Noether–Mei symmetry of a discrete mechanico-electrical system*

Zhang Wei-Wei(张伟伟) and Fang Jian-Hui(方建会)†

College of Science, China University of Petroleum, Qingdao 266580, *China*

(Received 5 January 2012; revised manuscript received 18 June 2012)

Noether–Mei symmetry of a discrete mechanico-electrical system on a regular lattice is investigated. Firstly, the Noether symmetry of a discrete mechanico-electrical system is reviewed, and the motion equations and energy equations are derived. Secondly, the definition of Noether–Mei symmetry for the system is presented, and the criterion is derived. Thirdly, conserved quantities induced by Noether–Mei symmetry with their existence conditions are obtained. Finally, an example is discussed to illustrate the results.

Keywords: discrete mechanico-electrical system, Noether–Mei symmetry, conserved quantity

PACS: 02.20.–a, 45.20.Jj, 45.05.+x **DOI:** 10.1088/1674-1056/21/11/110201

1. Introduction

Research on the symmetry and conserved quantities of mechanical systems has played a very important role in modern mathematics, mechanics, and physics. It has been a good method of solving a great amount of scientific problems. The symmetries of continuous systems have been extensively and deeply investigated, and many meaningful results have been obtained.[1−5] In order to cope with complex problems, more attention has been paid to discrete systems so that we can take advantage of a computer's high efficiency. Symmetries of discrete mechanical systems and difference equations have been investigated recently.[6−13] When building discrete mechanical equations, we should notice that the symmetries are intrinsic and fundamental features of the mechanical differential equations, so they should be retained. Dorodnitsyn considered group transformations acting both on difference equations and on the lattice. In this way, all the symmetries of the original differential equations can be preserved.[14−17] Based on this method, Fu *et al.*[18−22] investigated symmetry theories of discrete non-holonomic systems, nonconservative systems, and mechanico-electrical systems (including regular and irregular lattices), and obtained valuable results.

A mechanico-electrical system is a kind of system where a mechanical process connects with an electromagnetic process. Fu *et al.* extended the Noether symmetry and Lie symmetry theories to discrete mechanico-electrical systems, built the motion equations of such systems, and obtained corresponding conserved quantities.[18,20,22] Wang applied perturbation to the Noether symmetry and gained the Noether adiabatic invariants.[23] This paper aims to study Noether–Mei symmetry and corresponding conserved quantities of discrete mechanico-electrical systems.

2. Discrete motion equations

Let us consider a mechanico-electrical dynamical system consisting of N particles and m electric circuits. The mechanical part contains n generalized coordinates q^s $(s = 1, \ldots n)$, and the electromagnetic part contains j $(j = 1, \ldots m)$ circuits. For circuit j, u^j is the generalized electrical force, e^j is the charge of the capacitor, and $\dot{e}^j$ is the current. The parameters $W_{\rm e}$ and $W_{\rm m}$ are the electric-field energy and magnetic-field energy, respectively. The Lagrangian of the mechanico-electrical system is given by

$$L = T(\boldsymbol{q}, \dot{\boldsymbol{q}}) - V(\boldsymbol{q}) + W_{\rm m}(\boldsymbol{q}, \dot{\boldsymbol{e}}) - W_{\rm e}(\boldsymbol{q}, \boldsymbol{e}), \quad (1)$$

where T and V are the kinetic energy and potential energy, respectively. The Lagrange–Maxwell equations

*Project supported by the Natural Science Foundation of Shandong Province of China (Grant No. ZR2011AM012).

†Corresponding author. E-mail: fangjh@upc.edu.cn

© 2012 Chinese Physical Society and IOP Publishing Ltd

of the system are given by[24]

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}^s} - \frac{\partial L}{\partial q^s} + \frac{\partial F}{\partial \dot{q}^s} = Q^s, \tag{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{e}^j} - \frac{\partial L}{\partial e^j} + \frac{\partial F}{\partial \dot{e}^j} = u^j, \tag{3}$$

where Q^s are the generalized non-potential forces, and F is the dissipative function in the following form:

$$F = F_{\mathrm{e}}(\dot{\boldsymbol{e}}) + F_{\mathrm{m}}(\boldsymbol{q}, \dot{\boldsymbol{q}}). \tag{4}$$

