Fractional-order permanent magnet synchronous motor and its adaptive chaotic control*

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In this paper we investigate the chaotic behaviors of the fractional-order permanent magnet synchronous motor (PMSM). The necessary condition for the existence of chaos in the fractional-order PMSM is deduced. And an adaptive-feedback controller is developed based on the stability theory for fractional systems. The presented control scheme, which contains only one single state variable, is simple and flexible, and it is suitable both for design and for implementation in practice. Simulation is carried out to verify that the obtained scheme is efficient and robust against external interference for controlling the fractional-order PMSM system.

Keywords: fractional-order, permanent magnet synchronous motor, adaptive chaotic control

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1. Introduction

The investigation of chaos in permanent magnet synchronous motor (PMSM) is a field of active research due to its direct applications in many areas especially for industrial applications in low-medium power range, since it has excellent features such as simple structure, high torque-to-inertia ratio, high torque-to-weight ratio, and low manufacturing cost.[1] However, the performance of the PMSM is sensitive to system parameter and external load disturbance in the plant. Some investigations, for example, by Li *et al.*[2] and Jing *et al.*[3] show that with certain parameter values, the PMSM displays chaotic behavior. The chaos in the PMSM is highly undesirable and can result in intermittent ripples of torque, low-performance property speed control of motor, low-frequency oscillations of current, and even induce the motors collapse. Thus, it is indispensable to control or suppress chaotic oscillations in motor system. Up till now, several control strategies have been proposed for the chaotic control of PMSM.[4−8]

In the recent years, the research on fractional-order dynamical systems has been receiving increasing attention. It is found that with the help of fractional derivatives, many systems in interdisciplinary fields can be elegantly described.[9−12] Furthermore Lorenz, Rossler, Chen, and Lü chaotic systems of fractional order have been studied widely.[13−16] In fact, all the physical phenomena in nature exist in the form of fractional order,[17] integer order (classical) differential equation is just a special case of fractional differential equation. The importance of fractional-order models is that they can yield a more accurate description and give a deeper insight into the physical processes underlying a long range memory behavior. This, of course, is closer to the real world. An important challenge in the fractional-order chaos theory is the control. There are a few effective schemes for achieving the stabilization of fractional-order chaotic systems to steady states or regular behaviors, such as state feedback control,[18] sliding mode control.[19] It should be noted that most of these contributions are investigated through numerical simulations that are based on the stability criteria of the linear fractional-order dynamics systems or the fractional Routh–Hurwitz conditions, and all these controllers are nonlinear and complicated. Therefore, it is of practical significance to study the fractional-order PMSM and to seek for a

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simple but efficacious method for its chaotic control.

In this paper, we concentrate on the fractional version of the PMSM for the first time. The necessary condition (minimum effective dimension) of the fractional-order PMSM to keep chaotic is investigated. A Lyapunov stability theory of fractional-order dynamic systems is introduced, and an adaptive-feedback control method is proposed for controlling the fractional-order PMSM to its equilibrium. The proposed controller, which contains only one single state variable, to our knowledge, is the simplest control scheme for controlling fractional-order chaotic system. What is more, the control scheme is flexible, and is suitable both for design and for implementation in practice. Finally, simulation is carried out to verify that the obtained scheme is efficient and robust against external interference for controlling the fractional-order PMSM system.

2. Fractional-order PMSM

2.1. Model of PMSM

The equivalent circuit of the PMSM in nominal operating conditions is displayed in Fig. 1, while the parameters are summarized in Table 1. The dimensionless mathematical model of PMSM is given in Refs. [2], [3], and [20]

$$\begin{cases} \dfrac{\mathrm{d}i_d}{\mathrm{d}t} = -i_d + \omega i_q + u_d, \\ \dfrac{\mathrm{d}i_q}{\mathrm{d}t} = -i_q - \omega i_d + \gamma\omega + u_q, \\ \dfrac{\mathrm{d}\omega}{\mathrm{d}t} = \sigma(i_q - \omega) - T_L, \end{cases} \tag{1}$$

