Controlling group velocity in a superconductive quantum circuit*

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We investigate the controllable group velocity of a microwave probe field in a superconductive quantum circuit (SQC) pumped by microwave fields, and the use of such a SQC function as an artificial Λ-type three-level atom. The exchange between the subluminal and the superluminal states of the probe field can be realized simply by sweeping the pumping intensity, and the superluminal state is usually realized with a lower absorption. This work is one of the efforts to extend the study of electromagnetically induced transparency and its related properties from the lightwave band to the microwave band.

Keywords: subluminal, superluminal, superconductive quantum circuit, group velocity

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1. Introduction

During the last few decades, research into subluminal and superluminal propagations of light has attracted much attention.[1−5] Most analyis was made based on the principle of electromagnetically induced transparency (EIT),[6−8] and the light velocity control was testified in various media, such as the Bose–Einstein condensate,[1] atomic vapors,[4,5] magneto-optically trapped atoms, and solid-state quantum dots.[8] However, achieving a strong interaction between the spatial electromagnetic modes and natural atoms is not always easy in practice.[9−11] Recently, a superconductive quantum circuit (SQC) made of mesoscopic Josephson junctions comes into sight.[12−14] It behaves as an atom and is thus called the artificial atom, with which the EIT effect has been demonstrated.[14] In fact, because of its simple energy level structure and easy relevant measurements,[15] such an artificial atom has been widely employed as a testing ground for quantum optics in a nearly macroscopic system. Many fundamental quantum optical phenomena known in real atoms have been demonstrated.[16−18] In this paper, we discuss whether the light velocity control achieved with atomic media[19−21] can be realized in the SQC system.

2. Model and equations

The SQC considered in this paper is illustrated in Fig. 1(a).[22,23] A superconducting loop of macroscopic size (> 1 μm) is made of three Josephson junctions threaded by an external magnetic flux Φ_e through the loop and inductively coupled to a transmission line (open 1D space) via a mutual kinetic inductance. Two of the three junctions have equal Josephson energy, capacitance, and size. The potential energy of the SQC as a function of $\varphi_{p,m} = (\varphi_1 \pm \varphi_2)/2$ and flux bias $f = \Phi_e/\Phi_0$ (Φ_0 is the flux quantum) exhibits an asymmetric double-well profile when the controllable quantity f is taken to be away from 0.5 (Fig. 1(b)),[12,23] where φ_i $(i = 1, 2, 3)$ is the gauge-invariant phase across the i-th junction and constrained by $\varphi_1 - \varphi_2 + \varphi_3 + 2\pi f = 0$. When the lowest three energy levels of the f-dependent energy spectrum are well separated from the other upper energy levels,[23] then a three-level artificial atom is formed. Since the energy gap between the two levels is located in the microwave band, the fields coupling to this atom must come from microwave sources.[13,15,17]

As shown in Fig. 1, this artificial atom exhibits a Λ-like three-level structure, where the ground states $|1\rangle$ and $|2\rangle$ are effectively metastable with respect to

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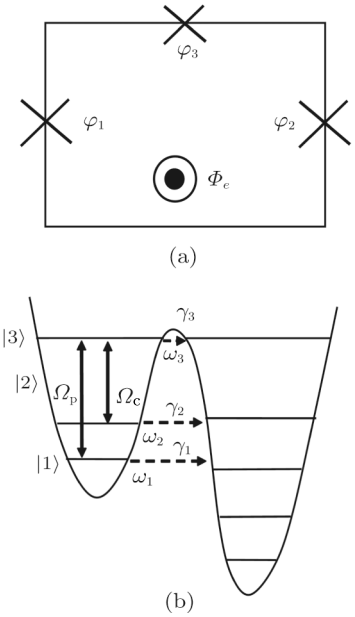


Fig. 1. (a) The schematic diagram of SQC. (b) One-dimensional double-well potential and energy-level diagram of the artificial three-level atom.