In the space Z of sequence (t, q^s, e^j, $\dot{q}^s$, $\dot{e}^j$), we introduce a first-order linear differential operator

$$D = \frac{\partial}{\partial t} + \frac{\partial}{\partial q^s}\dot{q}^s + \frac{\partial}{\partial e^j}\dot{e}^j + \frac{\partial}{\partial \dot{q}^s}\ddot{q}^s + \frac{\partial}{\partial \dot{e}^j}\ddot{e}^j + \cdots, \tag{5}$$

which acts according to the rules $D(t) = 1$, $D(q^s) = \dot{q}^s$, $D(e^j) = \dot{e}^j$, etc. Consider the uniform lattice h in the t direction. For arbitrary $h > 0$, we construct a pair of shift operators acting on the right and left as

$$\begin{aligned} \underset{+h}{S} &= e^{hD} = \sum_{i=0}^{\infty} \frac{h^i}{i!} D^i, \\ \underset{-h}{S} &= e^{-hD} = \sum_{i=0}^{\infty} \frac{(-h)^i}{i!} D^i, \end{aligned} \tag{6}$$

$$\underset{+h}{S}\, q^s = q^{s+}, \quad \underset{-h}{S}\, q^s = q^{s-},$$

$$\underset{+h}{S}\, e^j = e^{j+}, \quad \underset{-h}{S}\, e^j = e^{j-}. \tag{7}$$

Obviously, the operators satisfy $\underset{+h}{S} \cdot \underset{-h}{S} = \underset{-h}{S} \cdot \underset{+h}{S} = 1$. With the help of $\underset{-h}{S}$ and $\underset{+h}{S}$, we define a pair of discrete (finite-difference) differentiations acting on the right and left as

$$\begin{aligned} \underset{+h}{D} &= \frac{\underset{+h}{S} - 1}{h} = \sum_{i=1}^{\infty} \frac{h^{i-1}}{i!} D^i, \\ \underset{-h}{D} &= \frac{1 - \underset{-h}{S}}{h} = \sum_{i=1}^{\infty} \frac{(-h)^{i-1}}{i!} D^i, \end{aligned} \tag{8}$$

The operators $\underset{-h}{S}$, $\underset{+h}{S}$, $\underset{-h}{D}$, and $\underset{+h}{D}$ commute with each other in any combination, and $\underset{+h}{D} = \underset{-h}{D}\,\underset{+h}{S}$, $\underset{-h}{D} = \underset{+h}{D}\,\underset{-h}{S}$. The difference derivatives on the right and left are

$$\begin{aligned} q^s_k &= \underset{+h}{D}(q^s) = \frac{q^{s+} - q^s}{t^+ - t} = \frac{q^{s+} - q^s}{h}, \\ q^s_{\bar{k}} &= \underset{-h}{D}(q^s) = \frac{q^s - q^{s-}}{t - t^-} = \frac{q^s - q^{s-}}{h}, \\ e^j_k &= \underset{+h}{D}(e^j) = \frac{e^{j+} - e^j}{t^+ - t} = \frac{e^{j+} - e^j}{h}, \\ e^j_{\bar{k}} &= \underset{-h}{D}(e^j) = \frac{e^j - e^{j-}}{t - t^-} = \frac{e^j - e^{j-}}{h}, \end{aligned} \tag{9}$$

where the subscript k represents the first-order difference derivative. The second-order difference derivatives are given by

$$q^s_{k\bar{k}} = \underset{-h}{D}\,\underset{+h}{D}(q^s), \quad e^j_{k\bar{k}} = \underset{-h}{D}\,\underset{+h}{D}(e^j). \tag{10}$$

The Lagrangian of discrete mechanico-electrical system corresponding to systems (2) and (3) can be expressed as

$$\begin{aligned} &L_{\mathrm{d}} = T(q^s, q^s_k) - V(q^s) + W_{\mathrm{m}}(q^s, e^j_k) - W_{\mathrm{e}}(q^s, e^j), \\ &(s = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m). \end{aligned} \tag{11}$$

Introducing discrete infinitesimal transformations with respect to time, we obtain generalized coordinates and electrical quantities as

$$\begin{aligned} t^* &= t + \varepsilon\xi_0(t, q^s, e^j, q^s_k, e^j_k), \\ q^{s*}(t^*) &= q^s(t) + \varepsilon\xi_s(t, q^s, e^j, q^s_k, e^j_k), \\ e^{j*}(t^*) &= e^j(t) + \varepsilon\eta_j(t, q^s, e^j, q^s_k, e^j_k), \end{aligned} \tag{12}$$

where ε is an infinitesimal parameter, and ξ_0, ξ_s, and η_j are discrete infinitesimal generators, which constitute the generalized Noether operator[18]

$$\begin{aligned} X =\ & \xi_0\frac{\partial}{\partial t} + \xi_s\frac{\partial}{\partial q^s} + \eta_j\frac{\partial}{\partial e^j} \\ & + [\underset{+h}{D}(\xi_s) - q^s_k\,\underset{+h}{D}(\xi_0)]\frac{\partial}{\partial q^s_k} \\ & + [\underset{+h}{D}(\eta_j) - e^j_k\,\underset{+h}{D}(\xi_0)]\frac{\partial}{\partial e^j_k} + \cdots \\ & + h\,\underset{+h}{D}(\xi_0)\frac{\partial}{\partial h}. \end{aligned} \tag{13}$$

