The effects of self-fields on the electron trajectory in a two-stream free electron laser with a helical wiggler and an axial guiding magnetic field*

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A theory for the two-stream free-electron laser (TSFEL) with a helical wiggler and an axial guide magnetic field is developed. In the analysis, the effects of self-fields are taken into account. An analysis of the two-stream steady-state electron trajectories is given by solving the equation of motion. Numerical calculations show that there are seven groups of orbits in the presence of self-fields instead of two groups reported in the absence of self-fields. The stability of the trajectories is studied numerically.

Keywords: two-stream free electron laser, electron trajectory

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1. Introduction

The free-electron laser (FEL) is a high-intensity, continuously tunable source of coherent electromagnetic radiation. The radiation is generated by the passage of a relativistic electron beam through a spatially periodic wiggler magnetic field.[1−6] A uniform axial magnetic field is often employed to collimate the relativistic electron beam in the transverse direction. It is known that the self-electric and self-magnetic fields generated by charge and current densities of the electron beam have significant effects on the operation of an FEL. Self-fields can produce chaos in FELs, particularly in the vicinity of gyro resonance.[7−10] The first-order self-magnetic field generated by the wiggler-induced transverse velocity has been calculated by Freund *et al.*[11] Recently, more exact calculations of self-fields and analysis of their effects on electron orbits have been published.[12,13]

McNeil *et al.*[14] studied a model of the FEL amplifier operating with two-electron beams of different energies. Bekefi and Jacobs[15] have shown that the amplitudes of the electromagnetic and space-charge waves are considerably affected by the two-stream instability due to the prebunching of electron beams. Also, the effects of two-stream instability on the growth rate of FELs with a helical wiggler have been investigated recently by Mehdian and Abasi.[16] Liu *et al.*[17] have investigated the effects of two-stream instability on the growth rate of an FEL with a helical wiggler and an axial magnetic field. Finally, the effects of self-fields on gain and electron trajectories in a free electron laser with an electromagnetic wave wiggler and axial magnetic field has been investigated.[18]

The purpose of the present paper is to investigate the effects of self-fields on electron trajectory in a two-stream free-electron laser with a helical wiggler and an axial guiding field. The rest of the paper is organized as follows. In Section 2, the analyses of single-particle trajectories of TSFEL in the presence of an axial guiding field and self-field are given. The numerical results and conclusions are given in Section 3.

2. Single particle orbits

The physical configuration consists of a two-stream relativistic electron beam propagating through a helical wiggler in the presence of the axial-guide magnetic field, $B = B_0\hat{e}_z$ and

$$B_{\rm w} = B_{\rm w}(\hat{e}_x \cos k_{\rm w} z + \hat{e}_y \sin k_{\rm w} z), \tag{1}$$

is the idealized helical wiggler magnetic field with amplitude $B_{\rm w}$ and wavelength $\lambda_{\rm w} = 2\pi/k_{\rm w}$.

The beam equilibrium self-electric and self-magnetic fields, respectively, are

$$\boldsymbol{E}_s = -2\pi \sum_{i=1}^{2} e n_{b_i}(x\hat{e}_x + y\hat{e}_y), \tag{2}$$

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$$B_{s_i} = B_{s\|_i} = 2\pi \sum_{i=1}^{2} e n_{b_i} \beta_{\|_i} (y\hat{e}_x - x\hat{e}_y). \quad (3)$$

The relativistic equations of motion for an electron with rest mass m and charge $-e$ moving with velocity v in the combined field are

$$\frac{\mathrm{d}(m\gamma_i V_i)}{\mathrm{d}t} = -e\Big[E_s + \frac{1}{c} V \times (B_{\mathrm{w}} + B_0 \hat{e}_z + B_{s\|_i})\Big]. \quad (4)$$

Using the normalized conditions $\tau = k_{\mathrm{w}} ct$ and $\boldsymbol{x} = k_{\mathrm{w}} \boldsymbol{\chi}$, where $\boldsymbol{\chi}$ represents the transverse displacement

$$\chi_{1_i} = k_{\mathrm{w}} [x \cos k_{\mathrm{w}} z + y \sin k_{\mathrm{w}} z], \quad (5)$$

$$\chi_{2_i} = k_{\mathrm{w}} [-x \sin k_{\mathrm{w}} z + y \cos k_{\mathrm{w}} z]. \quad (6)$$

