Spiking sychronization regulated by noise in three types of Hodgkin–Huxley neuronal networks*

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In this paper, we study spiking synchronization in three different types of Hodgkin–Huxley neuronal networks, which are the small-world, regular, and random neuronal networks. All the neurons are subjected to subthreshold stimulus and external noise. It is found that in each of all the neuronal networks there is an optimal strength of noise to induce the maximal spiking synchronization. We further demonstrate that in each of the neuronal networks there is a range of synaptic conductance to induce the effect that an optimal strength of noise maximizes the spiking synchronization. Only when the magnitude of the synaptic conductance is moderate, will the effect be considerable. However, if the synaptic conductance is small or large, the effect vanishes. As the connections between neurons increase, the synaptic conductance to maximize the effect decreases. Therefore, we show quantitatively that the noise-induced maximal synchronization in the Hodgkin–Huxley neuronal network is a general effect, regardless of the specific type of neuronal network.

Keywords: spiking synchronization, neuronal network, noise

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1. Introduction

Synchronization is a basic phenomenon in nature, and has extensive applications in nonlinear systems,[1] living systems,[2] neural systems,[3] and so on. In neural systems, synchronization is widely used to describe interactions between coupled neurons demonstrating excitation.[3] It is believed that synchronization of neuronal excitations is one of the main mechanisms to induce neuronal oscillations in the central nervous system (CNS).[4,5] Many papers also show that synchronization may be the mechanism for information binding in the brain.[6−8] In recent years, a kind of novel network with intricate topology, called a small-world (SW) network, has attracted much attention.[9,10] An SW network has a small average distance between vertices and exhibits a high degree of clustering.[9] Many biological, technological, and social networks possess the behaviour of a SW network. In particular, some experimental papers show that many neuronal networks in a brain display the two main characteristics of the SW networks, i.e., a small average distance and a high clustering coefficient.[11−13] On the other hand, regular and random neuronal networks are the classic neuronal networks which have been the centre of extensive investigation, and have abundant theoretical meanings.[14−16]

Nervous systems are full of noises, and the noises play an important role in the electrical behaviour of nervous systems.[17−23] Noises can induce spontaneous spiking,[17] stochastic resonance,[18] and coherent resonance[19] in a single neuron. In coupled nonlinear systems, noises can induce some interesting phenomena, such as array enhanced stochastic resonance (AESR) and array enhanced coherence resonance (AECR).[24−27] During the last several years, synchronization of neural networks has attracted much attention from many scientists, and many interesting results have been obtained. Cao and Wang[28] investigated the global synchronization of delayed neural networks with constant or delayed coupling, and found that global synchronization can be approached by suitable inner coupled linking and suitable inner

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delayed linking. Li *et al.*[29] investigated the robust synchronization of coupled delayed networks through impulsive control, and estimated the predicted stable region for robust synchronization. The robust chaos synchronization of uncertain chaotic neural networks was also explored, and a control scheme for the robust adaptive synchronization was presented.[30] Research on heterogeneous small-world neural networks shows that the necessary coupling strength for phase and complete synchronization decreases with the increase of neuron number, node degree, and connection density.[31] The effect of heterogeneity was also studied, and it was found that the network heterogeneity has an adverse effect on synchronization.[31] As well as the symmetric neural networks, scholars also studied the global synchronization of the asymmetric neural networks with constant and delayed coupling.[32] Some global synchronization criteria for the asymmetric neural networks were obtained through combining the Lyapunov functional method and the dynamical error feedback theory.[32] The neural network coupling is usually uniform, however, in some cases the neural network coupling can be hybrid.[33] Tang *et al.*[33] studied the hybrid neural networks, and obtained several theoretical results guaranteeing the synchronization of the hybrid neural networks. Besides the network topology and coupling, the properties of a single node in the neural network is also an important factor affecting the synchronization behaviour of neural networks. Synchronization in the Hindmarsh–Rose neural network was also studied,[34] and it was shown that synchronization in these neural networks can be approached under suitable coupling strength and network topology. In the present paper, we study the spiking synchronization in the three different types of Hodgkin–Huxley (HH) neuronal networks, i.e., the SW, the regular, and the random neuronal networks.[28−36] The neurons in the network are subjected to subthreshold stimulus and external noise. We find that there is an optimal strength of noise to induce maximal spiking synchronization in each of the networks. It is further found that in each of the networks there is a range of synaptic conductance to cause the effect, i.e., the noise-induced maximal synchronization. If the synaptic conductance is small or large, the effect vanishes. Only when the synaptic conductance dwells in the middle range will the effect be considerable. With the increase of the connections between neurons, the synaptic conductance to maximize the effect decreases. Therefore, it is demonstrated quantitatively that the noise-induced maximal synchronization in the Hodgkin–Huxley (HH) neuronal networks is a general phenomenon, which is independent of the specific types of neuronal networks.