In Ref. [18], the Noether theorem of the discrete mechanico-electrical system has been studied. The discrete Noether identity and Lagrange–Maxwell equations of the system have been built. The discrete Noether identity of the system is expressed as

$$\begin{aligned} &X^{(1)}(L_{\mathrm{d}}) + L_{\mathrm{d}}\,\underset{+h}{D}(\xi_0) + (Q^s_{\mathrm{d}} - \frac{\partial F_{\mathrm{d}}}{\partial q^s_k})(\xi_s - q^s_k\xi_0) \\ &+ (u^j_{\mathrm{d}} - \frac{\partial F_{\mathrm{d}}}{\partial e^j_k})(\eta_j - e^j_k\xi_0) + \underset{+h}{D}(G_{N\mathrm{d}}) = 0, \end{aligned} \tag{14}$$

where $G_{N\mathrm{d}} = G_{N\mathrm{d}}(t, q^s, e^j, q^s_k, e^j_k)$ is the discrete gauge function, and

$$\begin{aligned} X^{(1)} =\ & \xi_0\frac{\partial}{\partial t} + \xi_s\frac{\partial}{\partial q^s} + \eta_j\frac{\partial}{\partial e^j} \\ & + [\underset{+h}{D}(\xi_s) - q^s_k\,\underset{+h}{D}(\xi_0)]\frac{\partial}{\partial q^s_k} \\ & + [\underset{+h}{D}(\eta_j) - e^j_k\,\underset{+h}{D}(\xi_0)]\frac{\partial}{\partial e^j_k}. \end{aligned} \tag{15}$$

The discrete Lagrange–Maxwell equations and energy-evolution equation of the system are given by

$$\underset{-h}{D}\left(\frac{\partial L_{\rm d}}{\partial q_k^s}\right)-\frac{\partial L_{\rm d}}{\partial q^s}+\frac{\partial F_{\rm d}}{\partial q_k^s}=Q_{\rm d}^s, \tag{16}$$

$$\underset{-h}{D}\left(\frac{\partial L_{\rm d}}{\partial e_k^j}\right)-\frac{\partial L_{\rm d}}{\partial e^j}+\frac{\partial F_{\rm d}}{\partial e_k^j}=u_{\rm d}^j, \tag{17}$$

$$\begin{aligned}&\frac{\partial L_{\rm d}}{\partial t}+\underset{-h}{D}\left(q_k^s\frac{\partial L_{\rm d}}{\partial q_k^s}+e_k^j\frac{\partial L_{\rm d}}{\partial e_k^j}-L_{\rm d}\right)\\&-(Q_{\rm d}^s-\frac{\partial F_{\rm d}}{\partial q_k^s})q_k^s-\left(u_{\rm d}^j-\frac{\partial F_{\rm d}}{\partial e_k^j}\right)e_k^j=0,\end{aligned} \tag{18}$$

where $Q_{\rm d}^s$ is the discrete non-potential generalized force, $u_{\rm d}^j$ is the discrete generalized electromotive force, and $F_{\rm d}$ is the discrete dissipative function in the following form:

$$F_{\rm d}=F_{\rm ed}(e_k^j)+F_{\rm md}(q^s,q_k^s). \tag{19}$$

3. Noether–Mei symmetry of the system

Under the infinitesimal transformations (12), functions $L_{\rm d}$, $Q_{\rm d}^s$, $u_{\rm d}^j$, and $F_{\rm d}$ become $L_{\rm d}^*$, $Q_{\rm d}^{s*}$, $u_{\rm d}^{j*}$, and $F_{\rm d}^*$, respectively. Using Taylor expansion, they will further become

$$\begin{aligned}L_{\rm d}^*&=L_{\rm d}(t^*,q^{s*},e^{j*},q_k^{s*},e_k^{j*})\\&=L_{\rm d}(t,q^s,e^j,q_k^s,e_k^j)+\varepsilon X^{(1)}(L_{\rm d})+O(\varepsilon^2),\\Q_{\rm d}^{s*}&=Q_{\rm d}^s(t^*,q^{s*},e^{j*},q_k^{s*},e_k^{j*})\\&=Q_{\rm d}^s(t,q^s,e^j,q_k^s,e_k^j)+\varepsilon X^{(1)}(Q_{\rm d}^s)+O(\varepsilon^2),\\u_{\rm d}^{j*}&=u_{\rm d}^j(t^*,q^{s*},e^{j*},q_k^{s*},e_k^{j*})\\&=u_{\rm d}^j(t,q^s,e^j,q_k^s,e_k^j)+\varepsilon X^{(1)}(u_{\rm d}^j)+O(\varepsilon^2),\\F_{\rm d}^*&=F_{\rm d}^(q^{s*},q_k^{s*},e_k^{j*})\\&=F_{\rm d}(q^s,q_k^s,e_k^j)+\varepsilon X^{(1)}(F_{\rm d})+O(\varepsilon^2).\end{aligned} \tag{20}$$

