)}(t) = k(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+), \tag{2}$$

$$H_{\rm I}^{(\rm d)}(t) = (\sigma_1^+ + \sigma_2^+)\sum_n g_n b_n {\rm e}^{{\rm i}(\varepsilon-\omega_n)t} + (\sigma_1^- + \sigma_2^-)\sum_n g_n^* b_n^+ {\rm e}^{-{\rm i}(\varepsilon-\omega_n)t}, \tag{3}$$

where $k$ is the interaction strength between qubits, $g_n$ is the interaction constant between the coupled qubits and an environmental thermal bath, $b_n$ and $b_n^+$ denote the creation operator and the annihilation operator, respectively, and $\sigma_j^{\pm}$ $(j = 1, 2)$ is the Pauli operator describing qubits and satisfies the commutation and anti-commutation algebraic relation.

In the Schrödinger picture, the master equation of reduced density operator is given by

$$\begin{aligned}\frac{\rm d}{{\rm d}t}\rho(t) =& -{\rm i}[H_{\rm I}^{(\rm s)}(t), \rho(0)] - t[H_{\rm I}^{(\rm s)}(t), [H_{\rm I}^{(\rm s)}(t), \rho(t)]] \\ &+ A(t)\left\{[\sigma_1^+, \rho(t)\sigma_1^-] + [\sigma_2^+, \rho(t)\sigma_2^-]\right\} \\ &+ \bar{A}(t)\left\{[\sigma_1^+\rho(t), \sigma_1^-] + [\sigma_2^+\rho(t), \sigma_2^-]\right\} \\ &+ B(t)\left\{[\sigma_1^-\rho(t), \sigma_1^+] + [\sigma_2^-\rho(t), \sigma_2^+]\right\} \\ &+ \bar{B}(t)\left\{[\sigma_1^-, \rho(t)\sigma_1^+] + [\sigma_2^-, \rho(t)\sigma_2^+]\right\}, \end{aligned} \tag{4}$$

where

$$A(t) = {\rm i}\sum_n |g_n|^2 n(\omega_n)\frac{1 - {\rm e}^{{\rm i}(\omega_0+\omega_n)t}}{\omega_0 + \omega_n},$$

$$B(t) = {\rm i}\sum_n |g_n|^2 [n(\omega_n) + 1]\frac{1 - {\rm e}^{{\rm i}(\omega_0+\omega_n)t}}{\omega_0 + \omega_n}.$$

Here, $\bar{A}(t)$ $(\bar{B}(t))$ is the complex conjugate of $A(t)$ $(B(t))$. In what follows, we consider the case for a zero temperature environment and suppose that the spectrum of the environmental thermal bath obeys Lorenz distribution,

$$J(\omega) = \frac{\gamma_0}{2\pi}\frac{\lambda^2}{\left(\omega + \frac{\omega_0}{2}\right)^2 + \lambda^2}, \tag{5}$$

where $\gamma_0$ is the coupling constant and $\lambda$ defines the spectral width of the coupling. This implies that the correlation functions reduce to

$$A(t) = 0, B(t) = \frac{\gamma_0}{2}\left(1 - {\rm e}^{-\lambda t}\right).$$

## 3. Measure of quantum entanglement and quantum correlation

Usually it is hard to analyse an arbitrary two-qubit state. In this paper, we perform an analysis on the special X-state. When the system is initially in an X-state, it can be said to be in the X-state during the dynamical evolution. Under the X-state, people have investigated the ESD phenomenon in the process of entanglement evolution and the dynamical process of quantum correlation. Recently, researchers have also discussed the sudden step of quantum correlation and

classical correlation in the decoherence tunnel. Give the X-state[20] as

$$\boldsymbol{\rho}_{\mathrm{AB}}(t)=\begin{pmatrix}\rho_{11}(t) & 0 & 0 & \rho_{14}(t)\\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0\\ 0 & \rho_{32}(t) & \rho_{33}(t) & 0\\ \rho_{41}(t) & 0 & 0 & \rho_{44}(t)\end{pmatrix}. \quad (6)$$

Based on the initial state, the solutions of the master equation are obtained (see Appendix A).