2. Model

The electrical behaviour of an excitable neuron is first described quantitatively through the HH equation. On the basis of the HH equation, the electrical dynamics of a coupled neuronal network are given by the following equation:[37]

$$C_{\rm m}\frac{{\rm d}V_i}{{\rm d}t} = g_{\rm Na}m_i^3h_i(V_{\rm Na}-V_i)+g_{\rm K}n_i^4(V_{\rm K}-V_i) + g_{\rm L}(V_{\rm L}-V_i)+I_{i({\rm ext})} \quad (1)$$

with

$$I_{i({\rm ext})} = I_0+\xi_i(t)+I_{\rm syn}^i, \quad (2)$$

where $C_{\rm m}$ is the specific capacitance of the neuronal membrane, V_i is the membrane potential of the i-th neuron in the network, $V_{\rm Na}$, $V_{\rm K}$, and $V_{\rm L}$ are the reversal potentials of the sodium, the potassium, and the leakage systems respectively, $g_{\rm Na}$, $g_{\rm K}$, and $g_{\rm L}$ are the corresponding maximal conductances, m_i, n_i, and h_i are the gating variables of m, n, and h gates in the i-th neuron respectively, and $I_{i({\rm ext})}$ is the total external current received by the i-th neuron. The dynamics of the gating variable n_i are described by Eq. (3), which is similar to that for m_i and h_i. In Eq. (3), α_{n_i} and β_{n_i} are the voltage-dependent transition rates. The specific values of parameters and rate functions used in the paper can be found in Table 1.[38]

$$\frac{{\rm d}n_i}{{\rm d}t} = \alpha_{n_i}(V_i)(1-n_i)-\beta_{n_i}(V_i)n_i. \quad (3)$$

As shown in Eq. (2), the external current $I_{i({\rm ext})}$ of the i-th neuron consists of external subthreshold stimulus I_0, external noise current $\xi_i(t)$, and synaptic current $I_{\rm syn}^i$. We consider the noise current $\xi_i(t)$ as Gaussian white noise with[38]

$$\langle\xi_i(t)\rangle = 0, \quad \langle\xi_i(t)\xi_i(t')\rangle = \sigma^2\delta(t-t'), \quad (4)$$

where σ is the strength of noise current.

The synaptic current $I_{\rm syn}^i$ has the following expression:[37]

$$I_{\rm syn}^i = \frac{1}{N}\sum_{j=1,j\neq i}^{N} g_{\rm syn}\alpha(t-t_j)(V_{\rm syn}^{ij}-V_i) \quad (5)$$

with

$$\alpha(t) = (t/\tau)\exp[-t/\tau],$$

where N is the neuron number in the network, $g_{\rm syn}$ is the synaptic conductance, τ is the characteristic time of the synaptic interaction, which is chosen to be 2 ms in the paper, $V_{\rm syn}^{ij}$ is the synaptic potential between the i-th and j-th neurons, and t_j is the initial action time of the synapses in the j-th neuron, i.e. the firing time of the j-th neuron. For the excitatory synapses, $V_{\rm syn}^{ij}$ is set to be 0 mV.[37] For the inhibitory synapses, $V_{\rm syn}^{ij}$ is set to be -80 mV.[37] The percentage of the excitatory synapses is 90% in the paper,[39,40] and the synapses are determined randomly to be excitatory and inhibitory.

Table 1. Parameters and the rate functions used in the paper.

Specific membrane capacitance ($C_{\rm m}$)	1 μF/cm^2
Potassium reversal potential ($V_{\rm K}$)	-77 mV
Sodium reversal potential ($V_{\rm Na}$)	50 mV
Leakage reversal potential ($V_{\rm L}$)	-65 mV
Maximal sodium channel conductance ($g_{\rm Na}$)	120 mS/cm^2
Maximal potassium channel conductance ($g_{\rm K}$)	36 mS/cm^2
Leakage conductance ($g_{\rm L}$)	0.3 mS/cm^2
α_{n_i}	$\frac{0.01(V+55)}{1-{\rm e}^{-(V+55)/10}}$
β_{n_i}	$0.125\,{\rm e}^{-(V+65)/80}$
α_{m_i}	$\frac{0.1(V+40)}{1-{\rm e}^{-(V+40)/10}}$
β_{m_i}	$4\,{\rm e}^{-(V+65)/18}$
α_{h_i}	$0.07\,{\rm e}^{-(V+65)/20}$
β_{h_i}	$\frac{1}{1-{\rm e}^{-(V+35)/10}}$