Definition 1 Under the infinitesimal transformations (12), if $L_{\rm d}$, $Q_{\rm d}^s$, $u_{\rm d}^j$, and $F_{\rm d}$ are replaced by $L_{\rm d}^*$, $Q_{\rm d}^{s*}$, $u_{\rm d}^{j*}$, and $F_{\rm d}^*$, the forms of Eqs. (16)–(18) are invariant, i.e.,

$$\underset{-h}{D}\left(\frac{\partial L_{\rm d}^*}{\partial q_k^s}\right)-\frac{\partial L_{\rm d}^*}{\partial q^s}+\frac{\partial F_{\rm d}^*}{\partial q_k^s}=Q_{\rm d}^{s*}, \tag{21}$$

$$\underset{-h}{D}\left(\frac{\partial L_{\rm d}^*}{\partial e_k^j}\right)-\frac{\partial L_{\rm d}^*}{\partial e^j}+\frac{\partial F_{\rm d}^*}{\partial e_k^j}=u_{\rm d}^{j*}, \tag{22}$$

$$\begin{aligned}&\frac{\partial L_{\rm d}^*}{\partial t}+\underset{-h}{D}\left(q_k^s\frac{\partial L_{\rm d}^*}{\partial q_k^s}+e_k^j\frac{\partial L_{\rm d}^*}{\partial e_k^j}-L_{\rm d}^*\right)\\&-\left(Q_{\rm d}^{s*}-\frac{\partial F_{\rm d}^*}{\partial q_k^s}\right)q_k^s-\left(u_{\rm d}^{j*}-\frac{\partial F_{\rm d}^*}{\partial e_k^j}\right)e_k^j=0.\end{aligned} \tag{23}$$

The invariance is named the Mei symmetry of the system.

According to Definition 1, ignoring the second- and higher-order terms of ε, we obtain the criterion of the Mei symmetry of the system.

Criterion 1 For the discrete mechanico-electrical systems (16)–(18), if the infinitesimal transformations (12) satisfy

$$\begin{aligned}&\underset{-h}{D}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial q_k^s}\right]-\frac{\partial X^{(1)}(L_{\rm d})}{\partial q^s}+\frac{\partial X^{(1)}(F_{\rm d})}{\partial q_k^s}\\&=X^{(1)}(Q_{\rm d}^s),\end{aligned} \tag{24}$$

$$\begin{aligned}&\underset{-h}{D}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial e_k^j}\right]-\frac{\partial X^{(1)}(L_{\rm d})}{\partial e^j}+\frac{\partial X^{(1)}(F_{\rm d})}{\partial e_k^j}\\&=X^{(1)}(u_{\rm d}^j),\end{aligned} \tag{25}$$

$$\begin{aligned}&\frac{\partial X^{(1)}(L_{\rm d})}{\partial t}+\underset{-h}{D}\left[q_k^s\frac{\partial X^{(1)}(L_{\rm d})}{\partial q_k^s}\right.\\&\left.+e_k^j\frac{\partial X^{(1)}(L_{\rm d})}{\partial e_k^j}-X^{(1)}(L_{\rm d})\right]\\&-\left[X^{(1)}(Q_{\rm d}^s)-\frac{\partial X^{(1)}(F_{\rm d})}{\partial q_k^s}\right]q_k^s\\&-\left[X^{(1)}(u_{\rm d}^j)-\frac{\partial X^{(1)}(F_{\rm d})}{\partial e_k^j}\right]e_k^j=0,\end{aligned} \tag{26}$$

then the corresponding symmetry is Mei symmetry of the system. Equations (24)–(26) are the criterion equations of the Mei symmetry.

Definition 2 If the discrete mechanico-electrical systems (16)–(18) are under the Noether symmetry and Mei symmetry simultaneously, the system would be under the Noether–Mei symmetry.

Considering the Noether identity (14) and the criterion equations (24)–(26) of Mei symmetry, we obtain the criterion of the Noether–Mei symmetry.

