A 2-stage strategy updating rule promotes cooperation in the prisoner's dilemma game*

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In this study, we propose a spatial prisoner's dilemma game model with a 2-stage strategy updating rule, and focus on the cooperation behavior of the system. In the first stage, i.e., the pre-learning stage, a focal player decides whether to update his strategy according to the pre-learning factor β and the payoff difference between himself and the average of his neighbors. If the player makes up his mind to update, he enters into the second stage, i.e., the learning stage, and adopts a strategy of a randomly selected neighbor according to the standard Fermi updating rule. The simulation results show that the cooperation level has a non-trivial dependence on the pre-learning factor. Generally, the cooperation frequency decreases as the pre-learning factor increases; but a high cooperation level can be obtained in the intermediate region of $-3 < \beta < -1$. We then give some explanations via studying the co-action of pre-learning and learning. Our results may sharpen the understanding of the influence of the strategy updating rule on evolutionary games.

Keywords: evolutionary game theory, strategy updating, social cooperation, prisoner's dilemma game

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1. Introduction

From biological systems to economic systems, cooperation plays a key role in evolution. However, understanding the emergence and persistence of cooperative behavior among selfish individuals remains a challenge.[1] The evolutionary game theory[2,3] provides a powerful tool to investigate the altruistic action among selfish individuals. The prisoner's dilemma game (PDG)[4] is frequently studied in this field. In the original PDG, two players simultaneously decide whether to cooperate (C) or defect (D). If they both choose C (D), each will get a payoff of R (P). If one of them chooses C, while the other chooses D, the defector will get a maximum payoff of T, and the cooperator gets a minimum payoff of S. The four payoff values are arranged as $T > R > S > P$ with precondition $2R > T + S$. As a result, the mutual defection is the only evolutionary stable strategy (ESS), even both players would be better off if they choose cooperation. The PDG model illustrates the conflict of interest: what is best for the individual and what is best for the group. Thus, it creates a social dilemma.

Traditionally, the evolutionary PDG is studied in an infinite well-mixed population, where all individuals are interacting with each other, and thus the cooperation cannot emerge under replicator dynamics.[5] However, observations in the real world usually show the opposite. To explain the emergence and maintenance of cooperative behavior, several mechanisms have been invoked in previous studies, including kin selection,[6] direct or indirect reciprocity,[7] group selection,[8] voluntary participation,[9] punishment,[10,11] and so on.[12]

By introducing a spatial structure, the pioneering work done by Nowak and May[13] has increasingly attracted interest from different fields (see Refs. [14]–[16] and references therein) as a significant extension of the traditional evolutionary game

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theory focusing on well-mixed populations. In spatial games, individuals are situated on the vertices of a graph, and the edges indicate the interactions among individuals. The evolutionary success of an individual is determined by its payoff accrued in the pairwise interactions with its neighbors. In this context, network topology[17,18] plays a key role in the evolution of cooperation, which has been widely studied over the past decade, e.g., regular networks,[19−26] small-world networks,[27−33] interdependent networks,[34] and scale-free networks.[35−41] Quite recently, the adaptive networks[42,43] and the mobility of players[44,45] have also been studied, and the mutual interaction between network topology and the evolution of strategies is considered.

Typically, the evolutionary game is carried out as follows. The player selects a neighbor according to a certain selection rule (e.g., random selection,[14] richest-following,[46] preferential selection,[47] and then adopts the chosen neighbor's current strategy according to a certain updating rule (e.g., deterministic rule,[13] stochastic evolutionary rule,[19] self-questioning,[48−50] aspiration-based learning[51,52]). However, in real social systems, the strategy updating process can be more complex, i.e., not all players have the passion to update their strategies. For example, if an employee is satisfied with his current state, he is unwilling to change his job. Thus, in our paper, we integrate a pre-learning process into the traditional strategy updating process, and introduce a 2-stage strategy updating rule to the spatial prisoner's dilemma game. Here the pre-learning means that whether or not to learn is decided according to the payoff difference between the focal player and its neighbors.