where i_d and i_q are the stator currents; ω is the rotor angular frequency; $u_d = K_T L U_d/bR^2 + K_T^2/bR + \gamma$ and $u_q = K_T L U_q/bR^2$ are the stator voltages, with $L = L_d = L_q$; T_L is the external load torque; $\sigma = bL/JR$ and γ are the operating parameters. In this paper we take the case $u_d = u_q = T_L = 0$ which was considered in the study of bifurcations as well.[2]

Applying the equilibrium condition to Eq. (1), it is determined that three equilibria exist. The previous results of investigation show that with system parameters σ and γ falling into a certain area, all the three equilibria become unstable, and the PMSM exhibits chaos.[2,3] The bifurcation diagram for $\gamma \in [40,\ 120]$ with $\sigma = 4$ and the typical chaotic attractor with $\sigma = 4$, $\gamma = 50$ are shown in Fig. 2.

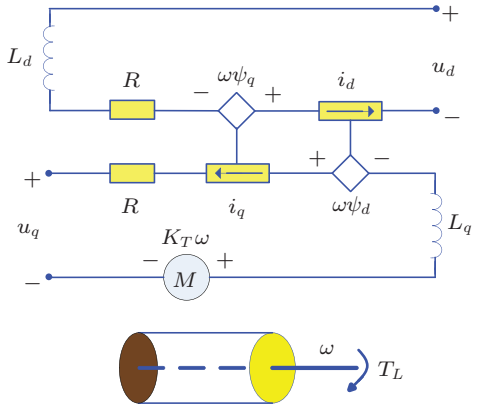


Fig. 1. (colour online) Equivalent circuit of the PMSM.

Table 1. The PMSM parameters.

Parameter	Nomenclature	Unit
L_d	stator inductance in the q axis	H
L_q	stator inductance in the d axis	H
K_T	torque constant	N·m/A
J	moment of inertia	Kg·m^2
b	viscous damping coefficient	N/(rad/s)
R	stator phase resistance	Ω
u_d	d-axis stator voltage	V
u_q	q-axis stator voltage	V
T_L	load torque	V
ψ_d	d-axis flux	Weber
ψ_q	q-axis flux	Weber

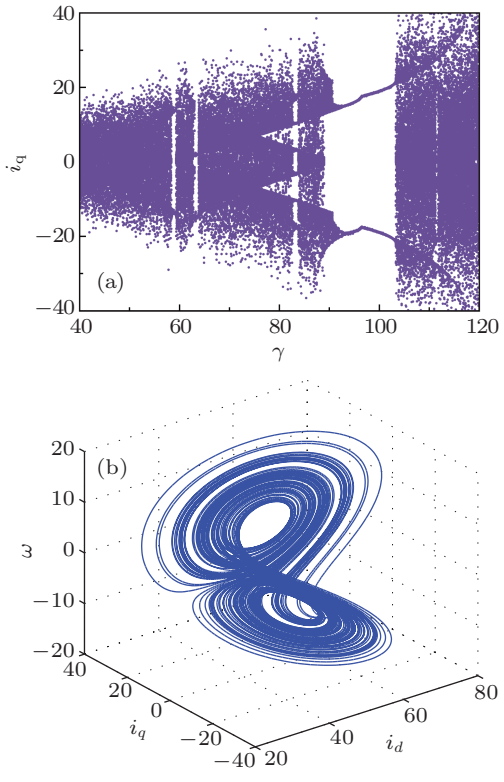


Fig. 2. (colour online) Bifurcation diagram and chaotic attractor of system (1): (a) bifurcation diagram; (b) chaotic attractor.