Criterion 2 For the discrete mechanico-electrical systems (16)–(18), if there exists a discrete gauge function $G_{N\rm d}=G_{N\rm d}(t,q^s,e^j,q_k^s,e_k^j)$ satisfying

$$\begin{aligned}&\left\{X^{(1)}(L_{\rm d})+L_{\rm d}\underset{+h}{D}(\xi_0)+\left(Q_{\rm d}^s-\frac{\partial F_{\rm d}}{\partial q_k^s}\right)(\xi_s-q_k^s\xi_0)\right.\\&\left.+\left(u_{\rm d}^j-\frac{\partial F_{\rm d}}{\partial e_k^j}\right)(\eta_j-e_k^j\xi_0)+\underset{+h}{D}(G_{N\rm d})\right\}^2\\&+\left\{\underset{-h}{D}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial q_k^s}\right]-\frac{\partial X^{(1)}(L_{\rm d})}{\partial q^s}\right.\\&\left.+\frac{\partial X^{(1)}(F_{\rm d})}{\partial q_k^s}-X^{(1)}(Q_{\rm d}^s)\right\}^2\\&+\left\{\underset{-h}{D}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial e_k^j}\right]-\frac{\partial X^{(1)}(L_{\rm d})}{\partial e^j}\right.\\&\left.+\frac{\partial X^{(1)}(F_{\rm d})}{\partial e_k^j}-X^{(1)}(u_{\rm d}^j)\right\}^2\end{aligned}$$

$$+\left\{\frac{\partial X^{(1)}(L_{\rm d})}{\partial t}+\underset{-h}{D}\left[q_k^s\frac{\partial X^{(1)}(L_{\rm d})}{\partial q_k^s}\right.\right.$$
$$\left.+e_k^j\frac{\partial X^{(1)}(L_{\rm d})}{\partial e_k^j}-X^{(1)}(L_{\rm d})\right]$$
$$-\left[X^{(1)}(Q_{\rm d}^s)-\frac{\partial X^{(1)}(F_{\rm d})}{\partial q_k^s}\right]q_k^s$$
$$\left.-\left[X^{(1)}(u_{\rm d}^j)-\frac{\partial X^{(1)}(F_{\rm d})}{\partial e_k^j}\right]e_k^j\right\}^2=0, \quad (27)$$

the corresponding symmetry would be Noether–Mei symmetry. Equation (27) is the criterion equation of Noether–Mei symmetry for the system.

4. Conserved quantities induced by Noether–Mei symmetry

In certain conditions, the Noether–Mei symmetry of a discrete mechanico-electrical system can lead to a discrete Noether conserved quantity and a Mei conserved quantity.

Proposition 1 For discrete mechanico-electrical systems (16)–(18), if the generators of Noether–Mei symmetry ξ_0, ξ_s, and η_j, as well as the discrete gauge function $G_{N\rm d}$ satisfy Noether identity (14), the Noether–Mei symmetry can lead to a discrete Noether conserved quantity in the following form:[18]

$$I_{N\rm d}=\xi_0\underset{-h}{S}(L_{\rm d})+\underset{-h}{S}\left(\frac{\partial L_{\rm d}}{\partial q_k^s}\right)[\xi_s-q_{\bar{k}}^s\xi_0]$$
$$+\underset{-h}{S}\left(\frac{\partial L_{\rm d}}{\partial e_k^j}\right)[\eta_j-e_{\bar{k}}^j\xi_0]+G_{N\rm d}=\text{const.} \quad (28)$$

Proof From definition 2, it can be deduced that if the system is in Noether–Mei symmetry, the system must be under Noether symmetry. Using Leibnitz rules[14]

$$\underset{+h}{D}(FG)=\underset{+h}{D}(F)G+F\underset{+h}{D}(G)+h\underset{+h}{D}(F)\underset{+h}{D}(G),$$
$$\underset{-h}{D}(FG)=\underset{-h}{D}(F)G+F\underset{-h}{D}(G)-h\underset{-h}{D}(F)\underset{-h}{D}(G), \quad (29)$$