With the aid of Eqs. (1)–(6) the normalized constant transverse velocity of each beam is given by

$$\beta_{w0_i} = \frac{\Omega_{w_i} \beta_{z0_i}^2}{\Omega_{0_i} \beta_{z0_i} + \Big(\sum\limits_{i=1}^{2} \omega_{b_i}^2 \beta_{z0_i}\Big) \beta_{z_i} - \sum\limits_{i=1}^{2} \omega_{b_i}^2 - \beta_{z0_i}^2}, \quad (7)$$

where

$$\Omega_{w_i} = \frac{eB_{\mathrm{w}}}{m\gamma_i k_{\mathrm{w}} c^2}, \quad \Omega_{0_i} = \frac{eB_0}{m\gamma_i k_{\mathrm{w}} c^2},$$

and

$$\omega_{b_i}^2 = \frac{2\pi e^2 n_{b_i}}{m\gamma_{0_i} k_{\mathrm{w}}^2 c^2}$$

denote the normalized wiggler frequency, normalized cyclotron frequency, normalized beam frequency respectively. Since energy is a constant for motion of the steady-state trajectory, we require that β_{w_i} and β_{z_i} are related via

$$\begin{aligned} \gamma_{0_i}^{-2} &= 1 - \beta_{x_i}^2 - \beta_{y_i}^2 - \beta_{z_i}^2 \\ &= 1 - \beta_{w_i}^2 - \beta_{z0_i}^2, \ i = 1, 2. \end{aligned} \quad (8)$$

For convenience, we shall work in the coordinate frame which rotates with the wiggler defined by the basis vectors:

$$\hat{e}_1 = \hat{e}_x \cos k_{\mathrm{w}} z + \hat{e}_y \sin k_{\mathrm{w}} z, \quad (9a)$$

$$\hat{e}_2 = -\hat{e}_x \sin k_{\mathrm{w}} z + \hat{e}_y \cos k_{\mathrm{w}} z, \quad (9b)$$

$$\hat{e}_3 = \hat{e}_z, \quad (9c)$$

$$\boldsymbol{v} = v_1 \hat{e}_1 + v_2 \hat{e}_2 + v_3 \hat{e}_3 = \boldsymbol{v}_\perp + \boldsymbol{v}_\|. \quad (9d)$$

In this coordinate frame, the substitution of the idealized wiggler model into the Lorentz force equation yields

$$\frac{\mathrm{d}\beta_{1_i}}{\mathrm{d}\tau} = A_{1i} \chi_{1_i} + B_{1i} \beta_{2_i}, \quad (10a)$$

$$\frac{\mathrm{d}\beta_{2_i}}{\mathrm{d}\tau} = A_{1i} \chi_{2_i} - B_{1i} \beta_{1_i} - \Omega_{w_i} \beta_{3_i}, \quad (10b)$$

$$\begin{aligned} \frac{\mathrm{d}\beta_{3_i}}{\mathrm{d}\tau} &= \Big(\sum_{i=1}^{2} \omega_{b_i}^2 \beta_{z0_i}\Big)(\beta_{1_i} \chi_{1_i} + \chi_{2_i} \beta_{2_i}) \\ &\quad + \Omega_{w_i} \beta_{2_i}. \end{aligned} \quad (10c)$$

The stability of orbits can be determined by small perturbations about the steady-state orbits. To investigate the orbital stability, we assume that

$$\chi_{1_i} = \delta\chi_{1_i}, \ \chi_{2_i} = \chi_{w0_i} + \delta\chi_{2_i}, \ \beta_{1_i} = \delta\beta_{1_i} + \beta_{w0_i},$$

$$\beta_{2_i} = \delta\beta_{2_i}, \ \beta_{3_i} = \delta\beta_{3_i} + \beta_{z0_i}, \ \chi_{w0_i} = -\frac{\beta_{w0_i}}{\beta_{z0_i}}. \quad (11)$$