The paper is organized as follows. In Section 2, we describe the evolutionary game model and the strategy-updating rule used in this work. The results and discussion are given in Section 3. The paper is concluded in Section 4.

2. Model

We consider a spatial PDG model. The players are located on the vertices of an $L \times L$ two-dimensional square lattice with periodic boundary conditions. Each individual is allowed to interact only with its four nearest neighbors, and self-interactions are excluded. A player located on site i is initially assigned a strategy s_i ($s_i = 1$ stands for cooperation C, and $s_i = 0$ indicates defection D) randomly and can change his strategy after each iteration cycle of the game. The strategies can be described in the form of vector

$$\theta = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{1}$$

Following the previous works,[35,53−55] we use a simplified version of the PDG model, which can be expressed by a payoff matrix

$$\chi = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}, \tag{2}$$

where $1 < b < 2$ represents the advantage of defectors over cooperators. The total payoff of player i is the sum of payoffs after interacting with his four neighbors, which is written as

$$P_i = \sum_{j \in \Omega_i} \theta_i^{\mathrm{T}} \chi \theta_j, \tag{3}$$

where Ω_i denotes the four neighbors of player i.

In our model, the strategy updating process is 2-stage. In the first stage, i.e., the pre-learning stage, player i will decide whether to update his strategy according to the pre-learning probability

$$\mathrm{Prob}^i = \frac{1}{1 + \exp[(P_i - P_i^{\mathrm{avg}}) * \beta]}, \tag{4}$$

where P_i^{avg} is the average payoff of the four nearest neighbors of player i, and the tunable factor β represents the rationality of pre-learning. If $P_i > P_i^{\mathrm{avg}}$ (or $P_i < P_i^{\mathrm{avg}}$), a larger value of β will result in a smaller (or larger) probability of pre-learning. The traditional evolutionary game model does not have this pre-learning process.

If player i passes the test of the pre-learning stage, he will enter into the learning stage. In this second stage, the player randomly selects a neighbor j from his four neighbors, and then adopts the strategy of player j according to the Fermi rule[19]

$$H_{i \to j} = \frac{1}{1 + \exp[(P_i - P_j)/\kappa]}. \tag{5}$$

Here κ characterizes the noise, including bounded rationality, individual trials, errors in decision, etc. The effect of noise κ on the stationary density of cooperation has been well studied in Refs. [56] and [57]. In this paper, κ is set to 0.1, as in many other works.

3. Simulation results and qualitative analysis

Initially, cooperators and defectors are randomly distributed on a 100×100 square lattice. After every iteration cycle of the game, all players simultaneously update their strategies according to Eqs. (4) and (5). The equilibrium frequency of cooperators is obtained by averaging the lastest 3000 generations after a transient time of 10000 generations. Each piece of data is averaged over 50 runs. The cooperation frequency ρ_c is defined as the ratio of cooperator number N_C to the total number of players N. Thus, $0 \leq \rho_c \leq 1$ holds, where 0 corresponds to all defectors, and 1 corresponds to all cooperators.

Figure 1(a) reports the relationship between cooperation frequency ρ_c and temptation parameter b. Compared with the result of the classical model in which the pre-learning stage is removed (the star symbol), ρ_c can be remarkably promoted by the pre-learning stage when $\beta < 0$, e.g., ρ_c can even be larger than 0.9 when $\beta = -1.5$ and $b = 1.0$. In addition, we also find that the cooperation level has a non-trivial dependance on β. To further investigate the effect of β, we show the relationship of ρ_c and the pre-learning parameter β in Fig. 1(b). Generally speaking, the cooperation frequency decreases as β increases; when β is slightly larger than 0, the cooperators are extinct. However, in the region of $-3 < \beta < -1$, we can also obtain a high cooperation level. In particular, the cooperation frequency reaches a local maximum level at the point of $\beta \approx -1.5$.

