Spatiotemporal chaos synchronization of an uncertain network based on sliding mode control*

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The sliding mode control method is used to study spatiotemporal chaos synchronization of an uncertain network. The method is extended from synchronization between two chaotic systems to the synchronization of complex network composed of N spatiotemporal chaotic systems. The sliding surface of the network and the control input are designed. Furthermore, the effectiveness of the method is analysed based on the stability theory. The Burgers equation with spatiotemporal chaos behavior is taken as an example to simulate the experiment. It is found that the synchronization performance of the network is very stable.

Keywords: spatiotemporal chaos synchronization, complex network, sliding mode control, Lyapunov theorem

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1. Introduction

Theoretical study of complex network synchronization is marked by the groundbreaking work of Winfree,[1] and a breakthrough has been made on the basis that small-world and scale-free network models were built by Watts,[2] Barabási and Albert,[3] and the characteristics of the network were studied as well. A complete synchronization method was proposed by Pecora and Carroll[4] in 1998 for the continuous-time coupling network, which attracted further attention to complex network synchronization. Together with the theoretical study of complex network synchronization, the unique advantage of network synchronization in many applied areas is also found, such as in information and communication, physics, computer, and the Internet, thus the study of complex network synchronization has a wider range of practical value. So far, many effective network synchronization schemes for random network, small-world network, scale-free network, and a variety of regular networks have been developed, and complete synchronization,[5−11] phase synchronization,[12−15] and generalized synchronization[16,17] of complex networks have been achieved.

The state equations of network nodes basically focused on continuous or discrete temporal chaos systems in the network synchronization reported. While in reality, the systems show chaos behaviors not only in time evolution but also in spatial evolution. Therefore, it is more practical to take spatiotemporal chaos systems as nodes of complex networks. Furthermore, the topological structure or parameters in the state equation of the network node are often uncertain in an actual network connection. Therefore, the network synchronization of uncertain spatiotemporal chaos would be more practical.

Sliding mode control method is widely used in the synchronization of two discrete or continuous chaotic systems because of its simple operation, robustness, and relatively mature synchronization technology. However, the synchronization of chaotic systems with multiple interrelated, that is, the synchronization of complex networks using sliding mode control method is less reported. Sliding mode control method is used to study spatiotemporal chaos synchronization of uncertain network in the paper. The method is extended from the synchronization between two chaotic systems to that of a complex network composed of N spatiotemporal chaotic systems first. Sliding surface of the network and the control input are further designed based on stability theory. The

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Burgers equation with spatiotemporal chaos behavior is taken for example to simulate experiment. It is found that the synchronization performance of the network is very stable.

2. Spatiotemporal chaos behavior of the Burgers equation

The Burgers equation is a universal equation with many practice physics processes, such as turbulence, fluid flow, and heat-transfer, which can show rich spatiotemporal chaotic behaviors. It is also used to describe many natural phenomena, such as the transport migration process of sediment and pollutant in rivers and lakes, the diffusion of the pollutant concentrations in the atmosphere and coastal temperature diffusion, which all obey the Burgers equation.

The form of one-dimensional Burgers equation is described as follows:[18]

$$\frac{\partial x(r,t)}{\partial t} = -k\frac{\partial x(r,t)}{\partial r} + \nabla^2 x(r,t), \tag{1}$$

where k is a system parameter and $x(r,t)$ is system variable, and $\nabla^2 = \partial^2/\partial r^2$.

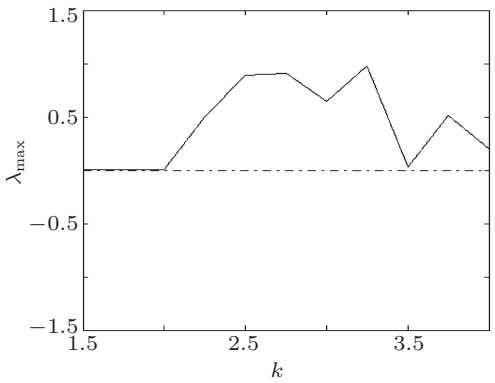


Fig. 1. Evolution of maximum Lyapunov exponent with parameter k.