The orbit equations for each of the streams are

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\delta\beta_{1_i} = A_{1_i}\delta\chi_{1_i} + B_{1_i}\delta\beta_{2_i}, \quad (12a)$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\delta\beta_{2_i} = A_{1_i}\delta\chi_{2_i} - B_{1_i}\delta\beta_{1_i} - C_{1_i}\delta\beta_{3_i}, \quad (12b)$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\delta\beta_{3_i} = A_{2_i}\delta\chi_{1_i} + B_{2_i}\delta\beta_{2_i}, \quad (12c)$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\delta\chi_{1_i} = \delta\beta_{1_i} + \beta_{3_i}\delta\chi_{2_i} + \chi_{w0_i}\delta\beta_{3_i}, \quad (12d)$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\delta\chi_{2_i} = \delta\beta_{2_i} - \beta_{3_i}\delta\chi_{1_i}, \quad (12e)$$

where

$$\begin{aligned} A_{1_i} &= \sum_{i=1}^{2} \omega_{b_i}^2 - \Big(\sum_{i=1}^{2} \omega_{b_i}^2 \beta_{z0_i}\Big)\beta_{z0_i}, \\ A_{2_i} &= \Big(\sum_{i=1}^{2} \omega_{b_i}^2 \beta_{z0_i}\Big)\beta_{w0_i}, \\ B_{1_i} &= \beta_{z0_i} - \Omega_{0_i}, \\ B_{2_i} &= \Omega_{w_i} + \chi_{w0_i}\Big(\sum_{i=1}^{2} \omega_{b_i}^2 \beta_{z0_i}\Big), \\ C_{1_i} &= \chi_{w0_i}\Big(\sum_{i=1}^{2} \omega_{b_i}^2 \beta_{z0_i}\Big) + \beta_{w0_i} + \Omega_{w_i}. \end{aligned} \quad (13)$$

Differentiating Eqs. (12b) and (12d) with respect to time and employing Eqs. (11)–(13), we obtain two homogeneous higher order differential equations as

$$\Big(\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} + \alpha_{1i}^2\Big)\delta\beta_{2_i} + M_{1i}^2 \delta\chi_{1_i} = 0, \quad (14a)$$

$$\Big(\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} + \alpha_{2i}^2\Big)\delta\chi_{1_i} + M_{2i}^2 \delta\beta_{2_i} = 0. \quad (14b)$$

Combining Eqs. (14a) and (14b) we obtain

$$\Big(\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} + \alpha_{1i}^2\Big)\Big(\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} + \alpha_{2i}^2\Big)\delta\beta_{2_i} - M_{1i}^2 M_{2i}^2 \delta\beta_{2_i} = 0, \quad (15)$$

where

$$\alpha_{1i}^2 = B_{2_i}^2 + B_{1_i}^2 - A_{1_i} + \Omega_{w_i}\beta_{w0_i} + A_{2_i}\chi_{w0_i}, \quad (16a)$$

$$\alpha_{2i}^2 = -A_{1_i} + \beta_{z0_i}^2 - A_{2_i}\chi_{w0_i}, \quad (16b)$$

$$M_{1i}^2 = 2A_{1_i}\beta_{z_i} - A_{1_i}\Omega_{0_i} + A_{2_i}C_{1_i}, \quad (16c)$$

$$M_{2i}^2 = -B_{1_i} - \beta_{z0_i} - B_{2_i}\chi_{w0_i}. \quad (16d)$$

Assume that all the variables oscillate with the same frequency ω, in the form of

$$\delta\beta_{2_i} = \beta_{2_i} \exp(\mathrm{i}\omega t), \quad (17)$$

then equation (15) will be

$$\bar{\omega}_{1,2} = \frac{(\alpha_{1i}^2 + \alpha_{2i}^2) \pm \sqrt{(\alpha_{1i}^2 + \alpha_{2i}^2)^2 - 4(\alpha_{1i}^2\alpha_{2i}^2 - M_{1i}^2 M_{2i}^2)}}{2}. \quad (18)$$

This equation has two roots for $\bar{\omega}$. The stability of the steady-state two-stream electron trajectory requires that these roots, $\bar{\omega}_1$ and $\bar{\omega}_2$, must be real and positive.