The neuronal network is comprised of 100 coupled HH neurons ($N = 100$). For the SW neuronal network, in the light of the Newman–Watts model we first initialize the network as a regular ring (connectivity $k = 2$), and then add new edges with probability p.[41,42] If the number of total added edges is M, then the probability p is equal to $2M/[N(N-1)]$. For a regular network with connectivity k, each neuron in the network connects to the nearest k neuron. In the random network, each neuron connects to the other neuron randomly with probability p. In order to measure the spiking synchronization of the neuronal network quantitatively, we introduce the phase of the i-th neuron[43,44]

$$\phi_i(t) = 2\pi \frac{t - \tau_{i,k}}{\tau_{i,k+1} - \tau_{i,k}} + 2\pi k, \tag{6}$$

where $\tau_{i,k}$ is the firing time of the k-th spike of the i-th neuron. The quantity[44,45]

$$s_{i,j} = \sin^2\left(\frac{\phi_i - \phi_j}{2}\right) \tag{7}$$

measures the spiking synchronization between the i-th neuron and the j-th neuron. A spatiotemporal average of $s_{i,j}$, i.e.,

$$S = \lim_{T\to\infty} \frac{1}{T} \int_0^T \left[\frac{2}{N(N-1)} \sum_{i=1,j=i+1}^{N} s_{i,j}\right] {\rm d}t, \tag{8}$$

provides a quantitative measure of the spiking synchronization in the neural network.[44] A smaller S means a higher degree of spiking synchronization, and *vice versa*.

3. Results

In all the simulations of this paper, the external subthreshold stimulus I_0 is fixed to be 6.0 μA/cm^2. In the first step, we investigate the spiking synchronization in the SW neuronal network. We first set the connection probability p to be 0.1 and the synaptic conductance $g_{\rm syn}$ to be 4.0 mS/cm^2, adjust the external noise strength σ from 0.7 μA/cm^2 to 10.0 μA/cm^2, and then plot the spatiotemporal graphs of neuronal spiking time in Fig. 1. As shown in Fig. 1, there is an optimal value of the noise strength σ to induce the maximal spiking synchronization. If σ is 0.7 μA/cm^2, almost no spiking synchronization appears. If σ increases to 2.0 μA/cm^2, the strong synchronization between neuronal spikes is present. If σ increases further to 5.0 μA/cm^2, just a slight synchronization between neuronal spikes occurs. If σ increases continually to 10.0 μA/cm^2, a very slight synchronization between neuronal spikes is achieved.

In order to investigate the spiking synchronization of the SW neuronal network quantitatively, we calculate the value of S, which is a quantitative measure of the spiking synchronization. As shown in Fig. 1, we also set the connection probability p to be 0.1. The curves of S versus the noise strength σ for different values of synaptic conductance $g_{\rm syn}$ are plotted in Fig. 2. Except $g_{\rm syn}$, all the parameters used in Fig. 2 are the same as those used in Fig. 1. As shown in Fig. 2, S decreases monotonically and the spiking synchronization increases monotonically with the increase of $g_{\rm syn}$. It is expected that coupling among neurons increases the spiking synchronization. From Fig. 2, we find that when the coupling among neurons is weak ($g_{\rm syn}$ = 1.0 mS/cm^2), noise strength σ has no effect on S. With the increase of $g_{\rm syn}$, the effect of the noise on the optimal strength is more considerable, reducing the value of S and increasing the spiking synchronization. When $g_{\rm syn}$ is equal to 4.0 mS/cm^2, the effect reaches its maximum. If $g_{\rm syn}$ increases continually, the effect decreases. When $g_{\rm syn}$ is equal to 8.0 mS/cm^2, the effect vanishes and the value of S increases monotonically with the increase of σ. Therefore, in order to induce the effect, i.e., the noise-induced maximal synchronization, synaptic conductance $g_{\rm syn}$ should dwell in a middle range. A small or large value of $g_{\rm syn}$ cannot induce the effect.