We adopt the concurrence entanglement defined by Wootters[21] to measure the system entanglement. Under the X-state,

$$C(t) = 2\max\{0, |\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)}, |\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}(t)}\}. \quad (7)$$

A bipartite state may include not only classical correlation but also quantum correlation. We describe the quantum correlation with the quantum discord put forward by Ali *et al.*[22] as

$$\mathrm{QD}\left(\boldsymbol{\rho}_{\mathrm{AB}}\right) = I\left(\boldsymbol{\rho}_{\mathrm{AB}}\right) - CD\left(\boldsymbol{\rho}_{\mathrm{AB}}\right), \quad (8)$$

where $I\left(\boldsymbol{\rho}_{\mathrm{AB}}\right) = S\left(\boldsymbol{\rho}_{\mathrm{A}}\right) + S\left(\boldsymbol{\rho}_{\mathrm{B}}\right) - S\left(\boldsymbol{\rho}_{\mathrm{AB}}\right)$ is the quantum mutual information and $CD\left(\boldsymbol{\rho}_{\mathrm{AB}}\right)$ is the classical correlation between the two subsystems. As discussed in Refs. [23] and [24], the classical correlation $CD\left(\boldsymbol{\rho}_{\mathrm{AB}}\right) = \max\limits_{\{B_k\}}\left[S\left(\boldsymbol{\rho}_{\mathrm{A}}\right) - S\left(\boldsymbol{\rho}_{\mathrm{AB}}\left|\boldsymbol{\rho}_{\mathrm{B}}\right.\right)\right]$, where $\{B_k\}$ is a set of von Neumann measurements performed on subsystem B locally, $S\left(\boldsymbol{\rho}_{\mathrm{AB}}\left|\{B_k\}\right.\right) = \sum\limits_k p_k S\left(\rho_k\right)$ is the quantum conditional entropy, $\rho_k = \left(I_{\mathrm{A}}\otimes B_k\right)\rho\left(I_{\mathrm{A}}\otimes B_k\right)/\mathrm{Tr}\left(I_{\mathrm{A}}\otimes B_k\right)\rho\left(I_{\mathrm{A}}\otimes B_k\right)$ is the conditional density operator corresponding to the outcome labeled by $k$, and $p_k = \mathrm{Tr}\left(I_{\mathrm{A}}\otimes B_k\right)\rho\left(I_{\mathrm{A}}\otimes B_k\right)$ with $I_{\mathrm{A}}$ being the identity operator performed on subsystem A.

## 4. Discussion

Without loss of generality, in the following discussion, we consider two kinds of initial states,[25]

$$\boldsymbol{\rho}_{\psi}\left(r,\theta\right) = r\left|\boldsymbol{\psi}(\theta)\right\rangle\left\langle\boldsymbol{\psi}(\theta)\right| + \frac{1-r}{4}\boldsymbol{I}_4, \quad (9)$$

where $r$ is the purity of the initial states, which ranges from 0 for maximally mixed states to 1 for pure states, $\boldsymbol{I}_4$ is the $4\times 4$ identity matrix, $\left|\boldsymbol{\psi}(\theta)\right\rangle = \cos\theta\left|00\right\rangle + \sin\theta\left|11\right\rangle$ is the Bell-like pure state and the parameter $\theta$ is sometimes called the degree of entanglement.

We plot the quantum discord (QD) and concurrence entanglement in Fig. 1. As the measure of quantum correlation, the quantum discord is very different from entanglement in this case. We find that the purity of quantum state $r$ does not have completely the same effects on the concurrence and quantum discord when other parameters keep invariant. With the value of purity $r$ decreasing, both concurrence and quantum discord decrease monotonically. But the attenuation of quantum discord is obviously slower than that of concurrence. In particular, according to Peres' criterion,[26] when $r$ is less than a certain value, the quantum state becomes a separate state and the concurrence will be zero. At this moment, however, the quantum discord is non-zero. Especially when $\theta = \pi/4$, the quantum discord will not be zero although the purity $r$ decreases.