2.2. Fractional-order PMSM

Now, we consider the fractional-order PMSM system as follows:

$$\begin{cases} \dfrac{\mathrm{d}^{\alpha_1} i_d}{\mathrm{d}t^{\alpha_1}} = -i_d + \omega i_q, \\ \dfrac{\mathrm{d}^{\alpha_2} i_q}{\mathrm{d}t^{\alpha_2}} = -i_q - \omega i_d + \gamma\omega, \\ \dfrac{\mathrm{d}^{\alpha_3} \omega}{\mathrm{d}t^{\alpha_3}} = \sigma(i_q - \omega), \end{cases} \tag{2}$$

where $\alpha_i \in (0,\ 1]$ $(i = 1,\ 2,\ 3)$ is the fractional order.

One can easily obtain three equilibria of system (2) as

$$S_1 = (0, 0, 0), S_{2,3} = (\gamma - 1, \pm\sqrt{\gamma - 1}, \pm\sqrt{\gamma - 1}). \tag{3}$$

And the corresponding Jacobian matrix of system (2) is depicted as

$$J = \begin{bmatrix} -1 & \omega_* & i_{q^*} \\ -\omega_* & -1 & \gamma - i_{d^*} \\ 0 & \sigma & -\sigma \end{bmatrix}, \tag{4}$$

where $(i_{d^*},\ i_{q^*},\ \omega_*)$ denote the equilibrium point of system (2).

In order to study the chaotic behavior of system (2), we should consider the stability of the equilibrium points to obtain the necessary condition of chaos occurrence. So we introduce an indispensable theorem in the first instance.

Theorem 1[21] For the following autonomous system:

$$\frac{\mathrm{d}^{\alpha} X}{\mathrm{d}t^{\alpha}} = AX, \quad X(0) = X_0, \tag{5}$$

where $0 < \alpha < 1$, $X \in R^m$, and $A \in R^{m\times m}$, if and only if $|\arg(eig(A))| > \alpha\pi/2$, the system will be asymptotically stable. In this case, each component of the states approaches to 0 with $t^{-\alpha}$ increasing. Also, this system is stable if and only if $|\arg(eig(A))| \geq \alpha\pi/2$ and the critical eigenvalues satisfying $|\arg(eig(A))| = \alpha\pi/2$ have geometric multiplicity 1.

With the aid of Theorem 1, we can analyse the stability of system (2) at its equilibrium points. Let $\lambda*$ be the eigenvalue of the Jacobian matrix (4) at a saddle point of index 2. Therefore, the necessary condition for instability region of these saddle points depends on

$$\alpha > \frac{2}{\pi} a\tan\left(\frac{|\mathrm{Im}(\lambda*)|}{\mathrm{Re}(\lambda*)}\right). \tag{6}$$

When system parameters $\sigma = 4$ and $\gamma = 50$, the three equilibria of system (2) are $S_1 = (0, 0, 0)$, $S_2 = (49, 7, 7)$, $S_3 = (49, -7, -7)$. And the eigenvalues corresponding to different equilibria are

$$\begin{aligned} &S_1 : \lambda_1 = -16.7215, \quad \lambda_2 = 11.7215, \quad \lambda_3 = -1; \\ &S_{2,3} : \lambda_1 = 0.3443 + 7.6477\mathrm{i}, \\ &\qquad \lambda_2 = 0.3443 - 7.6477\mathrm{i}, \quad \lambda_3 = -6.6887. \end{aligned}$$

Therefore, when $\sigma = 4$, $\gamma = 50$, $\alpha > 0.9714$ is the necessary condition for the existence of chaos in the fractional-order PMSM system, and the minimum effective dimension for the commensurate fractional-order PMSM system to keep chaotic is 2.9142.

For numerical simulations, we set $\sigma = 4$, $\gamma = 50$, and $\alpha_1 = 0.98$, $\alpha_2 = 1$, $\alpha_3 = 0.99$ to ensure chaotic motion. The initial conditions for the fractional-order PMSM system are selected to be $i_d = 2.5$, $i_q = 3$, and $\omega = 1$. The three-dimensional (3D) phase diagram is shown in Fig. 3 which reveals chaotic dynamics.