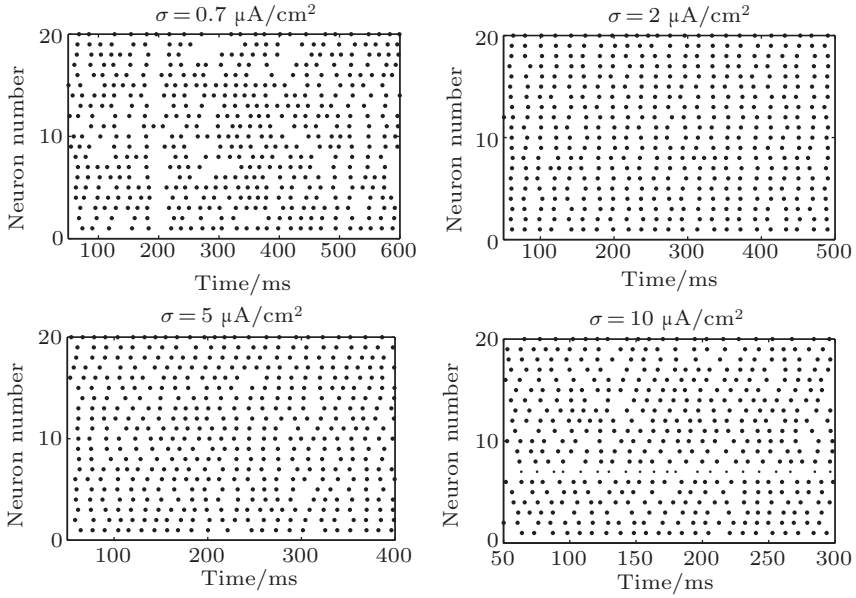


Fig. 1. Spatiotemporal graphs of neuronal spiking time versus σ of a SW neuronal network.

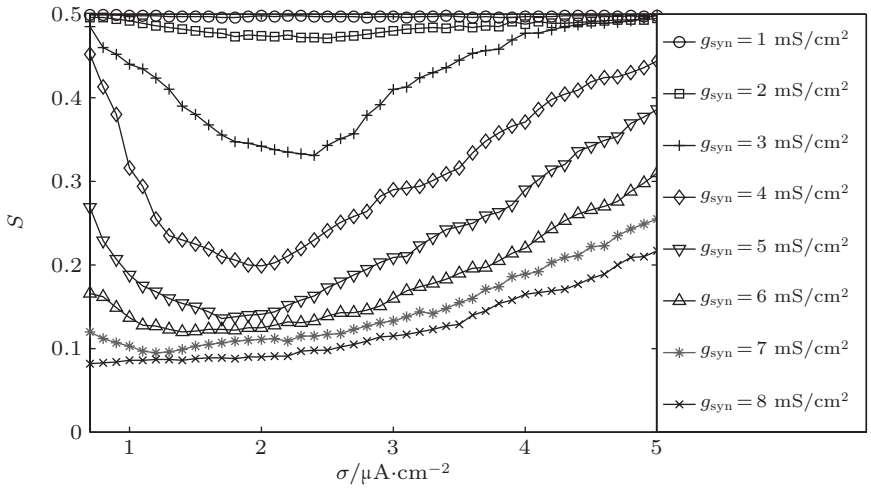


Fig. 2. Curves of S versus σ for different values of $g_{\rm syn}$ of the SW neuronal network.

In order to quantitatively measure the effect, i.e., the noise-induced maximal synchronization, we calculate the values of the maximal decreasing percentage of S for different noise strengths of the SW neuronal network. We set $g_{\rm syn}$ and p to be constant, and change σ to obtain minimal S. When σ is 0.7 μA/cm^2, it is close to the threshold strength of noise to generate spikes. Therefore, we choose 0.7 μA/cm^2 as the minimal noise strength to calculate the value of the maximal decreasing percentage of S. If the value of S for σ as 0.7 μA/cm^2 is denoted as $S_{\rm i}$ and the minimal value of S as $S_{\rm m}$, then the maximal decreasing percentage of S is obtained from the mathematic expression $(S_{\rm i} - S_{\rm m})/S_{\rm i}$. The curves of the maximal decreasing percentage of S versus $g_{\rm syn}$ for different values of p are plotted in Fig. 3. With the increase of $g_{\rm syn}$, the maximal decreasing percentage of S first increases and then decreases after reaching the maximum. At optimal $g_{\rm syn}$, the maximal decreasing percentage of S is as large as 60%. It is also shown in Fig. 3 that the optimal $g_{\rm syn}$ to induce the maximal percentage of S decreases with the increase of p.

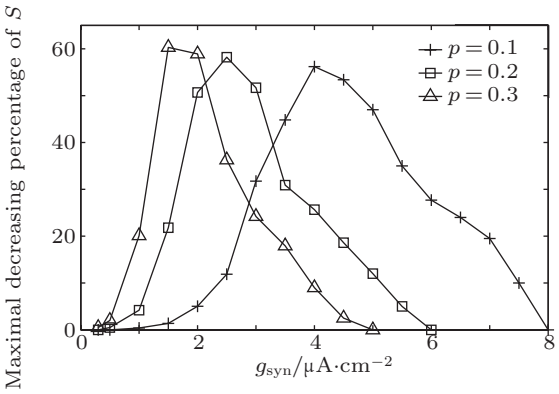


Fig. 3. Curves of maximal decreasing percentage of S versus $g_{\rm syn}$ for different values of p of the SW neuronal network.