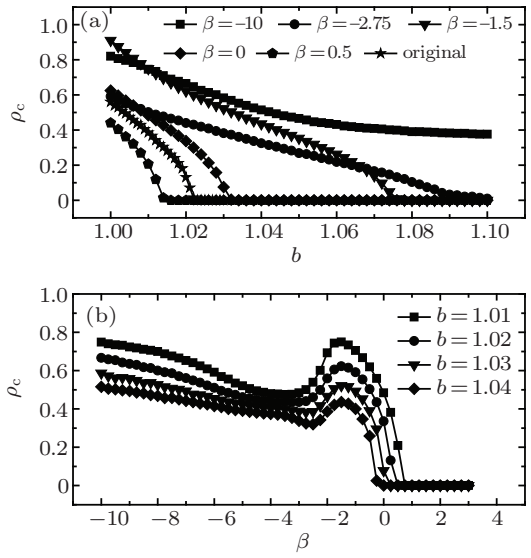


Fig. 1. (a) The relationship of cooperation frequency ρ_c and b. (b) The relationship of cooperation frequency ρ_c and pre-learning factor β for different values of b.

Next we will examine the evolution process from a microscopic view. Figure 2(a) shows the evolution time series, and figures 2(b)–2(u) show the evolutions of spatial patterns formed by cooperators and defectors. In the classical model, because defectors can get more payoffs under the condition that cooperators and defectors are randomly assigned, defectors perform better than cooperators in the early part of the time series. Thus, the cooperation level undergoes a decline. Afterwards, cooperators can form compact cooperator clusters to compete with defectors or even spread to the whole system, which is universally accepted as the main reason for the persistence of cooperation [e.g., Figs. 2(b)–2(e)]. When β is introduced, since the evolution is influenced not only by the payoff difference between cooperators and defectors but also by the individuals' relative states in their environments, the cooperation decline is not evident (Fig. 2(a)). For low values of β [e.g., Fig. 2(f)–2(i)], we can see that some isolated defectors are placed in the ocean of cooperators at the steady state. When the pre-learning factor β increases [e.g., Figs. 2(j)–2(m)], we can see that some isolated defectors invade their cooperator neighbors, which results in an increase of defectors. On the other hand, some cooperators fail to form compact clusters, and this causes a decrease in the cooperation level. However, as β further increases [e.g., Fig. 2(n)–2(q)], the clusters of cooperators begin to expand, and the cooperation level increases obviously. When β is large enough [e.g., Fig. 2(r)–2(u)], defectors tend to dominate eventually, the cooperators become extinct, and the lattice is occupied by defector clusters. The general observation is the same as that in Fig. 1 that the cooperation frequency ρ_c has a non-trivial dependence on the pre-learning factor β. Besides, we find that although the cooperation levels of $\beta = -10$ and $\beta = -1.5$ are close, the evolutionary spatial patterns are quite different.

To explain the phenomena in Figs. 1 and 2, we investigate the relationship of P_C, P_D, P_{NC}, and P_{ND} in the equilibrium state, where P_C (P_D) indicates the average payoff of cooperators (defectors); P_{NC} (P_{ND}) represents the average payoff of all cooperators' (defectors') neighbors. As shown in Fig. 3, we observe that both P_D and P_{NC} monotonically decrease with the increase in the pre-learning factor β, and the influence is more remarkable for P_D. P_C and P_{ND} have non-trivial dependence on the pre-learning factor: the payoffs decrease as the pre-learning factor increases, but a high payoff can be obtained in the intermediate region of

$-3 < \beta < -1$. In our model, a cooperator (defector) becomes a defector (cooperator) via two stages. In the pre-learning stage, players will acquire their right of learning from their neighbors according to Eq. (4), which is guided by β. For cooperators (defectors), Prob^i is determined by the difference of P_{C} and P_{NC} (P_{N} and P_{ND}) and the factor β. In the learning stage, players who successfully pass the pre-learning stage randomly choose one of their nearest neighbors and adopt their strategies according to Eq. (5), which is guided by κ. According to the above analyses, the cooperation behavior in Fig. 1 is jointly decided by the pre-learning (with probability Prob^i) and the learning (with probability $H_{i\to j}$) stages. Thus, we can explain the results via an understanding of the co-action of pre-learning and learning.