For one-dimensional Burgers equation, the maximum Lyapunov exponent is calculated to determine the divergence and convergence of system trajectories, and the dynamical behavior of the system is determined in this way. The system size is taken as $L = 100$ in simulation, and periodic boundary conditions are taken as $x(r,t) = x(r+L,t)$. The evolution of the maximum Lyapunov exponent of the system with parameter k is shown in Fig. 1. It is seen that positive region of Lyapunov exponent exists, which shows that the system is in spatiotemporal chaos when the parameter is in the region. The parameter is taken as $k = 4$ when the Lyapunov exponent is positive, and the spatiotemporal evolution of the corresponding state variables is shown in Fig. 2.

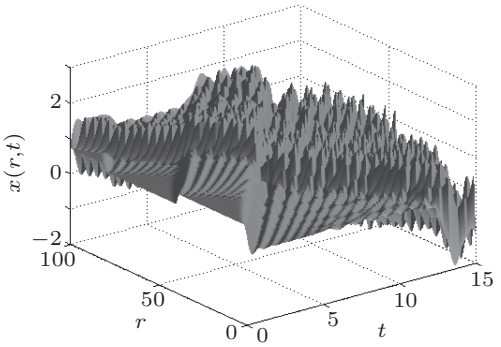


Fig. 2. Spatiotemporal evolution of state variable $x(r,t)$.

3. Synchronization mechanism for sliding mode control

Synchronization of complex network with N Burgers equations (1) as nodes is studied by the extended sliding mode control method. Thus, system (1) is written in the following general form of spatiotemporal chaotic system first:

$$\begin{aligned}\frac{\partial x(r,t)}{\partial t} &= F(x(r,t)) \\ &= f(x(r,t)) + kg(x(r,t). \end{aligned} \tag{2}$$

Obviously, there is $f(x(r,t)) = \nabla^2 x(r,t)$ and $g(x(r,t)) = -\partial x(r,t)/\partial r$, parameter k is assumed to be uncertain here.

The N spatiotemporal equations (2) are taken as nodes to construct a complex network, where the state equation of node i can be described as

$$\begin{aligned}\frac{\partial x_i(r,t)}{\partial t} &= f(x_i(r,t)) + kg(x_i(r,t)) \\ &\quad + \varepsilon\sum_{j=1}^{N} c_{ij}x_j(r,t) + u_i \quad (i = 1,2,\ldots,N), \end{aligned} \tag{3}$$

where ε is the coupling strength between the nodes of the network; c_{ij} is the matrix element of the coupling matrix c, whose specific form depends on the connection type of the network, and it is used to express the topological structure of the network; u_i is the control input of the network.

Errors between the state variables of spatiotemporal chaotic systems at nodes are defined as

$$e_i(r,t) = x_{i+1}(r,t) - x_i(r,t) \quad (i = 1,2,\ldots,N-1). \tag{4}$$

Error evolution equation can be further obtained as follows:

$$\begin{aligned}\frac{\partial e_i(r,t)}{\partial t} &= \Delta f(x_{i+1},x_i)+k\Delta g(x_{i+1},x_i)\\ &\quad+\varepsilon\sum_{j=1}^{N}c_{i+1j}x_j(r,t)\\ &\quad-\varepsilon\sum_{j=1}^{N}c_{ij}x_j(r,t)+u_{i+1}-u_i,\end{aligned}\tag{5}$$

where $\Delta f(x_{i+1},x_i)=f(x_{i+1}(r,t))-f(x_i(r,t))$, and $\Delta g(x_{i+1},x_i)=g(x_{i+1}(r,t))-g(x_i(r,t))$.

Two basic steps are required to achieve the synchronization of two chaotic systems by sliding mode control method. One is to choose an appropriate sliding surface, ensuring that it is asymptotically stable. The second is to design a controller or control input, and make the controlled system tend towards the sliding surface.