As demonstrated in the above section, there is an optimal strength of noise to maximize the spiking synchronization in the SW neuronal network, and the synaptic conductance should dwell in a suitable range to induce the maximizing effect. It is natural to ask whether the maximizing effect is unique for the SW neuronal network or a general effect regardless of the specific type of neuronal network. Therefore, we investigate the spiking synchronization in another kind of important neuronal network, i.e., a regular neuronal network. We set the connectivity k to be 10, which corresponds to p as 0.1 approximately. Curves of S versus σ for different values of $g_{\rm syn}$ are plotted in Fig. 4. As shown in Fig. 4, there is an optimal σ to maximize the spiking synchronization in the regular neuronal network, which accords with that in the SW neuronal network. In order to compare this with Fig. 3, we we calculate the values of the maximal decreasing percentage of S for different values of connectivity k of the regular neuronal networks, and show the corresponding results in Fig. 5. The values of connectivity k in Fig. 5 are 10, 20 or 30, which correspond to values of p as 0.1, 0.2 or 0.3 approximately. We can find that Fig. 5 matches Fig. 3 qualitatively but had some quantitative differences. As shown in Fig. 5, there also exists a suitable range of $g_{\rm syn}$ to induce the maximal effect and the optimal $g_{\rm syn}$ also decreases with the increase of the connectivity k in the regular neuronal network.

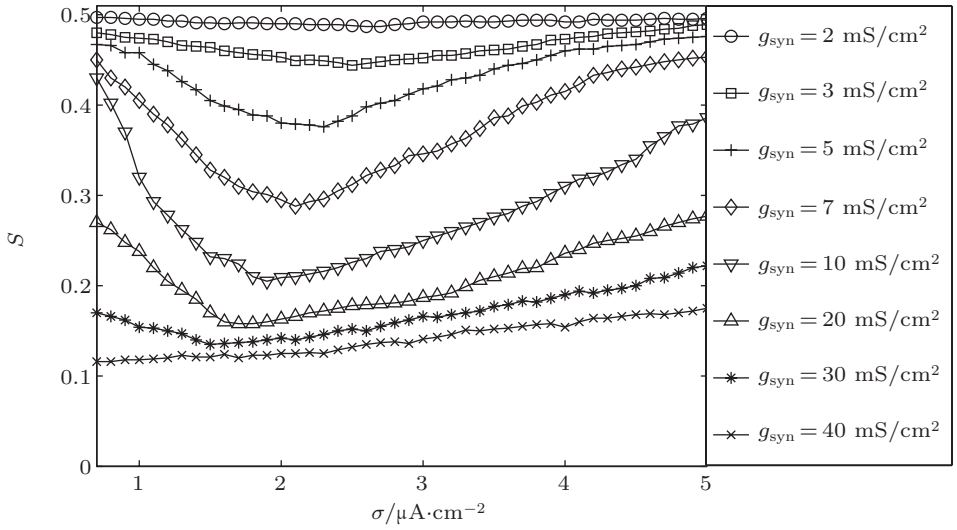


Fig. 4. Curves of S versus σ for different values of $g_{\rm syn}$ for the regular neuronal network.

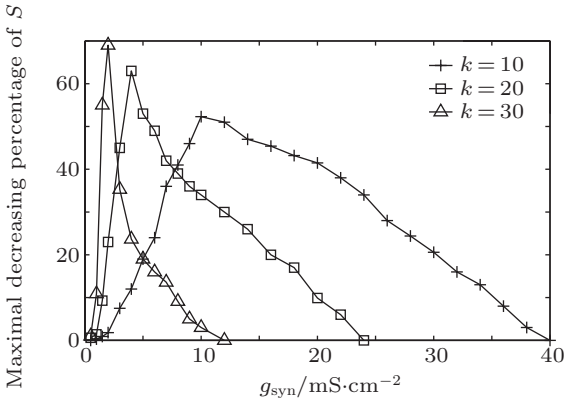


Fig. 5. Curves of the maximal decreasing percentage of S versus $g_{\rm syn}$ for different values of connectivity k for the regular neuronal network.