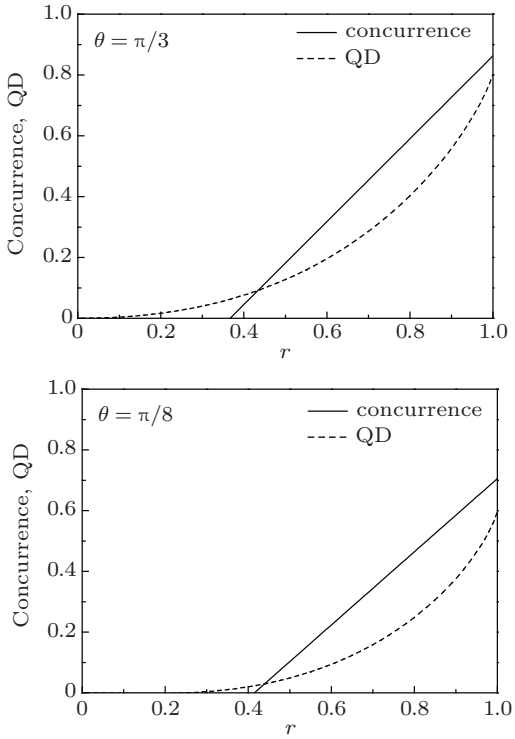



**Fig. 1.** Concurrence and quantum discord versus $r$ with $\lambda t = 0.1$ and $\gamma_0/\lambda = 0.1$ for $\rho_{\psi}(r,\theta)$.

Figure 2 shows that with the variation of $\theta$, death and rebirth happen to both quantum discord and concurrence periodically. The periods are identical. However, if the purity of the initial quantum state is higher, the survival time of quantum discord is shorter and the value of quantum discord is also a little smaller than that of concurrence. In contrast, if the purity of the initial quantum state is lower, the survival time of

quantum discord is longer and the value of quantum discord is also a little bigger than that of concurrence.

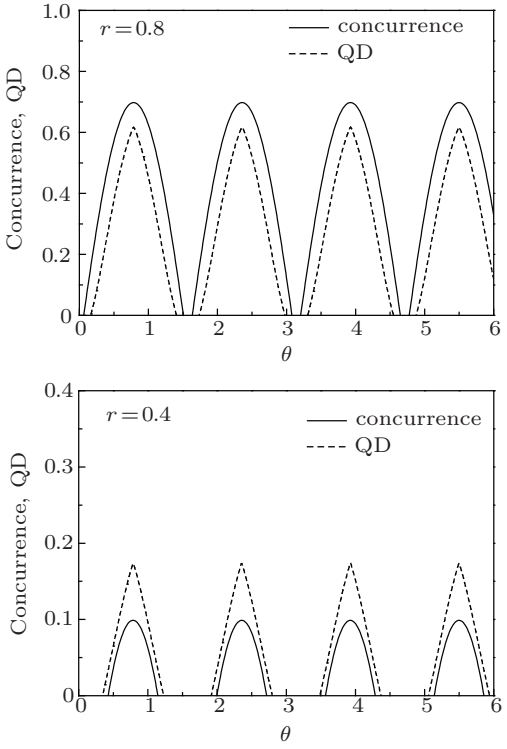



**Fig. 2.** Concurrence and quantum discord versus $\theta$ with $\gamma_0/\lambda = 0.1$ and $\lambda t = 0.1$ for $\rho_\psi(r, \theta)$.

In the Markovian approximation, through some simple calculations, the sudden-death time is given by

$$T_{\rm ESD} = \frac{1}{\gamma_0} \ln \left[ 1 + \frac{r\,|\sin 2\theta| - 0.5(1-r)}{2r\cos^2\theta + (1-r)} \right]. \quad (10)$$

From the above formula, it is easy to obtain that the provided system is kept in an entangled state, and the survival time of the entanglement relates to both $r$ and $\theta$ when the other parameters are invariant. It could exist longer at the expense of a definite initial entanglement degree. The results provide more direct approaches to the quantum regulation and design of a corresponding control scheme.