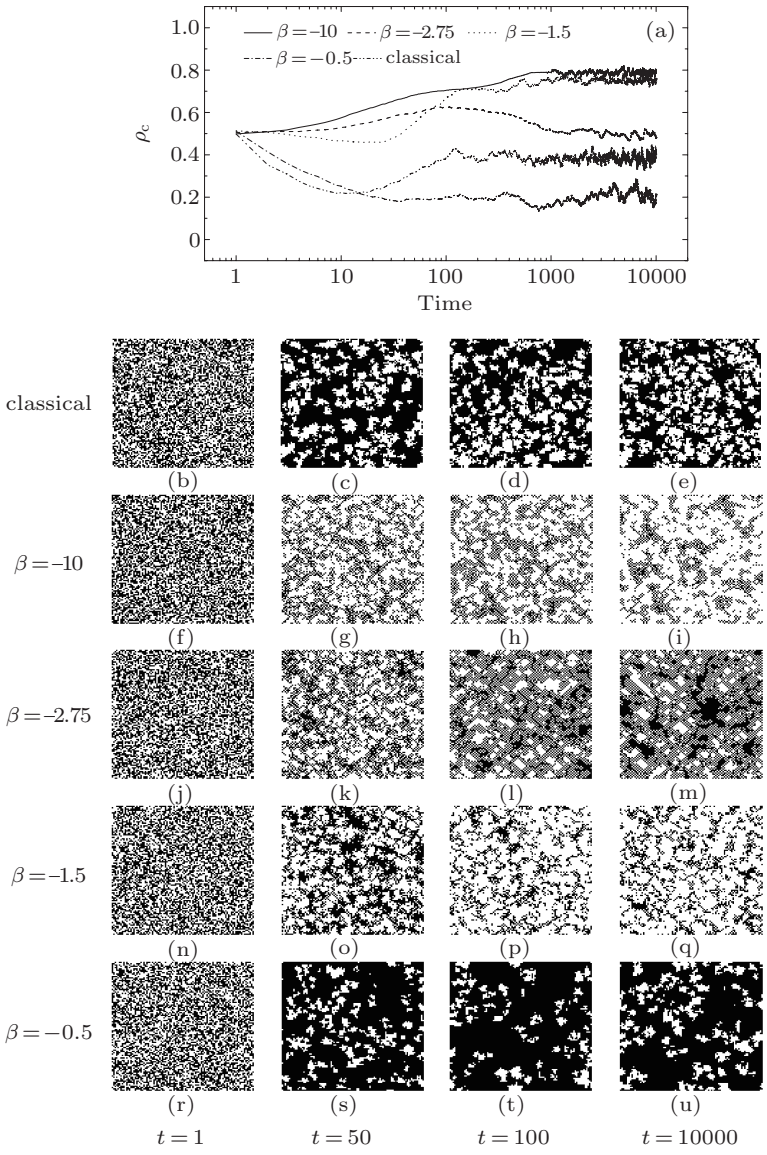


Fig. 2. (a) The evolution time series, (b)–(e) the evolution of the spatial patterns classical model, (f)–(i) the evolution of spatial patterns for $\beta = -10$, (j)–(m) the evolution of spatial patterns for $\beta = -2.75$, (n)–(q) the evolution of spatial patterns for $\beta = -1.5$, (r)–(u) the evolution of spatial patterns for $\beta = 0.5$. Here $b = 1.01$. The color coding is black for defectors and white for cooperators.