Besides the SW and the regular neuronal networks, the random neuronal network is also a type of important neuronal network. In the next step, we investigate the spiking synchronization of the random neuronal network. We set p to be 0.1. Curves of S versus σ for different values of $g_{\rm syn}$ are plotted in Fig. 6. As shown in Fig. 6, there also exists optimal σ to maximize the spiking synchronization in the random neuronal network, which accords with that in the SW and the regular neuronal networks. In order to compare this with Figs. 3 and 5, we calculate the values of the maximal decreasing percentage of S for different values of σ of the random neuronal network, and show the corresponding results in Fig. 7. The values of p in Fig. 7 are set to be 0.1, 0.2 or 0.3 respectively. We can find that Fig. 7 accords with Figs. 3 and 5 qualitatively, but with some quantitative differences. This means that a suitable range of $g_{\rm syn}$ also exists to induce the maximal effect, and the optimal $g_{\rm syn}$ also decreases with the increase of p in the random neuronal network.

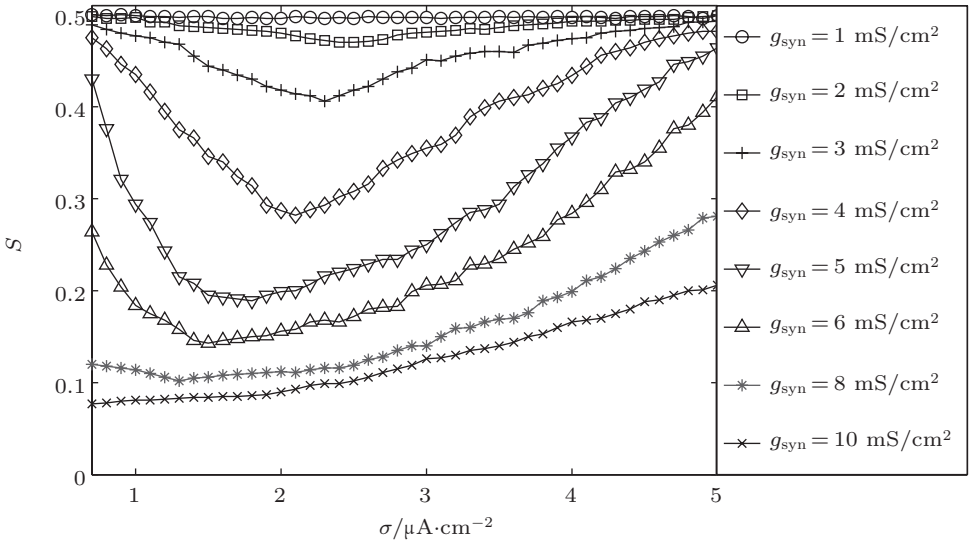


Fig. 6. Curves of S versus σ for different values of $g_{\rm syn}$ for the random neuronal network.

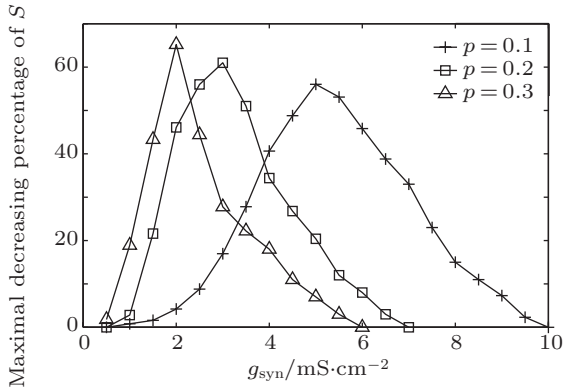


Fig. 7. Curves of maximal decreasing percentage of S versus $g_{\rm syn}$ for different values of p for the random neuronal network.

According to the above investigation, it is shown that there exists an optimal strength of noise to maximize the spiking synchronization and a suitable range of synaptic conductance to induce the maximizing effect for all the SW, regular, and random neuronal networks. Therefore, we can come to the conclusion that the maximizing effect is a general effect in all the neuronal networks, regardless of the specific type of neuronal network. It is natural to ask what the mechanism to induce the maximizing effect is. Similar to the synchronization in other excitable media, the mechanism of the noise-induced maximal synchronization is caused by coherence resonance (CR) of a single uncoupled HH neuron.[46] For a single uncoupled HH neu-

ron, there is an optimal strength of noise to cause the maximal spiking coherence. When the synaptic conductance is strong enough, the quality of the synaptic current is enhanced and the intensity is increased with the increase of noise strength due to the CR in a single neuron. The synaptic current received by the local neuron as an input signal will regulate the output, and the local neuron will be excited with the same frequency and phase as those of the input signal. Because the input signal of the local neuron is an average output of the other neurons and the output of the local neuron is the input signal of the other neurons, there is a process of positive feedback and the CR of a single neuron can promote the spiking coherence of the total network. Through the dynamical optimization process, an optimal spatiotemporal order can be reached when the spiking coherent of the neuronal network is the maximal under the optimal strength of noise, and the frequency of oscillation, which is just the frequency of CR, is selected. It means that the maximal spiking synchronization is reached under the optimal strength of noise. In order to show quantitatively that the CR of the neuronal network causes the maximal spiking synchronization, we calculate the coherence measure β of the neuronal network. The neuronal spike train can be approximated as a train of δ spikes at central positions t_n of action potentials, i.e.