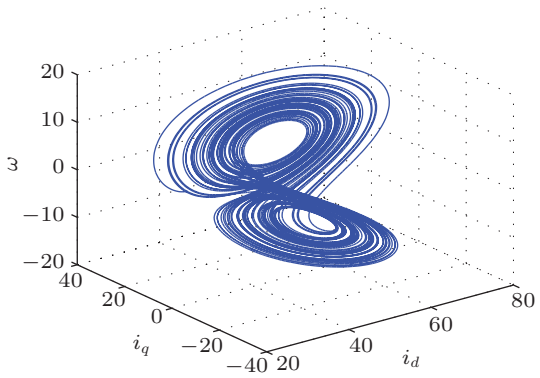


Fig. 3. (colour online) Chaotic attractor of fractional-order PMSM system.

3. Adaptive chaotic control for fractional-order PMSM

In this section, before proposing an adaptive-feedback control method for fractional-order PMSM, we first introduce a stability theory of fractional-order dynamic system.

3.1. Stability theory of fractional-order dynamic system

Theorem 2 Consider the following fractional-order autonomous system:

$$\frac{\mathrm{d}^{\alpha} X}{\mathrm{d}t^{\alpha}} = F(X) = A(X)X, \quad \alpha \in (0,\ 1], \tag{7}$$

where $X \in R^m$ is the state variable and $A(X) \in R^{m\times m}$ is the coefficient matrix. If there exists $M^{\mathrm{T}} = M > 0$ such that

$$\Lambda = X^{\mathrm{T}} M \frac{\mathrm{d}^{\alpha} X}{\mathrm{d}t^{\alpha}} \leq 0, \tag{8}$$

then system (7) is asymptotically stable.

Proof When $\alpha = 1$, this is the case of stability for integer-order autonomous system, the conclusion is obvious.

When $0 < \alpha < 1$, let $\lambda*$ be one of eigenvalues of the matrix $A(X)$, $\varsigma \in R^m$ is the corresponding nonzero eigenvector. Then we have

$$A(X)\varsigma = \lambda * \varsigma \tag{9}$$

and

$$\varsigma^H A(X)^H = \overline{\lambda*}\varsigma^H. \tag{10}$$

Further, we obtain

$$\varsigma^H M A(X)\varsigma = \lambda * \varsigma^H M\varsigma, \tag{11}$$

$$\varsigma^H A(X)^H M\varsigma = \overline{\lambda*}\varsigma^H M\varsigma. \tag{12}$$

From Eqs. (11) and (12), we gain

$$\varsigma^H (MA(X) + A(X)^H M)\varsigma = (\lambda * + \overline{\lambda*})\varsigma^H M\varsigma. \tag{13}$$

Since $\Lambda = X^{\mathrm{T}} MA(X)X = 0.5X^{\mathrm{T}}(MA(X) + A(X)^H M)X \leq 0$, we have

$$\lambda * + \overline{\lambda*} \leq 0. \tag{14}$$

Therefore, condition $|\arg(\lambda*)| \geq \pi/2 > \alpha\pi/2$ is satisfied, according to Theorem 1, system (7) is asymptotically stable.

3.2. Adaptive chaotic control for the fractional-order PMSM

The controlled fractional-order dynamic system is considered here

$$\frac{\mathrm{d}^{\alpha} X}{\mathrm{d}t^{\alpha}} = F(X) + u_i, \quad \alpha \in (0,\ 1]. \tag{15}$$

The adaptive-feedback control scheme is presented as

$$u_i = -\rho(x_i - x_{i*}), \tag{16}$$

$$\frac{\mathrm{d}^{\alpha_j}\rho}{\mathrm{d}t^{\alpha_j}} = \eta(x_i - x_{i*})^2, \tag{17}$$

where $X = (x_1, x_2, \ldots, x_m)^{\mathrm{T}}$ is the state variable; $F(X) = (f_1(X), f_2(X), \ldots, f_m(X))^{\mathrm{T}}$; u_i is the controller acting on the i-th term of Eq. (15); $\eta > 0$; $0 < \alpha_j \leq 1$; $X_* = (x_{1*}, x_{2*}, \ldots, x_{m*})$ is the equilibrium of the uncontrolled system.