Figure 4(a) shows the pre-learning probabilities of cooperators, defectors, and the average of all players, where P_{learn} represents the pre-learning probability. It is shown that for the cooperators, the pre-learning probability decreases as the pre-learning factor increases; but a local maximum pre-learning probability can be obtained in the intermediate region. The pre-learning probability approaches zero as the pre-

learning factor further increases, which is caused by the disappearance of cooperators. For the defectors, the pre-learning probability first decreases as the pre-learning factor increases. After ($\beta \approx -1$), the defectors maintain an equal probability of 50%. This is because the lattice is now (i.e., $\beta \geq -1$) filled with defectors, which is equipped with zero payoffs (see Fig. 3). According to Eq. (4), the defector will have a probability of 50% to enter the learning stage. For the average of all players, the pre-learning probability is 50% on average. To sum up, defectors always have a bias of learning from their neighbors, while the cooperators tend to keep their own strategies. In Fig. 4(b), we draw the learning probabilities of $W_{\mathrm{C}\to\mathrm{D}}$ and $W_{\mathrm{D}\to\mathrm{C}}$, where P_{change} represents the learning probability of $H_{i\to j}$. The main observation is that for both $W_{\mathrm{C}\to\mathrm{D}}$ and $W_{\mathrm{D}\to\mathrm{C}}$, as β increases, the learning probabilities will slowly increase to peak levels (about 0.175 for the maximum of $W_{\mathrm{C}\to\mathrm{D}}$ and 0.125 for the maximum of $W_{\mathrm{D}\to\mathrm{C}}$) from zero and then quickly decrease to zero again.

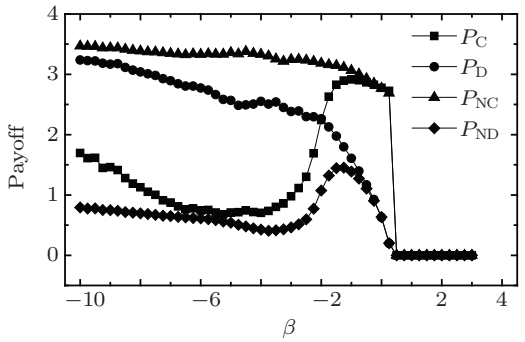


Fig. 3. The relationship of β and P_{C}, P_{D}, P_{NC}, and P_{ND} in the equilibrium state. Here $b = 1.02$.

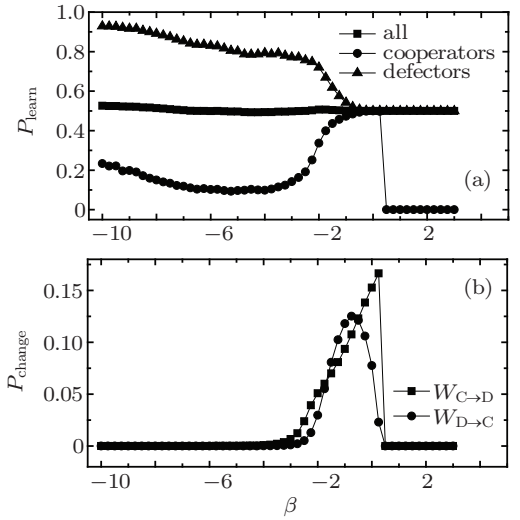


Fig. 4. (a) The pre-learning probability versus β; (b) the learning probability versus β. Here $b = 1.02$.

We now give the following explanation for Fig. 1. (i) $\beta \leq -3$: In this region, the defector has a larger payoff (P_{D}) than his neighbors (P_{ND}), which brings himself a great probability of entering into the learning stage; the cooperator has a smaller payoff (P_{C}) than his neighbors (P_{NC}), thus he will have a tiny probability of entering into the learning stage. The pre-learning stage now dominates the 2-stage strategy updating process. Even the cooperator's payoff is smaller than the defector's, and the actual probability of learning from a defection neighbor ($W_{\mathrm{C}\to\mathrm{D}}$) is very small. Thus, the players are more likely to change from defection to cooperation, resulting in a large cooperation frequency. (ii) $-3 < \beta < -1$: In this region, the absolute values of payoff differences (i.e., $|P_{\mathrm{C}}-P_{\mathrm{NC}}|$, $|P_{\mathrm{D}}-P_{\mathrm{ND}}|$) decrease, thus the cooperators' pre-learning probabilities increase, and the defectors' pre-learning probabilities decrease. At the same time, in the region of $-3 < \beta < -2$ ($-2 < \beta < -1$), the cooperator has a smaller (larger) payoff than the defector. Thus, in the learning stage, the defector will have a small (large) probability of learning from a cooperation neighbor in the region of $-3 < \beta < -2$ ($-2 < \beta < -1$). In this region, the pre-learning and learning stages jointly determine the 2-stage strategy updating process, and eventually result in an increase of ρ_{c}. (iii) $\beta \geq -1$: In this region, the learning stage dominates the 2-stage strategy updating process. Although the cooperator's payoff is larger than the defector's, the actual probability of learning from a defection neighbor ($W_{\mathrm{C}\to\mathrm{D}}$) is relatively large, and thus results in a low cooperation frequency.