the resonantly biased excited state $|3\rangle$. The weak microwave probe field of frequency $\omega_{\rm p}$ and the strong microwave pumping field of frequency $\omega_{\rm c}$ are applied to transitions $|3\rangle \leftrightarrow |1\rangle$ and $|3\rangle \leftrightarrow |2\rangle$, and the corresponding Rabi frequencies are $\Omega_{\rm p}$ and $\Omega_{\rm c}$ proportional respectively to the intensities of the applied microwave fields. We assume the decay rates of states $|1\rangle$, $|2\rangle$, and $|3\rangle$ to be γ_1, γ_2, and γ_3, respectively, which stem mainly from the interwell resonant-tunneling effects. In the rotating wave approximation and considering various dissipative processes, we derive the equations of the density matrix elements for the whole system

$$\begin{aligned}
\dot{\rho}_{11} &= -\frac{\mathrm{i}}{2}(\Omega_{\rm p}^*\rho_{31} - \Omega_{\rm p}\rho_{13}) - \gamma_1\rho_{11},\\
\dot{\rho}_{12} &= -\frac{\mathrm{i}}{2}(\Omega_{\rm p}^*\rho_{32} - \Omega_{\rm c}\rho_{13}) + \mathrm{i}(\Delta_{\rm c} - \Delta_{\rm p})\rho_{12}\\
&\quad - \Gamma_{12}\rho_{12},\\
\dot{\rho}_{22} &= -\frac{\mathrm{i}}{2}(\Omega_{\rm c}^*\rho_{32} - \Omega_{\rm c}\rho_{23}) - \gamma_2\rho_{22},\\
\dot{\rho}_{33} &= -\frac{\mathrm{i}}{2}(\Omega_{\rm p}\rho_{13} - \Omega_{\rm p}^*\rho_{31} + \Omega_{\rm c}\rho_{23} - \Omega_{\rm c}^*\rho_{32})\\
&\quad - \gamma_3\rho_{33},\\
\dot{\rho}_{31} &= -\frac{\mathrm{i}}{2}[\Omega_{\rm p}(\rho_{11} - \rho_{33}) + \Omega_{\rm c}\rho_{21}] + \mathrm{i}\Delta_{\rm p}\rho_{31}\\
&\quad - \Gamma_{13}\rho_{31},\\
\dot{\rho}_{32} &= -\frac{\mathrm{i}}{2}[\Omega_{\rm c}(\rho_{22} - \rho_{33}) + \Omega_{\rm p}\rho_{12}] + \mathrm{i}\Delta_{\rm c}\rho_{31}\\
&\quad - \Gamma_{32}\rho_{32}. \qquad (1)
\end{aligned}$$

Their conjugations are $\partial\rho_{ij}/\partial t = (\partial\rho_{ji}/\partial t)^*$, $(i, j = 1, 2, 3)$. We note here that the equations are expressed in the rotating frame of the time-dependent unitary transformation $W(t) = \exp[(\mathrm{i}/\hbar)(\hat{H}_0 + \hbar(\Delta_{\rm c} - \Delta_{\rm p})|2\rangle\langle 2| - \hbar\Delta_{\rm p})|3\rangle\langle 3|)t]$, where $\hat{H}_0$ is the Hamitonian of the free artificial atom, $\Delta_{\rm p} = \omega_{\rm p} - \omega_{31}$ and $\Delta_{\rm c} = \omega_{\rm c} - \omega_{32}$ are the detunings of the probe and the pumping fields from the atomic transition frequencies ω_{31} and ω_{32}, respectively. In Eq. (1), Γ_{ij} $(i, j = 1, 2, 3)$ describes the decoherence rate of the SQC and is defined as $\Gamma_{ij} = (\gamma_i + \gamma_j)/2 + \gamma_{ij}$ with γ_{ij} $(i \neq j)$ being the dephasing rate of the atomic coherence ρ_{ij}. Because in a SQC system, the decay and the dephasing rates usually meet the condition $\gamma_3 \gg \gamma_{ij} \gg \gamma_1, \gamma_2$, we can take $\gamma_1 = \gamma_2 \simeq 0$, so $\Gamma_{13} = \Gamma_{23} \approx \gamma_3/2$, and $\Gamma_{12} \approx \gamma_{12}$.

The SQC EIT property and the group velocity $v_{\rm g}$ of the probe field propagating in the SQC is closely related to its linear response to the field, which can be known from the linear susceptibility $\chi^{(1)}$ with respect to the probe frequency, $\chi^{(1)} = \Gamma|\boldsymbol{\mu}_{13}|^2/(V\varepsilon_0\hbar\Omega_{\rm p})\rho_{13}$, with Γ, $\boldsymbol{\mu}_{13}$, ε_0, V, and ρ_{13} being the optical confinement factor, the qubit dipole moment vector, the vacuum permittivity, the volume of a single SQC, and the density matrix element, respectively. Assuming that the artificial atom is initially prepared in a superposition state of the two metastable states $|1\rangle$ and $|2\rangle$, i.e., $|\Psi\rangle = (\Omega_{\rm c}/\Omega)|1\rangle + (\Omega_{\rm p}/\Omega)|2\rangle$ with $|\Omega|^2 = |\Omega_{\rm p}|^2 + |\Omega_{\rm c}|^2$, after the system reaches the quasisteady state, we obtain