The sliding mode control mechanism mentioned above is extended to the study of complex network synchronization. Therefore, for N spatiotemporal chaotic systems at the nodes of the network, a sliding surface is constructed by the two adjacent ones, and then $N-1$ sliding surfaces can be described as

$$S_i=e_i(r,t)+\varphi_i,\quad (i=1,2,\ldots,N-1),\tag{6}$$

where φ_i is the function to be determined whose variables are the errors between state variables of the spatiotemporal chaotic systems at the nodes. Assume that the adaptive law of φ_i is given as

$$\frac{\partial\varphi_i}{\partial t}=m_ie_i(r,t),\tag{7}$$

where m_i is a real number above zero.

According to the sliding mode control theory, $\partial S_i/\partial t=0$ must be met when the sliding surface moves, which is equivalent to the following equation when Eqs. (6) and (7) are taken into account

$$\frac{\partial e_i(r,t)}{\partial t}=-m_ie_i(r,t).\tag{8}$$

According to the above equation, when $t\to\infty$, then $e_i(r,t)\to 0$. The sliding surface designed is asymptotically stable.

In order to ensure the form of the control input of the network, the Lyapunov function is constructed as follows:

$$V=\frac{1}{2}\sum_{i=1}^{N-1}S_i^2+\frac{1}{2}(\hat{k}-k)^2,\tag{9}$$

where $\hat{k}$ is the identification of the uncertain parameter k. The derivative form of V can be further obtained as

$$\begin{aligned}\frac{\partial V}{\partial t} &= \sum_{i=1}^{N-1}S_i\frac{\partial S_i}{\partial t}+(\hat{k}-k)\frac{\partial\hat{k}}{\partial t}\\ &= \sum_{i=1}^{N-1}S_i[\Delta f(x_{i+1},x_i)+k\Delta g(x_{i+1},x_i)\\ &\quad+\varepsilon\sum_{j=1}^{N}c_{i+1j}x_j(r,t)-\varepsilon\sum_{j=1}^{N}c_{ij}x_j(r,t)+u_{i+1}\\ &\quad-u_i+m_ie_i(r,t)]+(\hat{k}-k)\frac{\partial\hat{k}}{\partial t}.\end{aligned}\tag{10}$$

If the parameter identification law is designed into the following form

$$\frac{\partial\hat{k}}{\partial t}=\sum_{i=1}^{N-1}S_i\Delta g(x_{i+1},x_i),\tag{11}$$

then, equation (10) can be simplified into

$$\begin{aligned}\frac{\partial V}{\partial t} &= \sum_{i=1}^{N-1}S_i[\Delta f(x_{i+1},x_i)+\hat{k}\Delta g(x_{i+1},x_i)\\ &\quad+\varepsilon\sum_{j=1}^{N}c_{i+1j}x_j(r,t)-\varepsilon\sum_{j=1}^{N}c_{ij}x_j(r,t)\\ &\quad+u_{i+1}-u_i+m_ie_i(r,t)].\end{aligned}\tag{12}$$

Take the control input as

$$\begin{aligned}u_{i+1} &= u_i-\eta\,\mathrm{sign}(S_i)-\Delta f(x_{i+1},x_i)\\ &\quad-\hat{k}\Delta g(x_{i+1},x_i)-\varepsilon\sum_{j=1}^{N}c_{i+1j}x_j(r,t)\\ &\quad+\varepsilon\sum_{j=1}^{N}c_{ij}x_j(r,t)-m_ie_i(r,t),\quad(\eta>0),\end{aligned}\tag{13}$$

then the following equation will be obtained

$$\frac{\partial V}{\partial t}=-\eta\sum_{i=1}^{N-1}|S_i|<0.\tag{14}$$

According to Lapunov theorem, global synchronization between nodes of the network is realized on the basis of Eq. (14).

4. Simulation results and discussion

The Burgers equation is taken as nodes of the network to test the effectiveness of the synchronization principle. $N=5$ is taken in simulation. The complex network is constructed by state equation (2), whose

single node is connected in the way of Eq. (3). The coupling matrix c in the coupling function which is used to connect each node can represent an arbitrary topology network, and a chain and a ring structure of network are taken for example in simulation.