the Noether identity (14) can be rewritten as

$$X^{(1)}(L_{\rm d})+L_{\rm d}\underset{+h}{D}(\xi_0)+\left(Q_{\rm d}^s-\frac{\partial F_{\rm d}}{\partial q_k^s}\right)(\xi_s-q_k^s\xi_0)$$
$$+\left(u_{\rm d}^j-\frac{\partial F_{\rm d}}{\partial e_k^j}\right)(\eta_j-e_k^j\xi_0)+\underset{+h}{D}(G_{N\rm d})$$
$$=\xi_0\left\{\frac{\partial L_{\rm d}}{\partial t}+\underset{-h}{D}\left(q_k^s\frac{\partial L_{\rm d}}{\partial q_k^s}+e_k^j\frac{\partial L_{\rm d}}{\partial e_k^j}-L_{\rm d}\right)\right.$$
$$\left.-(Q_{\rm d}^s-\frac{\partial F_{\rm d}}{\partial q_k^s})q_k^s-\left(u_{\rm d}^j-\frac{\partial F_{\rm d}}{\partial e_k^j}\right)e_k^j\right\}$$
$$+\xi_s\left\{\frac{\partial L_{\rm d}}{\partial q^s}-\underset{-h}{D}\left(\frac{\partial L_{\rm d}}{\partial q_k^s}\right)+\left(Q_{\rm d}^s-\frac{\partial F_{\rm d}}{\partial q_k^s}\right)\right\}$$
$$+\eta_j\left\{\frac{\partial L_{\rm d}}{\partial e^j}-\underset{-h}{D}\left(\frac{\partial L_{\rm d}}{\partial e_k^j}\right)+\left(u_{\rm d}^j-\frac{\partial F_{\rm d}}{\partial e_k^j}\right)\right\}$$
$$+\underset{+h}{D}\left\{\xi_0\underset{-h}{S}(L_{\rm d})+\underset{-h}{S}\left(\frac{\partial L_{\rm d}}{\partial q_k^s}\right)[\xi_s-q_{\bar{k}}^s\xi_0]\right.$$
$$\left.+\underset{-h}{S}\left(\frac{\partial L_{\rm d}}{\partial e_k^j}\right)[\eta_j-e_{\bar{k}}^j\xi_0]+G_{N\rm d}\right\}=0. \quad (30)$$

From Criterion 2 and Eqs. (16)–(18), if the infinitesimal generators of Noether–Mei symmetry ξ_0, ξ_s, and η_j, as well as the discrete gauge function $G_{N\rm d}$ satisfy the Noether identity or Eq. (30), we can obtain

$$\underset{+h}{D}\left\{\xi_0\underset{-h}{S}(L_{\rm d})+\underset{-h}{S}\left(\frac{\partial L_{\rm d}}{\partial q_k^s}\right)[\xi_s-q_{\bar{k}}^s\xi_0]\right.$$
$$\left.+\underset{-h}{S}\left(\frac{\partial L_{\rm d}}{\partial e_k^j}\right)[\eta_j-e_{\bar{k}}^j\xi_0]+G_{N\rm d}\right\}=0. \quad (31)$$

It can be easily deduced that $I_{N\rm d}$ is a Noether conserved quantity. Up to this point, Proposition 1 is proved.

Proposition 2 For the discrete mechanico-electrical systems (16)–(18), if the generators of Noether–Mei symmetry ξ_0, ξ_s, and η_j, as well as the discrete gauge function $G_{\rm Md}=G_{\rm Md}(t,q^s,e^j,q_k^s,e_k^j)$ satisfy

$$X^{(1)}(L_{\rm d})\underset{+h}{D}(\xi_0)+X^{(1)}[X^{(1)}(L_{\rm d})]$$
$$+\left[X^{(1)}(Q_{\rm d}^s)-\frac{\partial X^{(1)}(F_{\rm d})}{\partial q_k^s}\right](\xi_s-q_k^s\xi_0)$$
$$+\left[X^{(1)}(u_{\rm d}^j)-\frac{\partial X^{(1)}(F_{\rm d})}{\partial e_k^j}\right]$$
$$\times(\eta_j-e_k^j\xi_0)+\underset{+h}{D}(G_{\rm Md})=0, \quad (32)$$

the Noether–Mei symmetry can lead to discrete Mei-conserved quantity

$$I_{\rm Md}=\underset{-h}{S}[X^{(1)}(L_{\rm d})]\xi_0+\underset{-h}{S}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial q_k^s}\right](\xi_s-q_{\bar{k}}^s\xi_0)$$
$$+\underset{-h}{S}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial e_k^j}\right](\eta_j-e_{\bar{k}}^j\xi_0)+G_{\rm Md}$$
$$=\text{const.} \quad (33)$$