$$\rho_{13} = \mathrm{i}\frac{\Omega_{\rm p}|\Omega_{\rm c}|^2}{|\Omega|^2}\frac{2\gamma_3(\gamma_{12} + \mathrm{i}\Delta_{\rm p})}{[2(\gamma_{12} + \mathrm{i}\Delta_{\rm p})\gamma_3 + |\Omega_{\rm p}|^2](\gamma_3 + 2\mathrm{i}\Delta_{\rm p}) + |\Omega_{\rm c}|^2\gamma_3}, \qquad (2)$$

where we have considered the resonant coupling ($\Delta_{\rm c} = 0$) for simplicity. The imaginary part $\mathrm{Im}[\chi^{(1)}]$ determines the atomic absorption, and the real part $\mathrm{Re}[\chi^{(1)}]$ is the refractive index. In terms of $\chi^{(1)}$, group velocity $v_{\rm g}$ can be expressed as $v_{\rm g} = c/[n + \omega(\mathrm{d}n/\mathrm{d}\omega)]_{\omega_{\rm p}=\omega_{31}}$, where $n \approx 1 + 2\pi(\chi^{(1)})_{\omega_{\rm p}=\omega_{31}}$. The group index S is defined as the ratio of the light propagation time over the device length to that in the free space, which is a good quantity for describing the group velocity. It has the form $S = 1 + 2\pi\mathrm{Re}(\chi^{(1)})_{\omega_{\rm p}=\omega_{31}} + 2\pi\omega_{\rm p}\mathrm{Re}(\mathrm{d}\chi^{(1)}/\mathrm{d}\omega)_{\omega_{\rm p}=\omega_{31}}$ or

concretely

$$S = 1 + 4\pi\omega_{\rm p}\frac{\Gamma|\mu_{13}|^2}{V\varepsilon_0\hbar}|\Omega_{\rm c}|^2 \times \frac{2\gamma_{12}|\Omega_{\rm p}|^2 + 4\gamma_3\gamma_{12}^2 - \gamma_3|\Omega|^2}{\gamma_3|\Omega|^2(|\Omega|^2 + 2\gamma_3\gamma_{12})^2}. \quad (3)$$

It is clear from Eqs. (2) and (3) that the EIT and the group velocity of the SQC system can be adjusted by the pump field. The following numerical results are obtained based on the above two equations.

3. Results and discussion

Figures 2(a) and 2(b) show the real and the imaginary parts of the scaled linear susceptibility $\chi^{(1)}/[\Gamma|\mu_{13}|^2/(V\varepsilon_0\hbar)] \Rightarrow \chi^{(1)}$ as a function of the probe detuning $\Delta_{\rm p}$. When the pumping intensity is very weak, the artificial atomic dispersion in the region around the resonance $\Delta_{\rm p} = 0$ is normal (the solid curves in Fig. 2(a)). With the increase of the pumping intensity, the slopes of the refractive index curves rotate clockwisely around $\Delta_{\rm p} = 0$ (the dashed, dotted, and dash–dotted curves). Obviously, there is a critical value of the pumping intensity at which the dispersion changes from normal to anomalous. The inset of Fig. 2(a) provides an enlarged view of this variation process, where the critical value is $\Omega_{\rm c0} = 2.0$ under the current parametric condition (dotted curve in the inset) and can also be derived analytically from

$$\Omega_{\rm c0}^2 = \frac{1}{\gamma_3}[4\gamma_{12}^2\gamma_3 + (2\gamma_{12} - \gamma_3)\Omega_{\rm p}^2]. \quad (4)$$

On the other hand, the rotation of the refractive index curves around $\Delta_{\rm p} = 0$ will slow down in a further increase of the pumping intensity. If it can be strong enough, the propagation system will finally reach the anomalous state described by $\mathrm{Re}(\mathrm{d}\chi^{(1)}/\mathrm{d}\omega)_{\omega_{\rm p}=\omega_{31}} = -2\Gamma|\mu_{13}|^2/(V\varepsilon_0\hbar\Omega_{\rm c}^2)$. For the atomic absorption at $\Delta_{\rm p} = 0$, we can easily obtain

$$\mathrm{Im}[\rho_{13}]_{\omega_{\rm p}=\omega_{31}} = \frac{\Omega_{\rm p}|\Omega_{\rm c}|^2}{|\Omega_{\rm p}|^2 + |\Omega_{\rm c}|^2}\frac{2\gamma_{12}}{2\gamma_{12}\gamma_3 + |\Omega_{\rm p}|^2 + |\Omega_{\rm c}|^2}, \quad (5)$$

which means that the resonant absorption is inversely proportional to the pumping intensity. However, unlike the EIT in the real atomic system, in principle we cannot obtain the total transparency. Figure 2(b) shows the artificial atomic absorption spectrum, in which the absorption reduction appears at $\Delta_{\rm p} = 0$ and in the region around it.