Assumption 1 Function $f_i(X)$ is smooth in the neighbourhood of X_*, and there is a positive constant l such that

$$\begin{aligned} \|f_i(X) - f_i(X_*)\| &= \|f_i(X)\| \leq l\,\|x_i - x_{i^*}\| \\ &\leq l\,\|X - X_*\|_{\infty}, \end{aligned} \tag{18}$$

where $\|X - X_*\|_{\infty}$ is the ∞-norm of $X - X_*$, i.e., $\|X - X_*\|_{\infty} = \max_j \|x_j - x_{*j}\|$, $j = 1, 2, \ldots, m$.

For systems (15) and (17), we introduce a candidate function

$$\Lambda = (X - X_*)^{\mathrm{T}} \frac{\mathrm{d}^{\alpha}(X - X_*)}{\mathrm{d}t^{\alpha}} + \frac{(\rho - \rho_0)}{\eta} \frac{\mathrm{d}^{\alpha_j}(\rho - \rho_0)}{\mathrm{d}t^{\alpha_j}}, \tag{19}$$

where $\rho_0 \geq ml$.

Considering the Lipachitz condition, we obtain

$$\begin{aligned} \Lambda &= (X - X_*)^{\mathrm{T}} \frac{\mathrm{d}^{\alpha}(X - X_*)}{\mathrm{d}t^{\alpha}} + (\rho - \rho_0)(x_i - x_{i*})^2 \\ &= (X - X_*)^{\mathrm{T}}[F(X) - \rho(x_i - x_{i*})] \\ &\quad + (\rho - \rho_0)(x_i - x_{i*})^2 \\ &= (X - X_*)^{\mathrm{T}} F(X) - \rho(x_i - x_{i*})^2 \\ &\quad + (\rho - \rho_0)(x_i - x_{i*})^2 \\ &= (X - X_*)^{\mathrm{T}} F(X) - \rho_0(x_i - x_{i*})^2 \\ &\leq (ml - \rho_0)\,\|X - X_*\|_{\infty}^2 \\ &\leq 0. \end{aligned}$$

So, according to Theorem 2, systems (15) and (17) are asymptotically stable, that is, X converges to X_* and ρ converges to ρ_0 as $t \to \infty$.

Remark 1 The proposed scheme has the simplest expression for the controller and the adaptive updated algorithm. And this scheme is flexible which can be added to any term of the system equation. What is more, α_j can be any value taken from 0 to 1.

3.3. Numerical simulation

In this section, we numerically verify the proposed adaptive control scheme. We add the controller to the second term of Eq. (2), the controlled fractional-order PMSM is depicted as

$$\begin{cases} \dfrac{\mathrm{d}^{\alpha_1} i_d}{\mathrm{d}t^{\alpha_1}} = -i_d + \omega i_q, \\ \dfrac{\mathrm{d}^{\alpha_2} i_q}{\mathrm{d}t^{\alpha_2}} = -i_q - \omega i_d + \gamma\omega - \rho(i_q - i_{q^*}), \\ \dfrac{\mathrm{d}^{\alpha_3}\omega}{\mathrm{d}t^{\alpha_3}} = \sigma(i_q - \omega), \\ \dfrac{\mathrm{d}^{\alpha_j}\rho}{\mathrm{d}t^{\alpha_j}} = \eta(i_q - i_{q^*})^2. \end{cases} \tag{20}$$

In the numerical simulation, we set $\sigma = 4$, $\gamma = 50$, $\eta = 2$, and $\alpha_1 = 0.98$, $\alpha_2 = 1$, $\alpha_3 = 0.99$, $\alpha_j = 1$; the initial conditions for the fractional-order PMSM system are selected as $i_d = 2.5$, $i_q = 3$, $\omega = 1$; i_{q^*} is set to be 0, 7, −7, respectively; the control signal is put into effect at 20 s. Figures 4–6 show the control results, from which one can see clearly that the state of fractional-order PMSM reaches the desired equilibrium point.