Proof Equation (32) can be rewritten as

$$X^{(1)}(L_{\rm d})\underset{+h}{D}(\xi_0)+\frac{\partial X^{(1)}(L_{\rm d})}{\partial t}\xi_0$$
$$+\frac{\partial X^{(1)}(L_{\rm d})}{\partial q^s}\xi_s+\frac{\partial X^{(1)}(L_{\rm d})}{\partial e^j}\eta_j$$
$$+[\underset{+h}{D}(\xi_s)-q_k^s\underset{+h}{D}(\xi_0)]\frac{\partial X^{(1)}(L_{\rm d})}{\partial q_k^s}$$
$$+[\underset{+h}{D}(\eta_j)-e_k^j\underset{+h}{D}(\xi_0)]\frac{\partial X^{(1)}(L_{\rm d})}{\partial e_k^j}$$

$$+\left[X^{(1)}(Q_{\rm d}^s)-\frac{\partial X^{(1)}(F_{\rm d})}{\partial q_k^s}\right](\xi_s-q_k^s\xi_0)$$
$$+\left[X^{(1)}(u_{\rm d}^j)-\frac{\partial X^{(1)}(F_{\rm d})}{\partial e_k^j}\right](\eta_j-e_k^j\xi_0)$$
$$+\underset{+h}{D}(G_{\rm Md})=0. \tag{34}$$

Using rules (29), Eq. (34) becomes

$$\xi_0\left\{\frac{\partial X^{(1)}(L_{\rm d})}{\partial t}+\underset{-h}{D}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial q_k^s}q_k^s+\frac{\partial X^{(1)}(L_{\rm d})}{\partial e_k^j}e_k^j-X^{(1)}(L_{\rm d})\right]-\left[X^{(1)}(Q_{\rm d}^s)-\frac{\partial X^{(1)}(F_{\rm d})}{\partial q_k^s}\right]q_k^s-\left[X^{(1)}(u_{\rm d}^j)-\frac{\partial X^{(1)}(F_{\rm d})}{\partial e_k^j}\right]e_k^j\right\}$$
$$+\xi_s\left\{\frac{\partial X^{(1)}(L_{\rm d})}{\partial q^s}+\left[X^{(1)}(Q_{\rm d}^s)-\frac{\partial X^{(1)}(F_{\rm d})}{\partial q_k^s}\right]-\underset{-h}{D}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial q_k^s}\right]\right\}$$
$$+\eta_j\left\{\frac{\partial X^{(1)}(L_{\rm d})}{\partial e^j}+\left[X^{(1)}(u_{\rm d}^j)-\frac{\partial X^{(1)}(F_{\rm d})}{\partial e_k^j}\right]-\underset{-h}{D}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial e_k^j}\right]\right\}$$
$$+\underset{+h}{D}\left\{\underset{-h}{S}[X^{(1)}(L_{\rm d})]\xi_0+\underset{-h}{S}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial q_k^s}\right](\xi_s-q_k^s\xi_0)+\underset{-h}{S}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial e_k^j}\right](\eta_j-e_k^j\xi_0)+G_{\rm Md}\right\}=0. \tag{35}$$

From definition 2, we can know that if the system is in Noether–Mei symmetry, the system must be under Mei symmetry. Considering Eqs. (24)–(26), we obtain

$$\underset{+h}{D}\left\{\underset{-h}{S}[X^{(1)}(L_{\rm d})]\xi_0+\underset{-h}{S}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial q_k^s}\right](\xi_s-q_{\bar{k}}^s\xi_0)+\underset{-h}{S}\left[\frac{\partial X^{(1)}(L_{\rm d})}{\partial e_k^j}\right](\eta_j-e_{\bar{k}}^j\xi_0)+G_{\rm Md}\right\}=0. \tag{36}$$

Up to this point, Proposition 2 is proved.

5. An example

Now let us consider a mechanico-electrical system with a discrete Lagrangian

$$L_{\rm d}=\frac{1}{2}[(q_k)^2+(e_k)^2]+e_k-\frac{1}{2}\left(\frac{e^++e}{2}\right)^2. \tag{37}$$

The generalized electromotive force is

$$u_{\rm d}=e_k, \tag{38}$$

and the discrete dissipative function is

$$F_{\rm d}=\frac{1}{2}(e_k)^2. \tag{39}$$

From Eqs. (16) and (17), we can obtain the discrete motion equations

$$q_{k\bar{k}}=0,\quad e_{k\bar{k}}=-\left(\frac{e^++e}{2}\right), \tag{40}$$

then we have

$$\begin{aligned}X^{(1)}(L_{\rm d})&=q_k[\underset{+h}{D}(\xi)-q_k^1\underset{+h}{D}(\xi_0)]\\&\quad+e_k[\underset{+h}{D}(\eta)-e_k\underset{+h}{D}(\xi_0)]\\&\quad+[\underset{+h}{D}(\eta)-e_k\underset{+h}{D}(\xi_0)]-\left(\frac{e^++e}{2}\right)\eta,\\X^{(1)}(u_{\rm d})&=\underset{+h}{D}(\eta)-e_k\underset{+h}{D}(\xi_0),\\X^{(1)}(F_{\rm d})&=e_k[\underset{+h}{D}(\eta)-e_k\underset{+h}{D}(\xi_0)].\end{aligned} \tag{41}$$