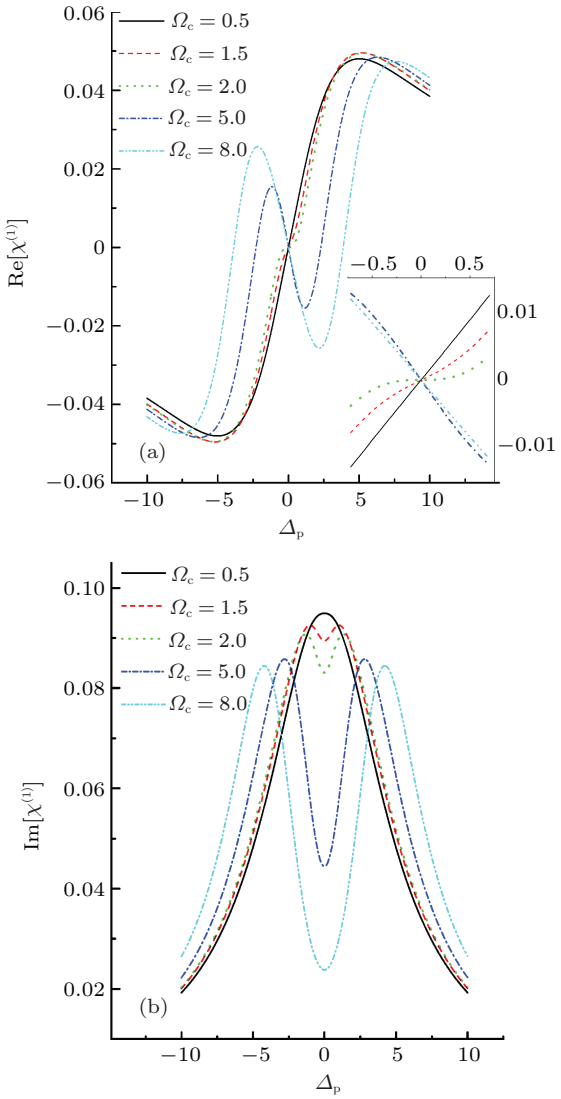


Fig. 2. (colour online) (a) Real and (b) imaginary parts of the scaled susceptibility $\chi^{(1)}$ as a function of the probe field detuning $\Delta_{\rm p}$ for different pumping intensities $\Omega_{\rm c}$ with $\Omega_{\rm p} = 0.1$, $\gamma_{12} = 1.0$, and $\gamma_3 = 10.0$.

Let us now consider the problem of controlling the group velocity by changing the pumping intensity. Obviously, $\Omega_{\rm c0}$ is also the critical point that divides the two propagation states of the probe field, the subluminal state as $\Omega_{\rm c} < \Omega_{\rm c0}$, and the superluminal state as $\Omega_{\rm c} > \Omega_{\rm c0}$. Because $\mathrm{Re}(\chi^{(1)})_{\omega_{\rm p}=\omega_{31}} = 0$, the subluminal or superluminal means the deviation of the group velocity from the light velocity in a vacuum. Furthermore, if the pumping field is strong enough, the probe field reaches the fixed group velocity described by

$$S = 1 - 4\pi\omega_{\rm p}\frac{\Gamma|\mu_{13}|^2}{V\varepsilon_0\hbar}\frac{1}{\Omega_{\rm c}^2}. \quad (6)$$