When synchronization of a chain structure of network simulation is made the coupling matrix is as follows:

$$c_l = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \qquad (15)$$

and $u_1 = 0$ is taken in simulation, that is, the first node of the network is the target system, which means it is the synchronization state. The remaining control input u_i $(i = 2, 3, 4, 5)$ are chosen to satisfy Eq. (13). Four sliding surfaces are constructed based on Eqs. (6) and (7). The coupling strength between the network nodes is arbitrarily taken as $\varepsilon = 1$, and the parameters are $\eta = 0.01$, $m_1 = m_2 = m_3 = m_4 = 1.5$, the coupling connection is made at any time, such as at the 5th second, the spatiotemporal evolutions of error variables of Burgers equations at the nodes of the chain network are shown in Figs. 3–6.

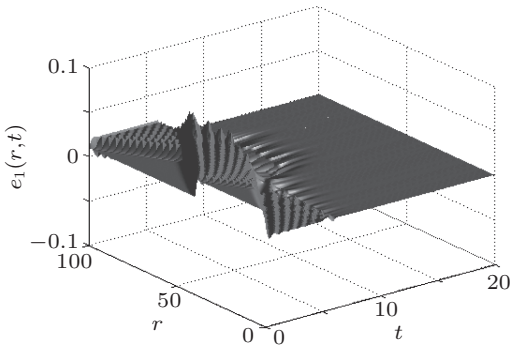


Fig. 3. Error spatiotemporal evolution of the chain network at nodes 1 and 2.

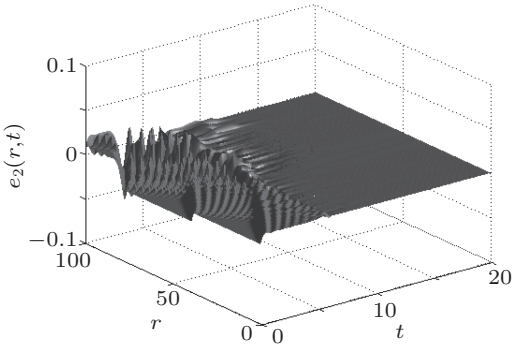


Fig. 4. Error spatiotemporal evolution of the chain network at nodes 2 and 3.

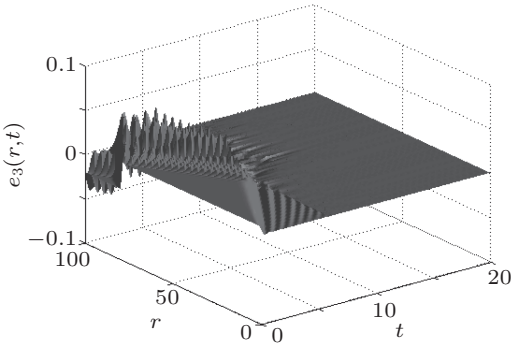


Fig. 5. Error spatiotemporal evolution of the chain network at nodes 3 and 4.

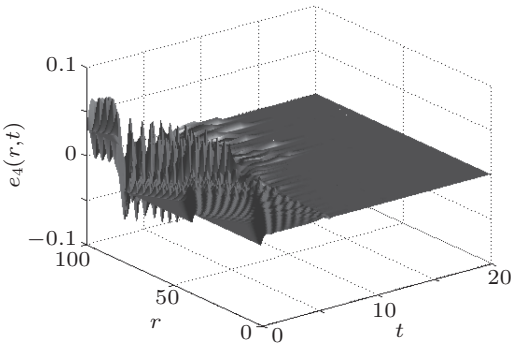


Fig. 6. Error spatiotemporal evolution of the chain network at nodes 4 and 5.

It is shown in Figs. 3–6 that after the coupling connection of the network, all the error signals $e_i(r,t)$ $(i = 1, 2, 3, 4)$ of the Burgers equation at the nodes approach to zero after a short series of time, which means complete synchronization of the network is realized.