From the above discussion, we know that the system behavior is determined by the pre-learning probability P_{learn} tuned through β and the learning probability P_{change} tuned through κ. The κ is used to adjust the effect of the pairwise payoff difference during the evolution. When $\kappa \to 0$, the low-payoff player will deterministically adopt the strategy of the high-payoff player, thus cooperators can hardly form clusters. When $\kappa \to \infty$, the payoff information is completely ignored, and the evolution becomes a stochastic process; and thus the cooperator frequency ρ_{c} reaches the highest level at a moderate value of κ, e.g., ρ_{c} can even reach 0.47 when $\kappa = 0.2$ and $b = 1.01$ (Fig. 5(a)). The β is employed to adjust the effect of environment information (the average payoff of its four neighbors is

actually a kind of environment information). We find that the introduction of β can further promote the cooperation level, e.g., ρ_c can even reach 0.80 when $\beta = -1.5$ and $b = 1.01$ (Fig. 5(b)). Figure 6 gives a clear picture for the joint influence of β and κ. We find that the cooperation frequency ρ_c has nonmonotonic dependences on β and κ, and the highest cooperation level is obtained at a small β and a large κ.

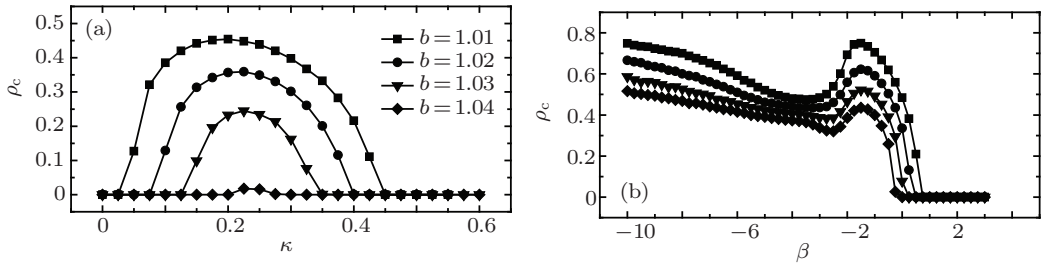


Fig. 5. (a) The relationship of cooperation frequency ρ_c and κ. Here the classical Fermi strategy updating rule without the pre-learning stage is adopted. (b) The relationship of cooperation frequency ρ_c and β under our 2-stage strategy updating rule. Here $\kappa = 0.1$.