Figure 3 presents the scaled group index $(S - 1)/(4\pi\omega_{\rm p}\Gamma|\mu_{13}|^2/(V\varepsilon_0\hbar))$ of the probe field as a function of the pumping intensity. The figures show how

the scaled group index changes from positive to negative or how the propagation of the weak probe field varies from the subluminal to the superluminal as the pumping intensity increases. The pumping intensities for the maximum group index in the slow light region and the minimum group index in the fast light region under each set of parametric condition satisfy the cubic equation of $|\Omega_c|^2$, $|\Omega_c|^6+A|\Omega_c|^4+B|\Omega_c|^2+C=0$, with $A(\gamma_{12},\gamma_3,\Omega_p)$, $B(\gamma_{12},\gamma_3,\Omega_p)$, and $C(\gamma_{12},\gamma_3,\Omega_p)$ derived from Eq. (3). For example, the two pumping intensities are $\Omega_c = 0.42$ and 5.29 when $\gamma_{12} = 1.0$, $\gamma_3 = 10.0$, and $\Omega_p = 0.1$ (the dashed curves in Figs. 3(a) and 3(b)). Furthermore, increasing the decay rate γ_3 reduces the velocity-controlling effect of the pumping field (Fig. 3(a)), because the escape of the atom from the excited state $|3\rangle$ plays a negative role in building the linear polarization response, while increasing the dephasing rate γ_{12} of the atomic coherence is helpful to keep the velocity-controlling ability in the slow light region (Fig. 3(b)). The concrete value of the group velocity corresponding to each pumping intensity is determined by the scaling factor $4\pi\omega_p\Gamma|\mu_{13}|^2/(V\varepsilon_0\hbar)$. Compared to the light velocity control in the EIT medium composed of real atoms, our control is simple and is easy to realize with the current experimental technique.

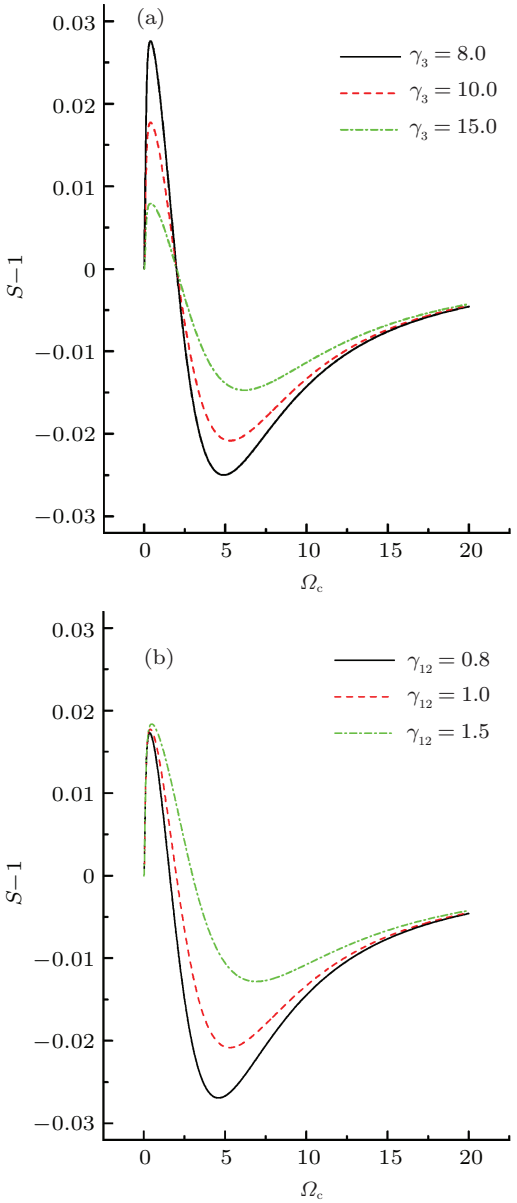


Fig. 3. (colour online) Scaled group index $S-1$ via pumping intensity Ω_c for (a) different decay rates γ_3 with fixed $\gamma_{12} = 1.0$ and (b) different dephasing rates γ_{12} with fixed $\gamma_3 = 10.0$. The other parameters are the same as those in Fig. 2.

4. Conclusion

In conclusion, we have investigated the controllable group velocity in a superconductive quantum circuit driven by microwaves. The system is modeled by a Λ-type three-level artificial atom with one of its two transitions coupling with a microwave probe field and the other coupling with a microwave pumping field. The results show that the EIT in such a microwave system can be easily controlled with the pumping field. For example, the absorption and the exchange between the subluminal and superluminal propagating states can be realized simply by sweeping the pumping intensity. All linear optical properties of this artificial atom come from the effective magnetic flux in the SQC system due to the current oscillation in the loop, which plays a role similar to that played by the atomic polarization induced by the dipole interaction of the atom with an electromagnetic wave. In the past years, we have seen many successful SQC experiments investigating the related effects known in real atomic systems, thus, from the point of view of technique, there are no major difficulties to realize the group velocity control in the SQC system. We expect that such an artificial atom with a controllable linear response will have applications in microwave-electronic devices, such as all-microwave switching, logic gates, and variable macroscopic optical buffers.

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