Figures 3 and 4 show the evolution performances of concurrence and quantum discord respectively versus time and coupling strength $\gamma_0$. With time passing or the value of coupling strength $\gamma_0$ increasing, the concurrence and quantum discord mainly decrease monotonically. The coupling qubit system evolves from certain initial states. The entanglement induced by environmental decoherence is zero while the quantum discord is non-zero, which shows that although these states are not entangled, they possess quantum correlation in this case. Therefore, quantum discord can be a useful resource for quantum computation since it is more robust than concurrence. However, Figs. 3 and 4 also indicate that quantum discord is not more robust than concurrence under all conditions.

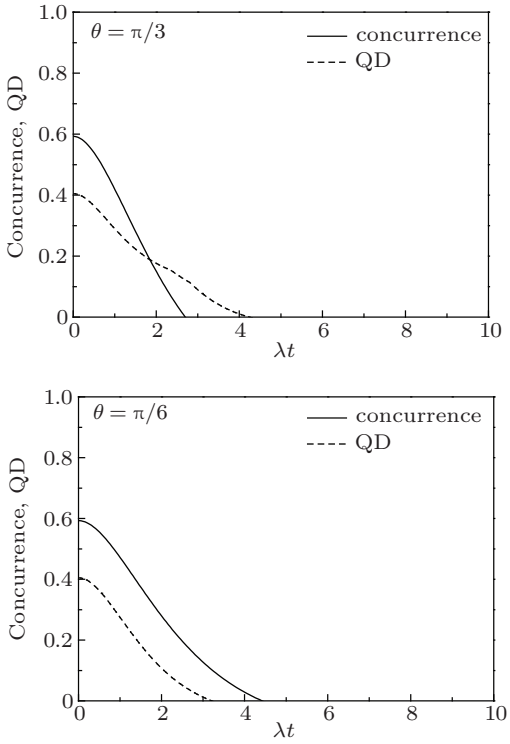



**Fig. 3.** Concurrence and quantum discord versus $\lambda t$ with $r = 0.8$ and $\gamma_0/\lambda = 0.1$ for $\rho_\psi(r, \theta)$.

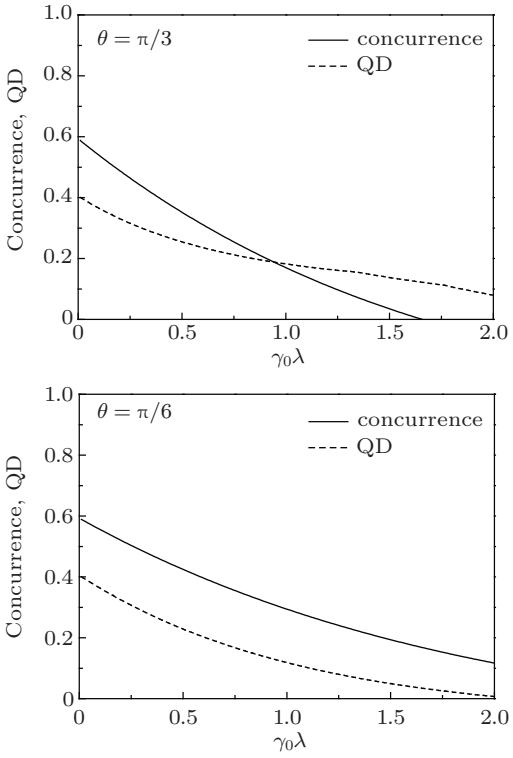



**Fig. 4.** Concurrence and quantum discord versus $\gamma_0/\lambda$ with $r = 0.8$ and $\lambda t = 0.5$ for $\rho_\psi(r, \theta)$.