$$v(t) = \sum_n \delta(t - t_n). \tag{9}$$

The power spectrum density of spike trains of the neuronal network is an average power spectrum density of the N (100) neurons. The coherence measure β can be obtained from the average power spectrum density as[47,48]

$$\beta = h(\Delta\omega/\omega_{\rm p})^{-1}, \tag{10}$$

where h and $\omega_{\rm p}$ are the height and the frequency of the peak of the power spectrum density, and $\Delta\omega$ is the width of the peak at height $h_1 = {\rm e}^{-0.5}h$. We chose the SW neuronal network as a research sample, set the connection probability p to be 0.1, the synaptic conductance $g_{\rm syn}$ to be 4.0 mS/cm^2, and plot β and S versus σ in Fig. 8. In order to show the results clearly, we multiply S 100 times in Fig. 8. From Fig. 8, we find that S is exactly minimal when β is maximal. This means that the maximal CR of the neuronal network matches exactly the maximal spiking synchronization of the neuronal network, therefore we show quantitatively that the CR of neurons causes the noise-induced maximal synchronization of the neuronal network.

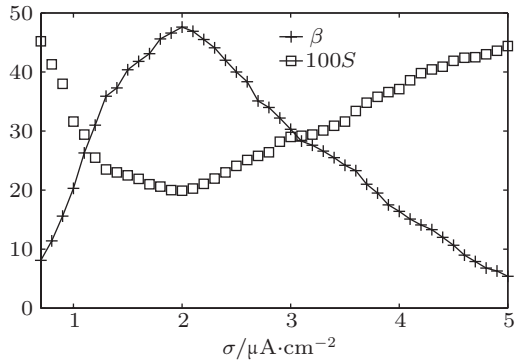


Fig. 8. Curves of β and S versus σ of the SW neuronal network.

Figures 3, 5, and 7 each show that there is a suitable range of synaptic conductance to cause the noise-induced maximal synchronization. If the synaptic conductance is small or large, the maximal effect vanishes. It is easy to understand that a synaptic conductance which is too small cannot cause the maximal effect. A synaptic conductance which is too small means a very small synaptic current, which is the regulation of mutual spiking. Therefore, the neuronal network cannot generate mutual spiking and neurons in the network spiking independently are too small for a synaptic current. As shown in Fig. 2, when the synaptic conductance is 1.0 mS/cm^2, all the values of S for different noise strengths are almost 0.5, i.e., the spiking is completely independent of the neurons. In the next step, we explain why a large synaptic conductance cannot cause the noised-induced maximal synchronization either. From the above section, we know that the noise-induced maximal synchronization is caused by the CR of the neurons. In the first paper demonstrating the CR in the HH neurons, Li *et al.*[49] explained the mechanism to induce CR. We first repeat the mechanism concisely. We consider the bifurcation in a single HH neuron without external noise. For a DC input of I_0, the firing threshold is approximately 6.2 μA/cm^2. If I_0 is less than the threshold, the neuron stays at the rest potential, owing to the fact that the fixed point is a global attractor of the system. The birth of the limit cycle, i.e. the neuronal firing, occurs at $I_0 \approx 6.2$ μA/cm^2 due to the saddle-node bifurcation. If external noise is taken into account, the system either fluctuates around the fixed point or makes an excursion into the limit cycle to generate spikes. The activation time $T_{\rm a}$ is the mean time to drive the system into the limit cycle region by external noise and is approximately inversely proportional to noise strength