When ring network synchronization is simulated, the coupling matrix is as follows:

$$c_r = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}. \qquad (16)$$

Keeping the same principle when the control input and the sliding surface are selected, all parameters of the network remain unchanged. The spatiotemporal evolutions of the error variables of Burgers equations at the nodes of the ring network are shown in Figs. 7–10. It is found that the synchronization performance of the network is still very stable.

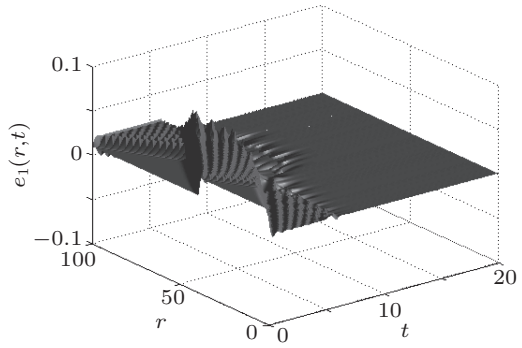


Fig. 7. Error spatiotemporal evolution of the ring network at nodes 1 and 2.

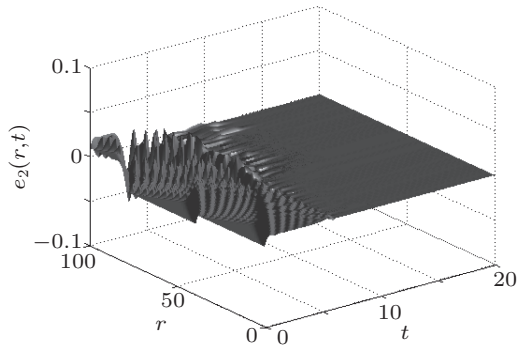


Fig. 8. Error spatiotemporal evolution of the ring network at nodes 2 and 3.

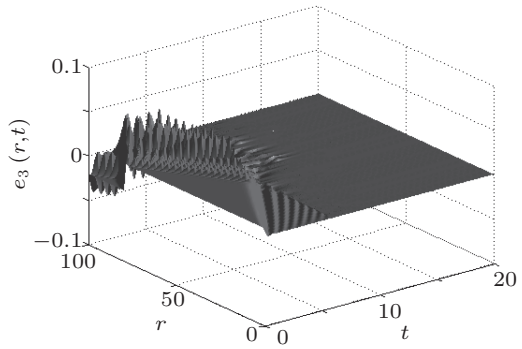


Fig. 9. Error spatiotemporal evolution of the ring network at nodes 3 and 4.

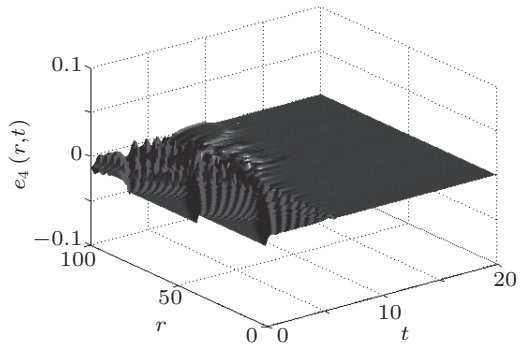


Fig. 10. Error spatiotemporal evolution of the ring network at nodes 4 and 5.

When parameters η and m_i are given other values, the synchronization performance of the network is still very stable. However the speed of the network synchronization is influenced by the values of parameters η and m_i. Furthermore, when the topological structure of the network is changed and the number of nodes is given other values, the synchronization stability of the entire network is not influenced. The simulation course is not repeated here.

5. Conclusion

The sliding mode control method is used to study spatiotemporal chaos synchronization of an uncertain network. The method is extended from synchronization between two chaotic systems to the synchronization of complex network constituted N spatiotemporal chaotic systems. Sliding surface of the network and the control input are designed. Furthermore, the effectiveness of the method is analysed based on the stability theory. The Burgers equation with spatiotemporal chaos behavior is taken for example to simulate experiment. It is shown in simulation that the speed of the network synchronization is influenced by the values of parameters η and m_i, while the stability of the entire network is influenced by neither the topological structure of the network nor the number of the nodes N.

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