In our physical model and initial quantum states, in the light of detailed analysis, we find that the robustness of quantum discord and concurrence relies on the entanglement degree of the initial quantum state. The higher the degree of entanglement, the more robust the quantum discord is than concurrence. And the reverse is equally true. Thus, careful comparison and investigation is required to decide whether or not to adopt the quantum discord or concurrence as the useful information resource in quantum information processing.

## 5. Conclusions

There must be a quantum correlation between quantum systems if they interact with each other. We discuss the quantum entanglement and quantum correlation of a bipartite quantum system according to a non-Markovian process in an open quantum system through two different measurements of concurrence and quantum discord. We analyse in detail the relationship between concurrence and the initial degree of entanglement or purity of the initial quantum state, and their relationship with quantum discord. We further illustrate the characteristics of quantum discord by comparing the evolutions of two measurements. The results indicate the attenuation of quantum discord versus the purity of initial quantum state and it is obviously slower than that of concurrence. In the case of a separating state, the survival time of quantum discord is relatively long. Further investigations also show that the robustness of quantum discord and concurrence depends on the entanglement degree of the initial quantum state. The higher the initial degree of entanglement, the more robust the quantum discord is than concurrence. In fact, the above results can also be obtained for other "X" initial states, for example, $\rho_{\Phi}(r,\theta) = r\,|\Phi(\theta)\rangle\langle\Phi(\theta)| + (1-r)I_4/4$, where $|\Phi(\theta)\rangle = \cos\theta\,|10\rangle + \sin\theta\,|01\rangle$. The results reveal that enhancing the initial entanglement degree between the coupling qubits or the purity of the quantum states is beneficial for efficiently suppressing the attenuation of the quantum correlation between the qubits.

## Appendix A

When the system is initially in an X-state, the solution of Eq. (4) in the initial state (6) is direct; the density matrix elements of the time evolution are given by

$$\rho_{11}(t) = \rho_{11}(0)\mathrm{e}^{-2f(t)}, \tag{A1}$$

$$\begin{aligned}\rho_{22}(t) = {} & \rho_{11}(0)\mathrm{e}^{-f(t)}\left(1-\mathrm{e}^{-f(t)}\right) \\ & + \rho_{22}(0)\mathrm{e}^{-f(t)}\cos^2(kt) \\ & + \rho_{33}(0)\mathrm{e}^{-f(t)}\sin^2(kt) \\ & - \mathrm{e}^{-f(t)}\sin(2kt)\,\mathrm{Im}\,\rho_{23}(0),\end{aligned} \tag{A2}$$

$$\begin{aligned}\rho_{33}(t) = {} & \rho_{11}(0)\mathrm{e}^{-f(t)}\left(1-\mathrm{e}^{-f(t)}\right) \\ & + \rho_{22}(0)\mathrm{e}^{-f(t)}\sin^2(kt) \\ & + \rho_{33}(0)\mathrm{e}^{-f(t)}\cos^2(kt) \\ & + \mathrm{e}^{-f(t)}\sin(2kt)\,\mathrm{Im}\,\rho_{23}(0),\end{aligned} \tag{A3}$$

$$\rho_{44}(t) = 1 - \rho_{11}(t) - \rho_{22}(t) - \rho_{33}(t), \tag{A4}$$

$$\rho_{14}(t) = \rho_{14}(0)e^{-2f(t)}, \tag{A5}$$

$$\begin{aligned}\rho_{23}(t) = {} & \mathrm{e}^{-f(t)}[\rho_{23}(0)\cos^2(kt) \\ & + \rho_{23}(0)^*\sin^2(kt)] \\ & + \mathrm{i}[\rho_{22}(0) - \rho_{33}(0)] \\ & \times \mathrm{e}^{-f(t)}\sin(kt)\cos(kt),\end{aligned} \tag{A6}$$

$$\rho_{32}(t) = \rho_{23}^*(t), \rho_{41}(t) = \rho_{14}^*(t), \tag{A7}$$

where

$$f(t) = \frac{\gamma_0}{\lambda}\left(\lambda t - 1 + \mathrm{e}^{-\lambda t}\right). \tag{A8}$$

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