σ. Another time scale is the oscillating period of the limit cycle, T_0. The time scale T_0 is mainly determined by the bifurcation behaviour of the system and only depends slightly on noise. For a weak noise, T_a is much larger than T_0, so that two branches of the time series of the membrane potential, i.e., the fluctuations around the fixed point and the periodic oscillation, are well separated from each other. In this case, the system is excited stochastically. When T_a increases rapidly and approaches T_0 with an increase in σ, the two processes, namely the noise-induced firing and the oscillation of the limit cycle, are almost matched, and then the spiking coherence is promoted and reaches a maximum. With the further increase of noise, the activation time T_a becomes very short and it is less than T_0, so that the periodic oscillation is affected mainly by noise and becomes stochastical. Now, we come to consider the neuronal network. Apart from the sub-threshold input I_0 and the external noise, the neurons are all subjected to a synaptic current. The synaptic current pushes the fixed point forward the limit cycle of the system and the necessary noise strength to cause a specific T_a will decrease. Therefore, the synaptic current reduces the necessary noise strength to cause $T_a \approx T_0$, which is the condition for the maximal spiking coherence, i.e. the CR. In other words, the synaptic current reduces the optimal noise strength to induce the CR. With the increase of synaptic conductance, the synaptic current increases and the optimal noise strength to induce the CR decreases gradually. Therefore, the CR of the neuronal network decreases gradually due to the reduction of the optimal noise strength, i.e., the optimal noise strength is moving forward the initial noise strength. It is shown in Ref. [49] that both the optimal noise strength to induce the CR and the CR of the neurons decrease with the increase of an external current. On the other hand, the CR of the neuronal network increases gradually, owing to AECR, with the increase of synaptic conductance.[44,50] Due to the interaction of the two effects, the CR of the neuronal network first increases, and then decreases after reaching the maximum with the increase of synaptic conductance. According to the phenomenon, we can explain that with the increase of synaptic conductance the effect of the noise-induced maximal synchronization first increases and then decreases after reaching the maximum. In order to demonstrate the phenomenon quantitatively, we choose the SW neuronal network as a research sample, set the connection probability p to be 0.1, and plot the curves of β versus σ for different values of g_{syn} in Fig. 9. It is shown in Fig. 9 that the optimal σ to induce the CR decreases continually with the increase of g_{syn}. When g_{syn} is 8.0 mS/cm^2, optimal σ approaches the initial strength, 0.7 μA/cm^2. This means that β increases gradually with the increase of g_{syn} and the obvious CR vanishes when $g_{syn} = 8.0$ mS/cm^2. The fading away of the CR induces the fading away of the noise-induced maximal synchronization. Therefore, the noise-induced maximal synchronization vanishes when the synaptic conductance is large enough. From Fig. 9, we can also find that with the increase of g_{syn} the CR first increases and then decreases after reaching the maximum. Finally, we explain qualitatively and quantitatively the mechanism of the noise-induced maximal synchronization by the combinative action of external noise and network coupling.

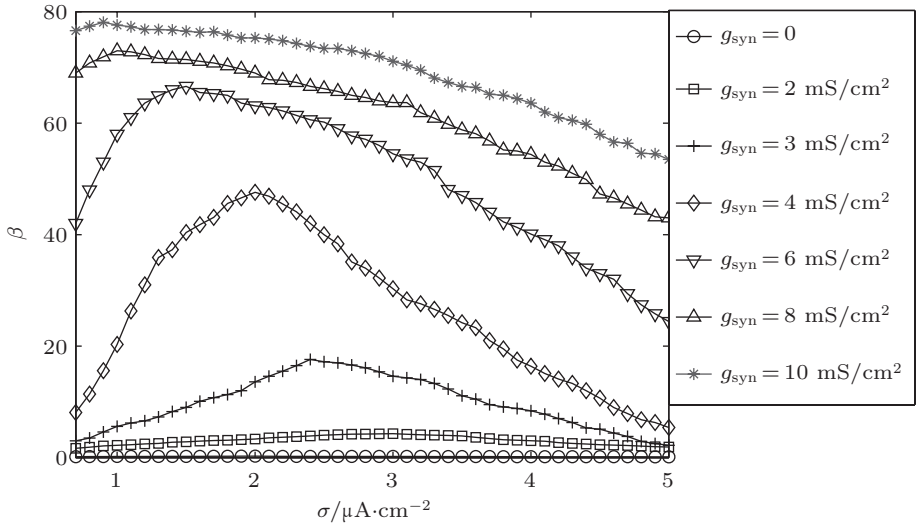


Fig. 9. Curves of β versus σ for different values of g_{syn} in the SW neuronal network.

4. Conclusions

Synchronization is an important phenomenon in neural systems, and may be the main mechanism to induce neuronal oscillations and information binding.[4−8] We studied the spiking synchronization of three types of neuronal networks, i.e., the SW, the regular, and the random neuronal networks. The neurons in the networks are subjected to subthreshold stimulus and external noise. We find that there is an optimal strength of noise to cause the maximal spiking synchronization in all of the neuronal networks. It is further shown that there is a range of synaptic conductance to induce the effect, i.e., the noise-induced maximal synchronization. Only when the strength of the synaptic conductance is a moderate value will the effect be considerable. If the synaptic conductance is small or large, the effect vanishes. The optimal synaptic conductance to maximize the effect decreases with the increase of connection probabilities between neurons. It is shown that the noise-induced maximal synchronization is a general phenomenon in the HH neuronal networks, regardless of the specific network type. Our research results are helpful for understanding the spiking synchronization of neuronal networks and providing the information for investigating neuronal oscillations and information binding in the brain.

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