3. Numerical results and conclusions

A numerical study of the steady-state relativistic two-stream electron trajectories in a helical wiggler in the presence of self-fields and axial magnetic field is conducted. The typical dependence of the axial velocity on Ω_{01} is obtained by simultaneously solving Eqs. (7) and (8). The electron energy is taken to be $\gamma_1 = 4$ and $\gamma_2 = 4.2$, the normalized wiggler magnetic field and the normalized electron beam frequency are chosen to be $\Omega_{w1} = 0.05$, and $\omega_{b1} = 0.2$, respectively. The normalized axial velocities of two beams, β_{z1} (solid line) and β_{z2} (dashed line), in the absence of self-fields for using axial magnetic field are shown in Fig. 1. As seen in these curves the electron trajectories in the absence of self-field have two distinguishable orbits. Figures 2(a) and 2(b) show the curves of the normalized axial velocity of two-beam, β_{z1} and β_{z2}, each as a function of the normalized cyclotron frequency, Ω_{01} in the presence of self-fields. The electron trajectories appearing in the presence of self-field consist of seven groups rather than two groups shown in Figs. 1(a) and 1(b).

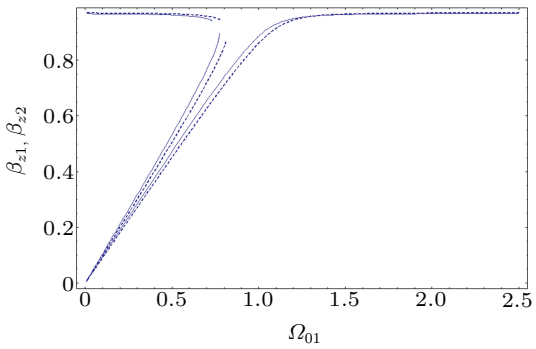


Fig. 1. (colour online) Curves of the normalized axial velocity, β_{z1} (solid line) and β_{z2} (dashed line), each as a function of normalized cyclotron frequency Ω_{01} in the presence of helical wiggler and axial guide field. The chosen parameters are as follows: $\gamma_1 = 4$ and $\gamma_2 = 4.2$, and $\Omega_{w1} = 0.05$.

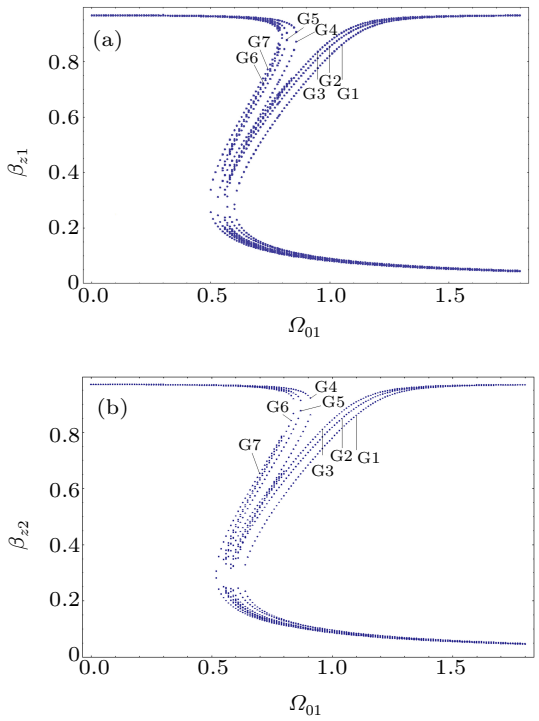


Fig. 2. (colour online) Curves of the normalized axial velocity (a) β_{z1} and (b) β_{z2} each as a function of normalized cyclotron frequency Ω_{01} in the presence of helical wiggler, axial guide field, and self-fields effects. The chosen parameters are as follows: $\gamma_1 = 4$, $\gamma_2 = 4.2$, $\Omega_{w1} = 0.05$, and $\omega_{b1} = 0.2$.