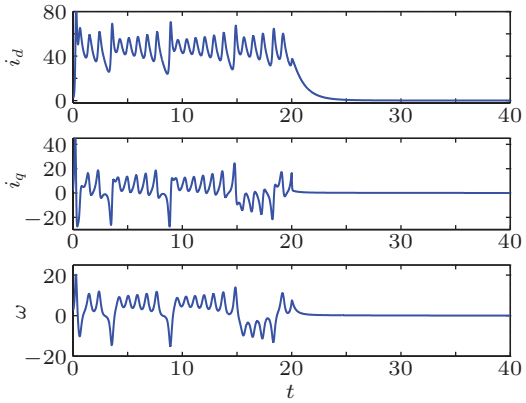


Fig. 4. (colour online) Time responses for the states of Eq. (20) stabilizing the equilibrium point S_1.

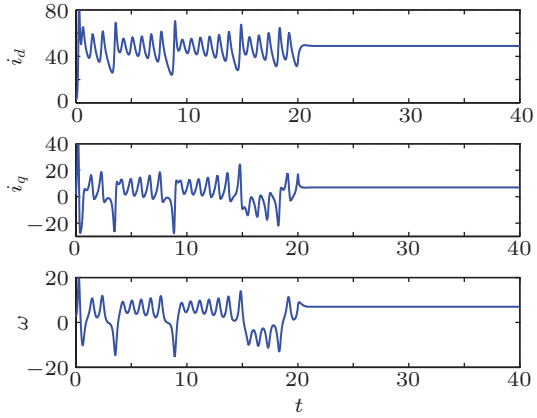


Fig. 5. (colour online) Time responses for the states of Eq. (20) stabilizing the equilibrium point S_2.

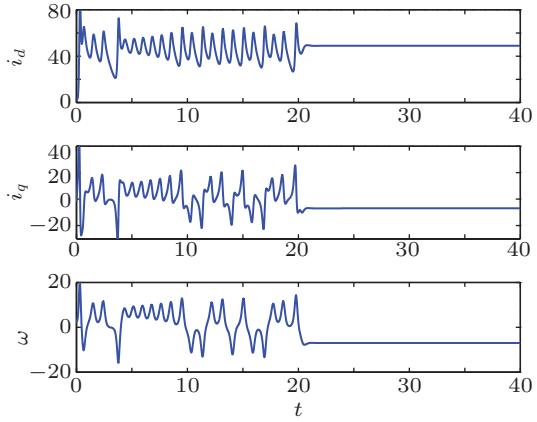


Fig. 6. (colour online) Time responses for the states of Eq. (20) stabilizing the equilibrium point S_3.