Choosing the discrete infinitesimal generators as

$$\xi_0=1,\quad \xi=\frac{t^++t}{2},\quad \eta=0, \tag{42}$$

we obtain

$$\begin{aligned}&X^{(1)}(L_{\rm d})=q_k,\\&X^{(1)}(Q_{\rm d})=X^{(1)}(u_{\rm d})=X^{(1)}(F_{\rm d})=0.\end{aligned} \tag{43}$$

Substituting Eqs. (37)–(43) into the criterion equation of Noether–Mei symmetry (27), we obtain the gauge function

$$G_{N\rm d}=-\left(\frac{q^++q}{2}\right). \tag{44}$$

From Proposition 1, we can obtain the discrete Noether-conserved quantity

$$\begin{aligned}I_{N\rm d}&=-\frac{1}{2}[(q_{\bar{k}})^2+(e_{\bar{k}})^2]\\&\quad+q_{\bar{k}}\left(\frac{t+t^-}{2}\right)-\frac{q+q^-}{2}\\&\quad-\frac{1}{2}\left(\frac{e+e^-}{2}\right)^2=\text{const.}\end{aligned} \tag{45}$$

According to Proposition 2, substituting Eqs. (42)–(43) into Eq. (32), we obtain the discrete gauge function

$$G_{\rm Md}=-\left(\frac{t^++t}{2}\right)+q_{\bar{k}}. \tag{46}$$

From Eq. (33), we can obtain the Mei-conserved quantity

$$I_{\rm Md}=q_{\bar{k}}=\text{const.} \tag{47}$$

6. Conclusion

In this paper, Noether–Mei symmetry and the corresponding conserved quantities of the discrete mechanico-electrical system are studied. The criterion of Noether–Mei symmetry is deduced, and the conserved quantities induced by Noether–Mei symmetry are also obtained. When $h \to 0$, the results presented in this paper will turn into the corresponding continuous situation naturally.

References

[1] Lutzky M 1979 *J. Phys.* A: *Math. Gen.* **12** 973

[2] Mei F X 1999 *Applications of Lie Groups and Lie Algebras to Constrained Mechanical Systems* (Beijing: Science Press) (in Chinese)

[3] Mei F X 2000 *J. Beijing Institute Tech.* **9** 120

[4] Mei F X 2004 *Symmetries and Conserved Quantities of Constrained Mechanical Systems* (Beijing: Beijing Institute of Technology Press) (in Chinese)

[5] Fu J L, Chen LQ, Jimnez S and Tang Y F 2006 *Phys. Lett.* A **358** 5

[6] Hydon P E 2000 *Proc. R. Soc.* A **456** 2835

[7] Levi D, Tremblay S and Winternitz P 2000 *J. Phys.* A: *Math. Gen.* **33** 8507

[8] Levi D and Winternitz P 2006 *J. Phys.* A: *Math. Gen.* **39** R1

[9] Zhang H B, Chen L Q and Liu R W 2005 *Chin. Phys.* **14** 1063

[10] McLachlan R and Perlmutter M 2006 *J. Nonlinear Sci.* **16** 283

[11] Zhang Y 2007 *Acta Phys. Sin.* **56** 1855 (in Chinese)

[12] Fu J L, Dai G D, Salvaolor J and Tang Y F 2007 *Chin. Phys.* **16** 570

[13] Shi S Y, Fu J L and Chen L Q 2007 *Acta Phys. Sin.* **56** 3060 (in Chinese)

[14] Dorodnitsyn V 1991 *J. Sov. Math.* **55** 1490

[15] Dorodnitsyn V 2001 *Appl. Numer. Math.* **39** 307

[16] Dorodnitsyn V 2004 *J. Math. Phys.* **45** 336

[17] Dorodnitsyn V 2010 *J. Eng. Math.* **66** 253

[18] Fu J L, Chen B Y and Xie F P 2008 *Chin. Phys.* B **17** 4354

[19] Fu J L, Chen B Y and Chen L Q 2009 *Phys. Lett.* A **373** 409

[20] Fu J L, Fu H and Liu R W 2010 *Phys. Lett.* A **374** 1812

[21] Fu J L, Chen L Q and Chen B Y 2010 *Sci. China-Phys. Mech. Astron.* **53** 545

[22] Fu J L, Chen L Q and Chen B Y 2010 *Sci. China-Phys. Mech. Astron.* **53** 1687

[23] Wang P 2011 *Chin. Phys. Lett.* **28** 040203

[24] Mei F X 1991 *Advanced Analytical Mechanics* (Beijing: Beijing Institute of Technology Press) (in Chinese)