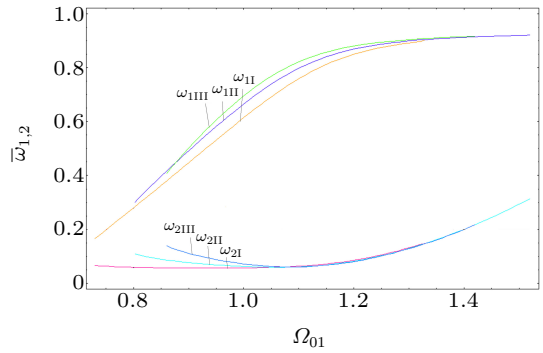


Fig. 3. (colour online) Curves of $\bar{\omega}_{1\text{I–III}}$ and $\bar{\omega}_{2\text{I-III}}$ versus Ω_{01} corresponding to the groups (I–III) orbits. The system parameters are chosen to be $\gamma_1 = 4$, $\gamma_2 = 4.2$, $\Omega_{w1} = 0.05$, and $\omega_{b1} = 0.2$.

To investigate the stability of the steady-state two-stream electron trajectory, we find the roots of Eq. (15), namely $\bar{\omega}_1$ and $\bar{\omega}_2$, through a numerical analysis. The roots of Eq. (18) ($\bar{\omega}_1$ and $\bar{\omega}_2$) each as a function of Ω_{01} are illustrated for groups (I–III) in Fig. 3. As seen in this figure in the given range of Ω_{01}, $\bar{\omega}_1$ and

$\bar{\omega}_2$ are real and positive. Therefore, groups (I–III) are stable, in the given range for Ω_{01}. The obtained results for groups (IV–VII) indicate that these groups are not completely stable. With the aid of Figs. 4(a) and 4(b), it can be seen that these groups are stable for the following regions: $\Omega_{01} < 0.86$ and $\beta_z > 0.908$ for group IV; $\Omega_{01} < 0.82$ and $\beta_z > 0.92$ for group V; $\Omega_{01} < 0.78$ and $\beta_z > 0.921$ for group VI; $\Omega_{01} < 0.78$ and $\beta_z > 0.922$ for group VII. In the other regions, groups (IV–VII) are unstable.

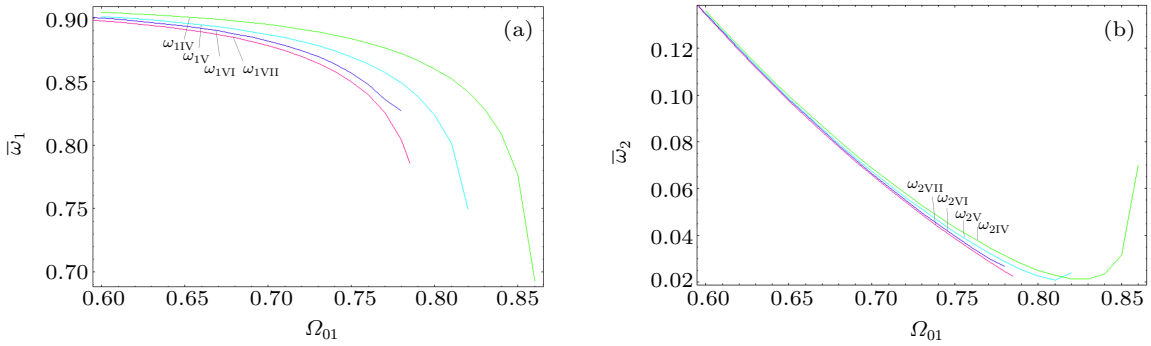


Fig. 4. (colour online) Curves of (a) $\bar{\omega}_{1\mathrm{IV–VII}}$ and (b) $\bar{\omega}_{2\mathrm{IV–VII}}$ versus Ω_{01} corresponding to the groups (IV–VII) orbits. The system parameters are chosen to be $\gamma_1 = 4$, $\gamma_2 = 4.2$, $\Omega_{w1} = 0.05$, and $\omega_{b1} = 0.2$.

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