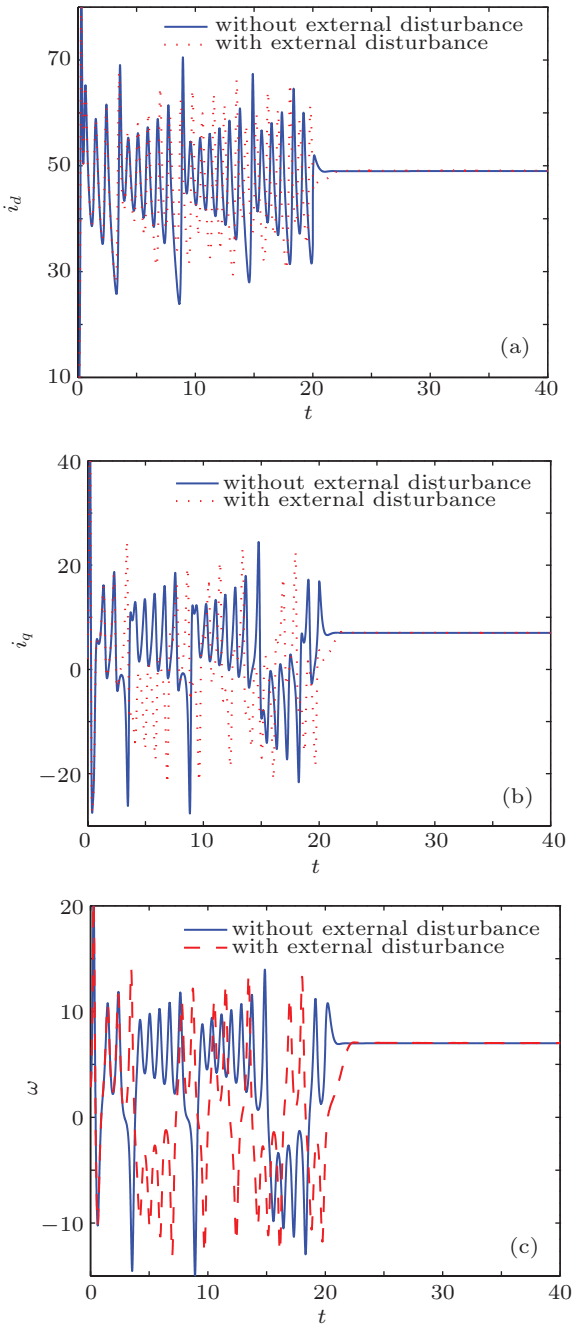


Fig. 7. (colour online) Time responses for the states of Eq. (21) in the presence of external disturbance.

Next, assume that the fractional-order PMSM is perturbed by the rand disturbances, and add the controller to the first term of Eq. (2), then we will have

$$\begin{cases} \dfrac{\mathrm{d}^{\alpha_1} i_d}{\mathrm{d}t^{\alpha_1}} = -i_d + \omega i_q + d_1 - \rho(i_d - i_{d^*}), \\ \dfrac{\mathrm{d}^{\alpha_2} i_q}{\mathrm{d}t^{\alpha_2}} = -i_q - \omega i_d + \gamma\omega + d_2, \\ \dfrac{\mathrm{d}^{\alpha_3} \omega}{\mathrm{d}t^{\alpha_3}} = \sigma(i_q - \omega) + d_3, \\ \dfrac{\mathrm{d}^{\alpha_j} \rho}{\mathrm{d}t^{\alpha_j}} = \eta(i_d - i_{d^*})^2, \end{cases} \tag{21}$$

where d_i is the rand disturbance satisfying $|d_1| \leq 0.5$, $|d_2| \leq 1$, and $|d_3| \leq 0.3$.

In the numerical simulation, we set $\sigma = 4$, $\gamma = 50$, $\eta = 2$, and $\alpha_1 = 0.98$, $\alpha_2 = 1$, $\alpha_3 = 0.99$; $i_{d^*} = 7$; but α_j is set to be 0.5; the control is active when $t \geq 20$ s. The time response of the states of controlled system (21) is illustrated in Fig. 7. The simulation results show that the proposed method is robust against random disturbance.

4. Conclusion

In this paper, we investigated the chaotic behaviors of the incommensurate fractional-order PMSM for the first time. The necessary condition of the fractional-order PMSM to keep chaotic is deduced. And an adaptive-feedback controller is developed based on the stability theory for fractional systems. The presented control scheme, which contains only one single state variable, is simple, flexible, and it is suitable both for design and for implementation in practice. Simulation results are presented to illustrate the effectiveness and robust of the proposed method.

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