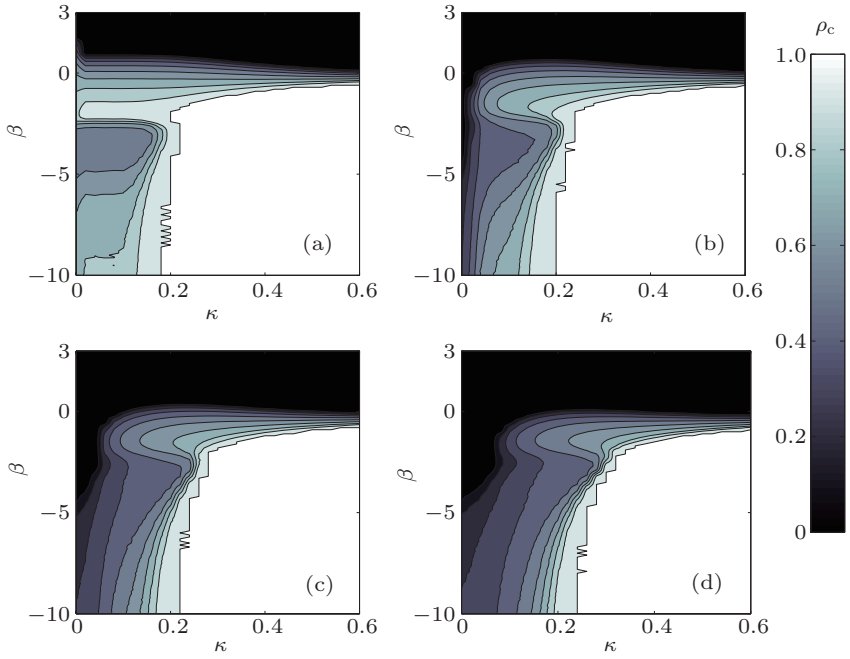


Fig. 6. (colour online) The cooperation frequency ρ_c for PDG in the (κ, β) parameter space. (a) $b = 1.0$, (b) $b = 1.01$, (c) $b = 1.03$, and (d) $b = 1.05$.

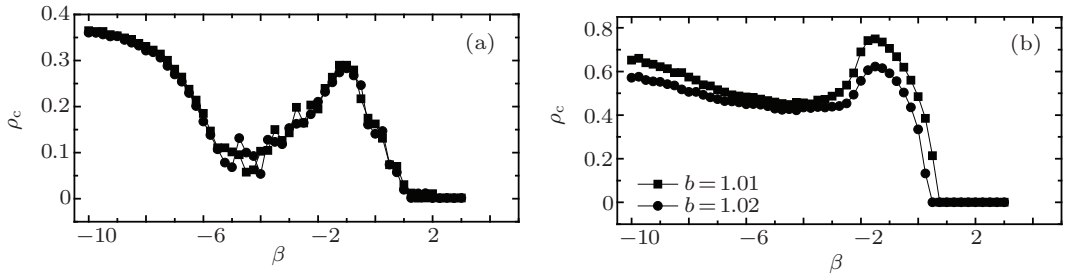


Fig. 7. Simulation results of our 2-stage strategy updating rule for: (a) the richest-following rule on a 100×100 regular lattice; (b) the Fermi updating rule on a NW network, which is generated by adding 80 links to a 100×100 regular lattice.

To further investigate the effect of our 2-stage strategy updating rule, we examine the richest-following rule[46] on the regular lattice [e.g., Fig. 7(a)] and the Fermi rule on the NW[58] network [e.g., Fig. 7(b)] with different values of b, and pay attention to the relationship of the cooperation frequency ρ_c and the pre-learning factor β. In Fig. 7(a), during the pre-learning stage, player i will decide whether to update his strategy according to Eq. (4); during the learning stage, when s_i is updated, a neighbor j with the largest payoff is drawn among its four neighbors, then player i will copy the strategy of the chosen neighbor j if $P_i < P_j$. In Fig. 7(b), the updating process is the same as the process described in the second section except that the regular lattice is replaced by the NW network. We find that the general tendency is the same as that shown in Fig. 1, indicating the fact that our model can be robust against different network topologies and strategy updating rules.

4. Conclusion

In this study, we proposed a spatial prisoner's dilemma game model with a 2-stage strategy updating rule, and focused on the cooperation behavior of the social system. In the first stage, the player decides whether to update his strategy according to the pre-learning factor. If the player passes the first stage, he enters into the second stage, i.e., he randomly selects a neighbor and adopts the chosen one's strategy according to the Fermi updating rule. The simulation results show that the cooperation level has a non-trivial dependence on the pre-learning factor. We explain the simulation results via studying the co-action of pre-learning and learning. Moreover, we find that the 2-stage strategy updating PDG model can be robust against different learning rules and different network topologies. Our results may sharpen the understanding of the influence of the strategy updating rule on evolutionary game cooperation.

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